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GENERATION OF ELASTIC WAVES IN A PIEZOELECTRIC PLATE BY INTERDIGITAL TRANSDUCERS

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Piezoelectric plate with periodic metal strips on both sides is considered. Propagation and Bragg scattering of plate modes and associated made conversion are analyzed. Excitation of acoustic plate modes by interdigital transducers is investigated and results are compared with experimental data.

1. Introduction

Excitation of surface acoustic waves (SAWs) in the piezoelectric halfspace by interdigital transducers (IDTs) was analysed in details in numerous papers [1–4]. In recent years, there is growing interest in applying of acoustic plate modes (APMs) instead of SAW in piezoelectric sensor and filters [5–8]. Theory of APM generation by IDT can be considered more difficult than that of Rayleigh waves [9]. It is caused mainly by

• multimodal propagation of APM, a number of different modes can propagate at the same frequency with different velocities,

• Bragg reflection of APM from transducers fingers can be accompanied with modes conversion,

• generation of APMs by a pair of IDts deposited on both surfaces of piezoelectric plate is unique problem for plates.

In this paper, a theory of generation of APM by such pair of IDTs is devoloped, using Bløtekjaer's method of analysis of waves propagating in the periodic system of metal strips [10].

In next section, we analyze electric properties of piezoelectric plate, with electric charge applied to its both surfaces. The immitance relation is derived which is the planar Green's function for piezelectric plate in spectral domain [9, 12]. The relation is a generalization of effective surafce permittivity, introduced in [12], to the case of piezoelectric plate.

In the following section, propagation of APM in the piezoelectric plate with electrodes deposited on both sides of plate is analyzed. Dispersion relations for most important cases, open and short-circuited strips on both sides of plate are discussed. Numerical results are presented for plates made of some known piezoelectric materials (quartz and $LiNbO_3$)

In Section 3, a theory of APM excitation by IDTs deposited on both sides of plate is presented. The corresponding inhomogenous problem is solved using method proposed in [2, 3, 10]. Numerical results are compared with experimental data presented in [13].

2. Immitance relations

Let us consider an infinite piezoelectric plate bounded by planes $x_2 = \pm d/2$ (Fig.1) and made of material characterized by mass density ρ and material constants ε_{ij}, e_{qij}, c_{ijpq}. Vacuum (ε_0) is outside the plate. We consider harmonic waves propagating along x_3, that is $\exp(j\omega t - jkx_3)$, where k is wavenumber and ω is angular frequency.

