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TEMPERATURE INCREASE IN A TWO-LAYER OBSTETRIC MODEL OF TISSUES IN THE CASE OF LINEAR AND NONLINEAR PROPAGATION OF A CONTINUOUS ULTRASONIC WAVE

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The object of the present study was the analysis of temperature effects which arise in a two-layer tissue model applied in obstetrics, from the point of view of continuous wave radiation of a focussed Gaussian ultrasound beam. In particular, the authors considered nonlinear propagation using the weak shock theory and compared the results of the analysis with those obtained assuming linear propagation. It was demonstrated that for 3 MHz, 10 cm focal length of the beam, the transducer diameter of 1 cm and the intensity of 0.1 W/cm² the schock parameter does ot then exceed 1.66⁰ cm. For a radiated intensity equal to 1 W/cm² the shock coefficient is higher than unity, causing losses related to nonlinear propagation. The temperature distributions were determined along the beam axis using both the weak shock theory and the linear propagation procedure.

1. Introduction

In the past years the first of the authors showed that ultrasonic Doppler equipment could cause a temperature increase on the body surface up to about 10°C [6, 7]. The measurements were taken using thermographic equipment showing decidedly that ultrasonic diagnostic equipment could cause relatively large temperature increase.

It is an important problem from the point of view of a threat caused by ultrasound to a foetus. In keeping with recommendation of the World Federation for Ultrasound and Biology [13], the permissible increase of the foetus temperature is 1.5°C above the physiological temperature of 37°. If the temperature rises to 41°C higher the nervous system of the child being born can be endangered, with e.g. tragic effects for it (teratism).

For ethical reasons direct measurements in the pregnant woman's uterus are impossible. Therefore, for several years attempts have been made to determine temperature increase analytically.

The purpose of this study is to determine temperature increases for diagnostic c.w. ultrasound beams. In obstetrics c.w. equipment is very broadly used to monitor and alert physicians in the case of a threat to a foetus in his mother's womb. According to Japanese data [11], intensities applied in c.w. diagnostic Doppler equipment vary between 10 and 330 mW/cm^2. In the present study both linear and nonlinear types of propagation are considered; the purpose is to find a limiting ultrasound intensity at which relatively simple and proven calculation methods developed for linear propagation may still be used.

2. A diagnostic ultrasound beam and a two-layer tissue model

In a previous work [9] the authors calculated temperature increases in a two-layer tissue for 6 ultrasonic probes. The frequencies of these probes were 3, 5 and 7.5 MHz the focal lengths were 10, 5, 4, 3 i 2 cm, and the transducer diameters were 20, 13, 9.6 and 6 mm. The calculated results showed that the greatest temperature increases occurred when a 3 MHz beam was applied, one which was generated by a piezoelectric transducer with radius $d=1$ cm and a focal length of 10 cm. Therefore, our further considerations will be restricted to such an ultrasonic beam which is the worst case.

Just as in the study cited above, the Gaussian beam model was assumed. It was for such a beam that such temperature increases as occurred for a continuous wave were published [14]. Moreover, the Gaussian beam is convenient because of the simplicity of mathematical transformations; additionally, it gives results which, as Wu and Nyborg showed, are very close to the real beams [15].

A temperature increase depends on the absorption and thermal conductivity of tissues as well on the heat generated by the ultrasonic probe. The latter may be ignored, for it has only a superficial effect, whereas the heat sources caused by absorption are volumetric. The cooling effect of blood is also neglected so as to consider the worst patient case.

The tissue model under consideration consists of two layers. The first one is water, analogous to the physiological liquids which fill the space between the body surface and the foetus. The other layer is the soft tissue of the foetus itself. It was shown in study [9] that in the case of a continuous wave there was the highest temperature increase in the beam focus (see Fig. 6.1 in the study cited above). It is a result of no attenuation of ultrasonic beam in the first (liquid) layer.

Moreover, it is assumed here that the thickness z_l of the first liquid layer is equal to the physical focal length $z_f=10$ am. It is than that an almost maximum temperature increase occurs (see Fig. 4 of study [15]).