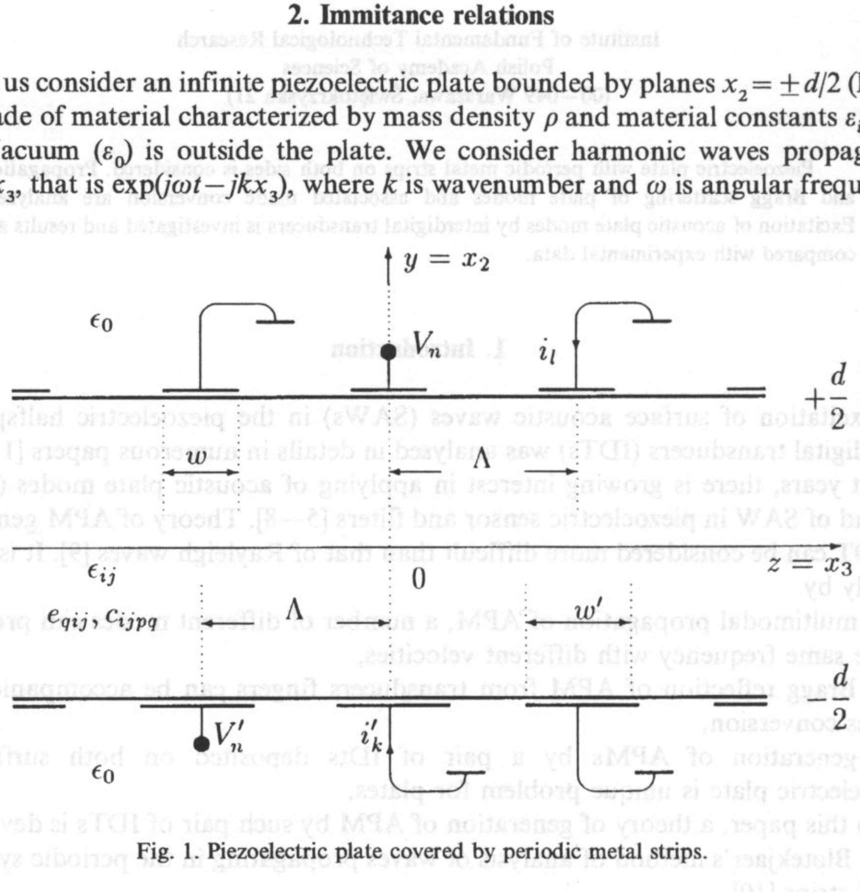

Fig. 1. Piezoelectric plate covered by periodic metal strips.

In the piezoelectric material, the coupled acoustic wave equations are

$$
\begin{aligned}
-\rho\omega^2 u_i &= c_{ijpq} u_{p,jq} + e_{qij} \varphi_{,ji} \\
0 &= e_{jpq} u_{p,jq} - \varepsilon_{jq} \varphi_{,jq},
\end{aligned}
\tag{2.1}
$$

where u_n is particle displacement component and φ is electric potential, both inside the plate. Electric potential outside the plate (in vacuum, $D_i'' = -\varepsilon_0 \varphi_{,i}''$) satisfy Laplace equation

$$D_{i,i}'' = 0. \tag{2.2}$$

Certain electric and mechanical conditions must be satisfied on both surfaces

$$T_{2i} = 0 \quad at \quad x_2 = \pm d/2, \tag{2.3}$$

and

$$\begin{aligned} \varphi = \varphi'', \quad D_2 - D_2'' = \Delta D_\perp, \text{ at } x_2 = d/2, \\ \varphi = \varphi'', \quad D_2 - D_2'' = \Delta D_\perp', \text{ at } x_2 = -d/2, \end{aligned} \tag{2.4}$$

where $\Delta D_\perp$ and $\Delta D_\perp'$ are electric charges induced on uper and bottom surfaces of the plate, respectivelly. Solving the corresponding boundary problem in the way presented in {14], we obtain a set of immitance relations for piezoelectric plate. This set is a generalization to effective electric surface permittivity introduced in [12] that involves that electric charges, and electric fields at both sides of the plate $E_\| = jk\varphi(x_2 = d/2)$, and $E_\|' = jk\varphi(x_2 = -d/2)$

$$\begin{aligned} E_\| &= jS_k X \Delta D_\perp + jS_k X' \Delta D_\perp' \text{ (for } x_2 = d/2), \\ E_\|' &= -jS_k X' \Delta D_\perp - jS_k X \Delta D_\perp' \text{ (for } x_2 = -d/2), \end{aligned} \tag{2.5}$$

where X and X' are functions of k, and

$$S_k = \begin{cases} 1 & \text{for } k \geqslant 0, \\ -1 & \text{for } k < 0. \end{cases} \tag{2.6}$$

In the above equations, we accounted for the symmetry relations {14, 15] (rotation of a plate by 180⁰ does not change its equations, but note that $\Delta D_\perp$ and $\Delta D_\perp'$ include vector components differently oriented with respect to the plate). Generally, the matrix elements of immitance relations (2.5) which can also be considered as a surface Green's matrix function in spectral domain k, can be evaluated only numerically. The matrix elements are singular at k being the wave numbers of plate modes.

An asymptotic behaviour of $X(|k|)$ and $X'(|k)$ for $|k| \to \infty$ are following

$$X \to X_\infty, \; X' \to 0, \tag{2.7}$$

which shows that the system of Eqs. (2.5) separates at large $|k|$. This is because of fast decaying of the wave-field in depth of the plate if the applied electric charge to the plate has large $|k|$.

3. A plate covered by periodic electrodes

3.1. Eigenvalue boundary problem

We consider infinite piezoelectric plate of thickness d. The plate surfaces are covered by periodic systems of weightless, ideally conducting metal strips (Fig. 1) which period Λ is the same on both plate sides but the electrode widths can be different, w and w' at $x_2=y=d/2$ and $-d/2$, correspondingly (in what follows, all quantities at $y=-d/2$ will be marked by 'prime'). The considered problem is 2-dimensional, waves in the system are assumed propagating in $z=x_3$ direction perpendicular to strips.

There are mixed electric boundary conditions on both surfaces

$$E_{\parallel}=0,\quad E'_{\parallel}=0,\quad \text{on electrodes,}$$

$$\Delta D_{\perp}=0,\quad \Delta D'_{\perp}=0,\quad \text{between electodes,} \tag{3.1}$$

where $E_{\parallel}$ and $\Delta D_{\perp}$ are defined as in previous Section.

Accordingly to the Floquet theorem [17], a solution to the eigenvalue boundary problem stated by Eqs. (2.5), (3.1) is searched in form ($K=2\pi/\Lambda$ is the wave-number of periodic strips)

$$E_{\parallel}=\sum_{n=-\infty}^{\infty} E_n e^{-j(s+nK)z},$$

$$\Delta D_{\perp}=\sum_{n=-\infty}^{\infty} D_n e^{-j(s+nK)z}, \tag{3.2}$$

at the upper plate surface, and similarly at the bottom surface, where $E_{\parallel}$, E_n and $\Delta D_{\perp}$, D_n should be replaced by corresponding 'primed' quantities. The time dependence $\exp(j\omega t)$ is dropped thoughout the paper. There is certain ambiguity concerning spectral parameter s, in what follows we will assume its value in the domain $(0,K)$.

Taking into account Eqs. (2.5) we obtain following relations for amplitudes of Bloch waves included in the above solution

$$E_n=jS_nX_nD_n+jS_nX'_nD'_n,$$

$$E'_n=-jS_nX'_nD_n+jS_nX_nD'_n, \tag{3.3}$$

where $X_n=X(s+nK)$, and $X'_n=X'(s+nK)$, similarly $S_n=S_{s+nK}$.

Asymptotic properties of X and X' (Eqs. (7)) allow us to find such integer numbers N_1 i N_2, that Eqs. (3.3) become separated if $n\notin[N_1,N_2]$

$$E_n=jS_nX_\infty D_n,\quad E'_n=-jS_nX_\infty D'_n. \tag{3.4}$$

This will be exploited below in expanding the Bloch amplitudes into another series which, according to the method presented in [10], make the solution (3.2) to satisfy the boundary conditions (3.1). The expansion is following

$$E_n=\sum_{m=M_1}^{M_2}\alpha_m S_{n-m}P_{n-m}(\cos\Delta),$$
$$D_n=\sum_{m=M_1}^{M_2}\beta_m P_{n-m}(\cos\Delta), \tag{3.5}$$

and similarly for 'primed' amplitudes in which relations α' and b$'$ substitute α and β, and $\Delta'=\pi\omega'/\Lambda$ substitutes $\Delta=\pi\omega/\Lambda$ in corresponding arguments of Legendre polynomials P_k. Taking into account Eqs. (3.4), we obtain that

$$\alpha_m=jX_\infty\beta_m,\quad \alpha'_m=-jX_\infty\beta'_m. \tag{3.6}$$

Following the method [10], we apply sufficiently large summation limits in expansions (3.5), $M_1=M_3=N_1$, and $M_2=M_4=N_2+1$. The solution given in Eqs. (3.2), (3.5) satisfies the boundary conditions (3.1) and Eqs. (3.3), but only at $n\notin[N_1, N_2]$ so that we must still consider Eqs. (3.3) which are explicitly

$$\alpha_m(S_{n-m}-S_nZ_n)P_{n-m}(\cos\Delta)-\alpha'_mS_nZ'_nP_{n-m}(\cos\Delta')=0,$$
$$-\alpha_mS_nZ'_nP_{n-m}(\cos\Delta)+\alpha'_m(S_{n-m}-S_nZ_n)P_{n-m}(\cos\Delta')=0, \tag{3.7}$$

at $n\in[N_1,N_2]$, in order to satisfy Eqs. (3.3) for any $n\in(-\infty,\infty)$. The above set of equations, where $Z_n=X_n/X_\infty$ and $Z'_n=X'_n/X_\infty$ including $2N$ linear equations for $2N+2$ unknowns α_m, α'_m, $N=N_2-N_1+1$, can be solved for any given α_0 and α'_0

$$\alpha_m=a_m\alpha_0+b_m\alpha'_0,\quad \alpha'_m=a'_m\alpha_0+b'_m\alpha'_0, \tag{3.8}$$

where $a_0=1$, $b_0=0$ and $a'_0=0$, $b'_0=1$. The efficients a_m, b_m, a'_m, b'_m are evaluated numerically from Eq. (3.7).