3. Basic formulae

Given the beams and tissues thus selected, the nonhomogenous equation of thermal conductivity can be solved in a form which neglects the cooling blood flow (perfusion)

$$\partial T/\partial t = a \nabla^2 T + Q/\rho c_s, \tag{3.1}$$

where T is the tissue temperature, t is time, $a = \lambda_c/\rho$ is a thermal conductivity coefficient, ∇^2 is the Laplace operator, ρ is the tissue density, c_s is the specific heat, Q is the power density of the heat sources, depending on the attenuation coefficient α, which is a function of frequency and the intensity I, according to the relation

$$Q = 2\alpha(f)\, I(x, y, z). \tag{3.2}$$

This relation is valid only for linear wave propagation. In a general nonlinear case the coefficient α is replaced by the attenuation parameter α_r, which is defined using the general relation [4]

$$\alpha_p = -(\mathrm{div}\, \mathbf{I})/2\mathrm{I}, \tag{3.3}$$

where $\mathbf{I}$, I are respectively the vector and value of a local wave intensity.

The power density of the heat sources (the power density absorbed in tissues) has the general form [3], [12] of

$$Q = -\mathrm{div}\, \mathbf{I}. \tag{3.4}$$

The intensity vector in a focussed Gaussian beam in a cylindrical coordinate system z, r is in the form (see equation A4 in [4]) of

$$\mathbf{I}(z, r) = \mathbf{I}(o, o) G^2 \exp[-2r^2/a_e^2(1+R^2)]\{e_r rR/r_0(1+R^2)^2 + e_z 1/(1+R^2)\} \times \tag{3.5}$$

$$\sum_{n=1}^{\infty} B_n^2[\sigma(z, r)] = \mathbf{I}'(z, r) \sum_{n=1}^{\infty} B_n^2[\sigma(z, r)],$$

where z, r — coordinates of the cylindrical system, $R = (z - z_f)h^{1/2}$, z_f — physical focal length, $k\text{-}2\pi f/c_0$ — wave number, $h = (kR_0^2/2z_a)^2$, c_0 — wave velocity of low amplitude waves, σ — shock parameter [1], $a_0^2 = R_0^2/(1+h)$ — beam radius at the focus for the amplitude level $\exp(-1)$, R_0 — radius for which the vibration velocity on the transducer surface falls to $e^{-1} \cong 0.37$ of its maximum amplitude [8], $r_0 = ka_0^2/2$, e_r, e_z — respective unit vectors of the cylindrical coordinate system, G — amplitude gain in the focus.

In the geometrical focus the amount of the gain in a Gaussian beam can be determined from formula (23) in study [8]. Then

$$G_g = \mid p,(z_0, 0)/p(0,0) \mid = kR_0^2/2z_0, \tag{3.5a}$$

p is the acoustic pressure in the beam. The gain in the physical focus, where the maximum amplitude occurs, is a slightly larger

$$G = (h+1)^{1/2} = G_f = \mid p(z_g, 0)/p(0,0) \mid > G_g. \tag{3.5b}$$

It can be calculated by determining the value of z_f given by formula (5a) in study [9]

$$z_f = z_g[1+4z_g^2/k^2R_0^4)]^{-1} = z_g h/(1+h) \cong z_g, \tag{3.5c}$$

In study [4] the authors assumed a different value of the amplitude gain

$$G_D = d/a = dkR_0/2z_g, \tag{3.5d}$$

which seems to be doubtfull since for the Gaussian beams exact are the values given by (3.5a), (3.5b), (3.5c).

According to [4], the last term of expression (3.5) is determined in terms of the weak shock theory proposed by BLACKSTOCK [1]. B_n is the harmonic coefficient of the distorted wave.

$$B_n[\sigma(z\,r)] = 2[n\pi\sigma]^{-1}\left\{\Phi_{\min} + \int_{\Phi_{\min}}^{\pi} \cos n[\Phi - \sigma\sin\Phi]d\Phi\right\}, \tag{3.6}$$

$\Phi_{\min}$ is the root of the equation $\Phi = \sigma(z,r)\ \sin\Phi_{\min}$ for $\sigma > 1$; $\Phi_{\min} = 0$ for $\sigma < 1$.

In a two-layer model with attenuation occurring for $z \geqslant z_l$ the intensity vector is

$$\mathbf{I}(z,r) = \mathbf{I}'(z\,r)\begin{cases} \sum_{n=1}^{\infty} B_n^2(z,r) & \text{for } z < z_l, \quad (3.7\text{a}) \\ \sum_{n=1}^{\infty} B_n^2(z,r)\exp[-2\sigma(n)(z-z_l)] & \text{for } z \geqslant z_l, \quad (3.7\text{b}) \end{cases}$$

where the vector field $I'(z,r)$ is lossless. The attenuation of the beam is represented in the second right term of Eq. (3.7 b). The attenuation coefficient α (for low amplitudes) is proportional to the (harmonic) frequencies $a(n) = n\alpha$.

The shock parameter σ for the Gaussian beam equals [4]

$$\sigma(z,r) = \beta\varepsilon k z_0 G(G^2-1)^{-1/2}\{\ln[G+(G^2-1)^{1/2})(R+(1+R^2)^{1/2})]\}\times \exp[-r^2/a_0^2(1+R^2)], \tag{3.8}$$

where $\beta = 1 + B/2A$, B/A — coefficient of nonlinearity equal for the muscle tissue and liver 7.5, for fat 11, for water 5.5, $\varepsilon = (2I_0/\rho c_0^3)^{1/2}$ — ratio of the particle velocity to c_0, $I_0 = I(0,0)$.

The value of Q can now be determined

$$Q = -\operatorname{div} I(z,r). \tag{3.9}$$

The vector $\mathbf{I}'$ in expressions (3.5) and (3.7 a, b) is a quantity formulated on the basis of the linear theory of the Gaussian beam, without any losses. Therefore, due to the energy conservation and Gauss laws

$$\int \mathbf{I}'ds = \int \operatorname{div}\mathbf{I}'\,dv = 0, \tag{3.10}$$

where ds and dv are surface and volumen elements. Hence div $\mathbf{I}' = 0$. On the other hand, the expression under the summation sign in Eq. (3.7) is a scalar U. Using then the vector relation one obtains

$$\operatorname{div} U\mathbf{I}' = U \operatorname{div} \mathbf{I}' + \mathbf{I}' \operatorname{grad} U = \mathbf{I}' \cdot \operatorname{grad} U. \tag{3.10b}$$

Therefore one obtains from relations (3.10b), (3.8), (3.7) and (3.5)

$$\text{for } z < z_l \text{ the value } \quad Q = Q_1, \tag{3.11a}$$

where

$$Q_1 = -C \exp\{-3r^2/a_0^2[1+R^2(z)]\}[1+R^2(z)]^{-1/2} \times \sum_{n=1}^{\infty} B_n[\sigma(z,r)]BP_n[\sigma(z,r)] \tag{3.11b}$$

$$\text{for } z \geqslant z_l \text{ the value } \quad Q = Q_L + Q_2, \tag{3.12a}$$

where

$$Q_L = 2\Gamma^2 G^2[1+R^2(z)]^{-1}\exp\{-2r^2/a_0^2[1+R^2(z)]\} \times \sum_{n=1}^{\infty} \sigma_n B_n^2[\sigma(z,r)]\exp[-2\alpha(n)(z-z_l)], \tag{3.12b}$$

$$Q_2 = -C \exp\{-3r^2/a_0^2[1+R^2(z)]\}[1+R^2(z)]^{-1/2} \times \sum_{n=1}^{\infty} B_n[\sigma(z,r)]BP_n[\sigma(z,r)] \exp[-2\sigma(n)(z-z_l)], \tag{3.12c}$$

and

$$C = 2\mathrm{I}_0\beta\varepsilon k G^3; \quad BP_n = B_{1n} - B_n/\sigma(z,r), \tag{3.12d, e}$$

$$B_{1n} = 2(\pi\sigma)^{-1} \int_{\Phi_{\min}}^{\pi} \sin[n(\phi - \sigma\sin(\phi))]\sin(\phi)d\phi. \tag{3.12f}$$