Integrating electric field $E_\parallel$ and $E'_\parallel$, represented by Eqs. (3.2), (3.5), over the domain between strips, we obtain relations for electric potential of electrodes on the upper ($\hat{V}(s)$), and bottom ($\hat{V}'(s)$) sides of the plate at $z=0$

$$\hat{V}(s)=\frac{-j\pi}{K\sin\pi s/K}(\alpha_0A_{11}+\alpha'_0A_{12}),$$
$$\hat{V}'(s)=\frac{-j\pi}{K\sin\pi s/K}(\alpha_0A_{21}+\alpha'_0A_{22}). \tag{3.9}$$

Analogously, integrating electric charge over strips placed on the plate surfaces at $z=0$, we obtain currents flowing to strips on upper ($\hat{I}(s)$) and bottom ($\hat{I}'(s)$) sides

$$\hat{I}(s)=2\pi\frac{\omega}{KX_\infty}(\alpha_0 B_{11}+\alpha_0' B_{12}),$$
$$\hat{I}'(s)=-2\pi\frac{\omega}{KX_\infty}(\alpha_0 B_{21}+\alpha_0' B_{22}), \tag{3.10}$$

where A_{ij} and B_{ij} are defined as follows ($\nu=s/K$, summation sumbols over m dropped to shorten notations)

$$\begin{aligned}
A_{11}&=(-1)^m a_m P_{m+\nu-1}(-\cos\Delta), & B_{11}&=a_m P_{m+\nu-1}(\cos\Delta),\\
A_{12}&=(-1)^m b_m P_{m+\nu-1}(-\cos\Delta), & B_{12}&=b_m P_{m+\nu-1}(\cos\Delta),\\
A_{21}&=(-1)^m a_m' P_{m+\nu-1}(-\cos\Delta'), & B_{21}&=a_m' P_{m+\nu-1}(\cos\Delta'),\\
A_{22}&=(-1)^m b_m' P_{m+\nu-1}(-\cos\Delta'), & B_{22}&=b_m' P_{m+\nu-1}(\cos\Delta'),
\end{aligned} \tag{3.11}$$

3.2. Dispersion relations

Equations (3.9), (3.10) are sufficient for analysis of propagation of APM in piezoelectric plate covered by strips on both sides. Generally, there are four possibilities

- all electrodes connected to ground, $\hat{V}=0$ and $\hat{V}'=0$,
- open electrodes on both surfaces, $\hat{I}=0$ and $\hat{I}'=0$,
- short-circuited strips on one side, and open strips on the other side of the plate, $\hat{V}=0$ and $\hat{I}'=0$,
- and vice-versa, $\hat{I}=0$ and $\hat{V}'=0$.

Corresponding dispersive relations resulting from Eqs. (3.9), (3.10) are

$$\begin{aligned}
A_{11}A_{22}-A_{12}A_{21}&=0,\\
B_{11}B_{22}-B_{12}B_{21}&=0,\\
B_{11}A_{22}-B_{12}A_{21}&=0,\\
A_{11}B_{22}-A_{12}B_{21}&=0,
\end{aligned} \tag{3.12}$$

which should be solved for s at given ω. Generally, it can be done only numerically. The most interesting feature of the solution for s is the existence, at certain frequency domain called a stopband, of complex s. The imaginary value of s makes the wave-field decaying along its propagation path. The reason of this decaying, which is generally faster for stronger piezoelectrics, is the Bragg reflection of APMs from strips which bring periodic electric perturbation into the elastic waveguide. Similar phenomenon, but caused by mechanical perturbation of plate by shallow grooves, was discussed in [16], for instance.

4. Excitation of APM by a pair of IDTs

4.1. Imhomogeneous problem for metal strips

In Eqs. (3.9) and (3.10), there are two arbitrary constants, α_0 and α_0' which are, in fact, functions of spectral variable s. Evaluation of these functions is the subject of inhomogeneous problem considered below.

In the considered inhomogeneous problem (Fig. 1), two electrodes, one on upper and the second on the bottom side of plate, have given voltages V_n and V_m' correspondingly, and the others are grounded. We will evaluate the transadmittance relations for strips,

$$\begin{aligned} i_l &= y^u_{lm} V_m + v^u_{lm} V'_m, \\ i'_l &= v^b_{lm} V_m + y^b_{lm} V'_m, \end{aligned} \tag{4.1}$$

which describe signal transmission between strips by both means of electric interaction [11], and APMs. The evaluation of transadmittance will be carried out on the way similar to that applied in [2–4] for Rayleigh waves.