In the case of a plane wave the following dependence should then be satisfied

$$\beta\varepsilon k \gg \alpha. \tag{3.13}$$

When relation (3.13) is not valid shock can be neglected [1]. Study [10] also makes a similar suggestion. It demonstrates that for a plane wave in soft tissues, harmonic frequencies are very rapidly attenuated, preventing shock [12]. In a weakly focussed Gaussian beam this phenomenon must occur in a similar way when attenuation is strong, which, as in other studies, is assumed to be 5 Np/(m.MHz); particularly in the focal area, where the wave is close to a plane one. The solution of equation (3.1) under the initial conditions

$$T(x,y,z,t) = 0 \quad \text{for } t = 0, \tag{3.14}$$

and the boundary ones

$$\partial T/\partial z = 0 \quad \text{for } z = 0, \tag{3.15}$$

which are the same as in the previous author's study [9], a general solution is obtained in the form given there (3.14)

$$T=(4\pi\rho c_s)^{-1/2}\int_{-\infty}^{\infty} d\zeta \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\zeta \int_0^t d\tau\,[4\pi a^3(t-\tau)^3]^{1/2}\cdot$$
$$\exp[-\gamma^2(\zeta,\zeta,\eta)/4a(t-\tau)]\cdot Q(\zeta,\eta,|\zeta|), \tag{3.16}$$

where

$$\gamma^2=(x-\zeta)^2+(y-\eta)^2+(z-\zeta)^2. \tag{3.17}$$

After integration with respect to τ (see equation A6 in study [9]) and considering the relation erfc(0)=1 for t which tends to infinity, equation (3.16) becomes

$$T=(4\pi a\rho c_s)^{-1}\int_{\infty}^{\infty} d\zeta \int_{\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\zeta\,\gamma^{-1}Q(\zeta,\eta,|\zeta|). \tag{3.18}$$

Integral (3.18) can be expressed in a cylindrical coordinate system introducing the dependencies

$$\zeta=r'\cos\phi',\quad \eta=r'\sin\phi',\quad x=r\cos\phi,\quad y=r\sin\phi \tag{3.19}$$

then

$$r'^2=\zeta^2+\eta^2,\quad r^2=x^2+y^2,\quad d\zeta d\eta=r'dr'd\phi'. \tag{3.20}$$

The temperature distribution on the beam axis ($r=0$) is taken into account. Then, considering (3.19) and (3.20) in expression (3.18) integration is carried out with respect to ϕ' within the limits of 0, 2π, finally to give the form

$$T(r',\zeta,t=\infty)=(2a\rho c_s)^{-1}\int_{-\infty}^{\infty} d\zeta \int_0^{\infty} [r'^2+(z-\zeta)^2]^{-1}Q(r',|\zeta|)\,r'dr'. \tag{3.21}$$

Integral (3.21) was solved numerically for the following data: $I_0(0,0)=0.01$, 0.1, 1 W/cm, 3 W/cm, 5 W/cm, $6=5$ Np/(m·MHz), $c=1500$ m/s, $c_s=4190$ J/(kg°C), $a=0.0015$ cm/s, $f=3$ MHz, $G=G_f=3.67$ ($G_g=3.53$), $d=1$ cm (the transducer radius), $R_0=0.75$ cm (see [9]), $z_f=10$ cm and $t=\infty$.

Table 1 shows the results of current calculations of the temperature increases T and the results determined for T_L using the previous linear procedure as applied in study [9]. These calculations were carried out for a two-layer TL model, where the

Table 1. Comaprison of temperature increases determined for a two-layer model in the cases of nonlinear T and linear T_L propagation

$I(z=0, r=0)$	W/cm²	0.01	0.1	1	3	5
T	°C	0.166	1.66	18.0	61.5	104.4
T_L	°C	0.166	1.66	16.6	49.8	83.0
T/T_L		1	1	1.08	1.23	1.26

first 10 cm thick layer was a liquid ($\alpha = 0$) and the other was soft tissue ($a = 5$ Np/(mMHz). The table also shows the maximum temperature found on the beam axis ($r = 0$) close to the focus ($z \cong z_f$).