The given strip voltages V_n and V_m' are following inverse Fourier transforms defined for discrete functions over periodic strips

$$V_n = \frac{1}{K}\int_0^K \hat{V}(s) e^{-jns\Lambda}\, ds, \quad V'_m = \frac{1}{K}\int_0^K \hat{V}'(s) e^{-jms\Lambda}\, ds, \tag{4.2}$$

where $\hat{V}(s)$ and $\hat{V}'(s)$ are as given in Eqs. (3.9). To satisfy the above relations, we must apply that

$$\begin{aligned} V_n e^{jsn\Lambda} &= \frac{-j\pi}{K \sin \pi s/K}(\alpha_0(s) A_{11} + \alpha_0'(s) A_{12}), \\ V'_m e^{jsm\Lambda} &= \frac{-j\pi}{K \sin \pi s/K}(\alpha_0(s) A_{21} + \alpha_0'(s) A_{22}), \end{aligned} \tag{4.3}$$

which can be solved for unknown $\alpha_0(s)$ and $\alpha_0'(s)$

$$\begin{aligned} \alpha_0(s) &= j\frac{K}{\pi} \sin \pi v \frac{V_m A_{22} - V'_m A_{12}}{A_{11}A_{22} - A_{12}A_{21}} e^{jsm\Lambda}, \\ \alpha_0'(s) &= j\frac{K}{\pi} \sin \pi v \frac{V'_m A_{11} - V_m A_{21}}{A_{11}A_{22} - A_{12}A_{21}} e^{jsm\Lambda}. \end{aligned} \tag{4.4}$$

The currents I flowing to upper electrodes, and I' flowing to bottom ones can be evaluated by applying similar inverse Fourier transforms to Eqs. (3.10)

$$I_l=\frac{1}{K}\int_0^K \hat{I}(s)e^{-jls\Lambda}\,ds,\quad I_l'=\frac{1}{K}\int_0^K \hat{I}'(s)e^{-jls\Lambda}\,ds, \tag{4.5}$$

which, applying solutions (4.4) yield ($v_{lm}=v_{lm}^u=-v_{lm}^b$ on the principle of virtual works)

$$y_{lm}^u=\frac{j2\omega}{KX_\infty}\int_0^K \frac{B_{11}A_{22}-B_{12}A_{12}}{A_{11}A_{22}-A_{12}A_{21}}\sin\pi v e^{-js(l-m)\Lambda}\,ds,$$

$$v_{lm}=\frac{j2\omega}{KX_\infty}\int_0^K \frac{B_{12}A_{11}-B_{11}A_{12}}{A_{11}A_{22}-A_{12}A_{21}}\sin\pi v e^{-js(l-m)\Lambda}\,ds=$$

$$\frac{j2\omega}{KX_\infty}\int_0^K \frac{B_{21}A_{22}-B_{22}A_{12}}{A_{11}A_{22}-A_{12}A_{21}}\sin\pi v e^{-js(l-m)\Lambda}\,ds, \tag{4.6}$$

$$y_{lm}^b=-\frac{j2\omega}{KX_\infty}\int_0^K \frac{B_{22}A_{11}-B_{21}A_{12}}{A_{11}A_{22}-A_{12}A_{21}}\sin\pi v e^{-js(l-m)\Lambda}\,ds$$

4.2. Radiation admittances

Integrals in Eqs. (4.6) have following general form

$$Y_{lm}=\frac{j2\omega}{KX_\infty}\int_0^K R(s)\sin\pi s/K e^{-js(l-m)\Lambda}ds \tag{4.7}$$

where function $R(s)$ which is different for different y_{lm}^x, but in all cases the denominator is the same in Eqs. (4.6), as singular at single poles for s being the solutions of dispersion equation for short-circuited strips. $R(s)$, and Y can be decomposed as follows

$$R(s)=R^c(s)+R^r(s),\quad Y_{lm}=Y_{lm}^c+Y_{lm}^r, \tag{4.8}$$

where $R^c(s)=R(s)-R^r(s)$ is assumed regular function of s, thus the corresponding integral for Y^c can be easily evaluated numerically. It describes mutual capacitance of electrodes l and m, placed on the same or different sides of the plate [11].

The function $R^r(s)$ that includes all singularities, is defined as follows

$$R^r(s)=\sum_i \frac{b_i}{s-s_i}+\sum_i \frac{\bar{b}_i}{s-K+s_i}, \tag{4.9}$$

where we accounted for that both s_i and $K-s_i$ are solutions to dispersion equations in the considered system, s_i-K being the wave-number of APM propagating backward. Corresponding integrals can be evaluated approximately by expanding the integration path to infinity on the complex plane s, and thus applying Jordan's lemma and residual theorem (see [3], for instance). We obtain

$$Y^r_{ll}=j2\frac{\omega\Lambda}{X_\infty}\sum_i b_i e^{-js_i\Lambda/2},$$

$$Y^r_{lm}=2\frac{\omega\Lambda}{x_\infty}\sum_i b_i e^{-js_i\,|\,l-m\,|\,\Lambda}\sin\pi s_i/K, \quad \text{for} \quad l\neq m. \tag{4.10}$$

This is similar equation to that presented in [2, 3] for Rayleigh waves. The main difference is in the number of propagating modes generated in plate which contribute to the strip radiation admittance.

5. Some numerical and experimental results

Typical interdigital transducers are composed of a number of metal strips connected to transducers bar-buses [1], which buses are connected to external voltage sources, in generating IDT, or to loading impedance, in receiving IDT. In piezoelectric plate covered by strips on both its sides, there is interesting possibility of APM excitation by a pair of transducers having their fingers on different sides of the plate.

In [13], an experiment is described where a pair of IDTs were placed face-to face on two sides of YX quartz, 64 μm thick plate. Both IDTs had 40 pairs of split Al fingers (strip period $\Lambda=40$ μm and $w=w'=20$ μm). There are measurements presented for IDTs connected in parallel and antiparallel, which means that corresponding strips on two sides of plate had the same, or oposite electric potentials. The measurements have not been interpreted as concern waves excited by transducers in the measured frequency band (10–100 MHz). This will be provided below, by comparison with numerical results. Let us note that the discussed plate is relatively thick as compared to the strip period, its normalized thickness is $Kd=10.035$, thus several modes can be observed in the measured frequency band. The numerical calculations presented here will include A_0, SH_0, S_0 and SH_1 modes only.

We introduce notations A, B for IDT bus-bars on the upper side, and A', B' — for corresponding bus-bars of IDT on the bottom surface of the plate. The voltages of these bus-bars will be noted V_A, V_B, $V_{A'}=V'_A$, $V_{B'}=V'_B$, and similarly for currents. Eqs. (4.6) results in following relations for the discussed IDTs

$$I_A=W\sum_{m=1}^{80}\sum_{l=1}^{80} w_m w_l y^u_{lm} V_A + w_l(1-w_m) y^u_{lm} V_B +$$

$$w_m w_l v^u_{lm} V'_A + w_l(1-w_m) v^u_{lm} V'_B\,, \tag{5.1}$$

$$I_B=W\sum_{m=1}^{80}\sum_{l=1}^{80}(1-w_l)w_m y^u_{lm} V_A + (1-w_l)(1-w_m) y^u_{lm} V_B +$$

$$(1-w_l)w_m v_{lm}^u V_A' + (1-w_m)(1-w_l) v_{lm}^u V_B',$$

and similarly for I_A' and I_B', with w_k defined as follows

$$w_k = \begin{cases} 1 & \text{if electrode } k \text{ is connected to } A \text{ or } A' \text{ bus-bars,} \\ 0 & \text{elsewhere, and } W \text{ is IDT aperture width,} \end{cases} \tag{5.2}$$

In the analyzed configurations, we have

- in symmetric configuration, $I=((I_A+I_A')-(I_B+I_B'))/2$, and $V_A=V_B'=V/2$, $V_B=V_B'=-V/2$,
- in antisymmetric configuration, $I=((I_A+I_B')-(I_B+I_B'))/2$, and $V_A=V_B'=V/2$, $V_B=V_A'=-V/2$,

and the measured admitance of transducer pairs is

$$Y=I/V \tag{5.3}$$

Its values are computed in following frequency bands: $13-16$ MHz, $32-34$ MHz, and $54-58$ MHz, and presented in Figs. 2, 3.

In conclusion, we recognize that the measured radiation conductances result from excitation of following APMs

- Lamb A_0 mode for $f \approx 14.5$ MHz, in antisymmetric configuration,
- transvers SH_0 mode for $f \approx 33.4$ MHz, in symmetric configuration,
- Lamb S_0 mode for $f \approx 56$ MHz, in both configurations, but in antisymmetric case more efficiently,
- transverse SH_1 mode for $f \approx 54.5$ MHz, in both configurations but for symmetric case more efficiently, and modes S_0 and SH_1 overlap in this case what makes the measured conductance of IDT highly distorted.

6. Conclusions

An analysis of propagation of plate modes in piezoelectric plate covered by periodic strips is presented. Bragg reflection and mode conversion is discussed. Inhomogeneous problem of generation of plate modes is solved and experimentally verified. Physical interpretation of measurement is provided.

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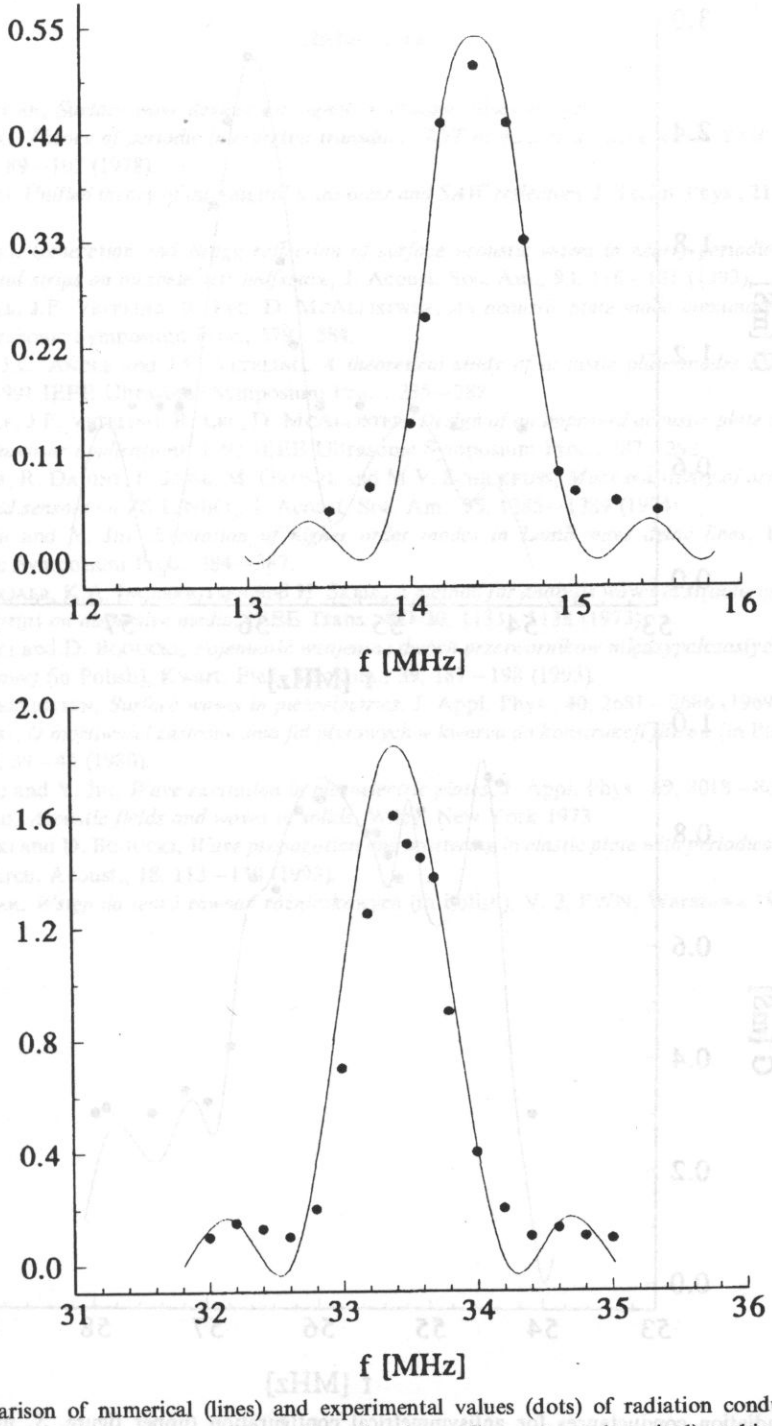

Fig. 2. Comparison of numerical (lines) and experimental values (dots) of radiation conductance of a two-sided pair of IDTs in antisymmetric configuration (upper figure, A_0 mode excited), and in symmetric configuration (below, SH_0 mode excited).

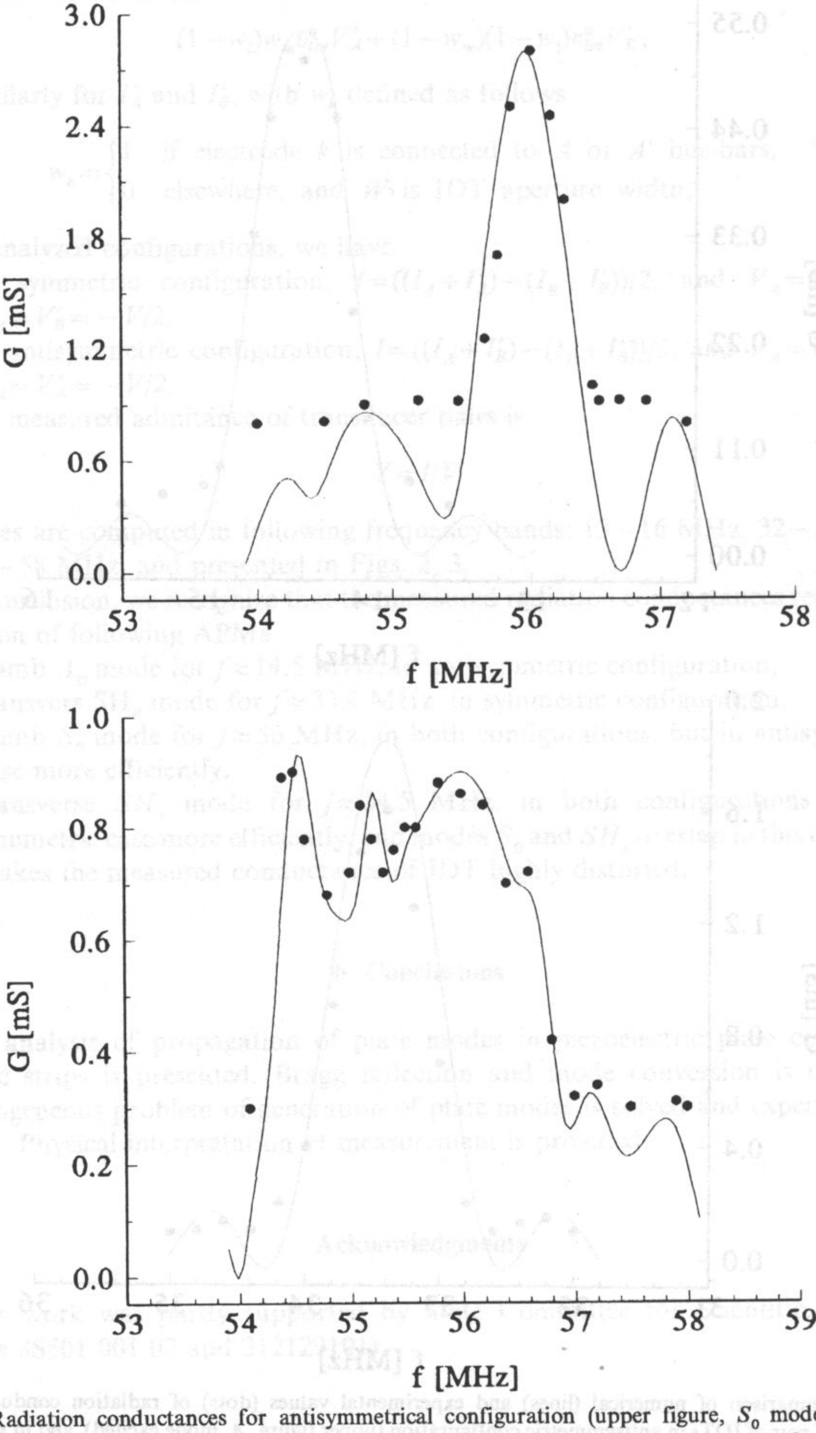

Fig. 3. Radiation conductances for antisymmetrical configuration (upper figure, S_0 mode), and for symmetric configuration of IDTs (below, S_0 and SH_1 modes excited almost at the same frequencies).

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