The linear procedure made it possible to apply simple extrapolation of temperature to any intensities I(0,0). For in the case of linear propagation a temperature increase is proportional to the intensity generated by the transducer (see Eq. (3.18) and A22 in study [9]). The temperature values determined in this way were denoted as T_L.

Figure 1 shows the distributions of the temperature increases T along the beam axis, determined by the weak shock method and based on formula (3.21) for the intensities $I(0,0) = 0.1$ W/cm² and 1 W/cm².

Figure 2 shows analogous distributions of temperature increases T_L determined for the intensity $I(0,0) = 0.1$ W/cm², assuming linear propagation (see Fig. 6.1 in study [9]).

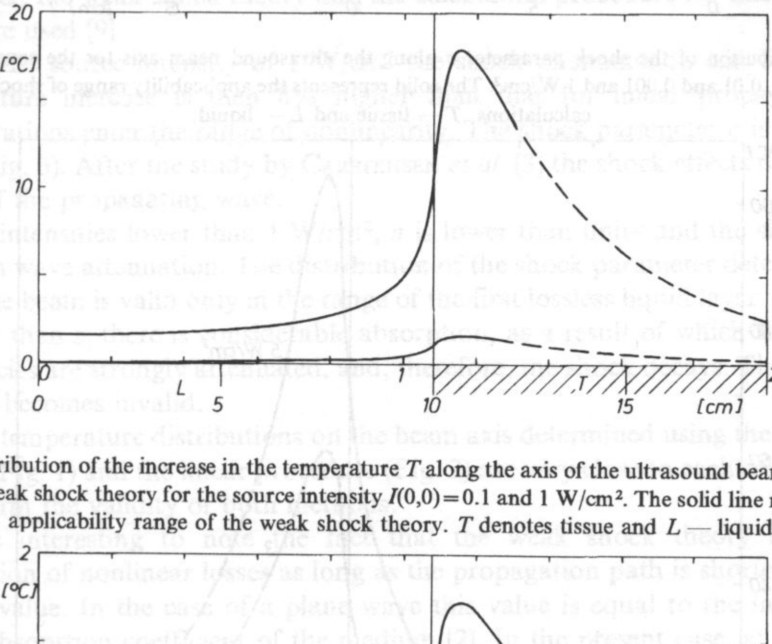

Fig. 1. Distribution of the increase in the temperature T along the axis of the ultrasound beam determined using the weak shock theory for the source intensity $I(0,0) = 0.1$ and 1 W/cm². The solid line represents the applicability range of the weak shock theory. T denotes tissue and L — liquid.

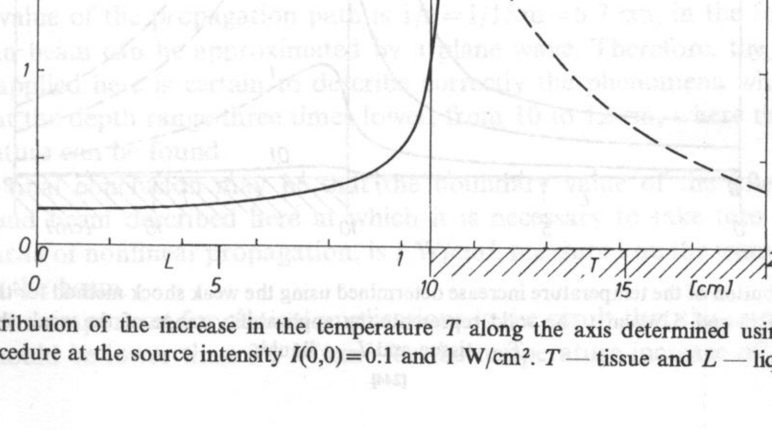

Fig. 2. Distribution of the increase in the temperature T along the axis determined using the linear procedure at the source intensity $I(0,0) = 0.1$ and 1 W/cm². T — tissue and L — liquid.

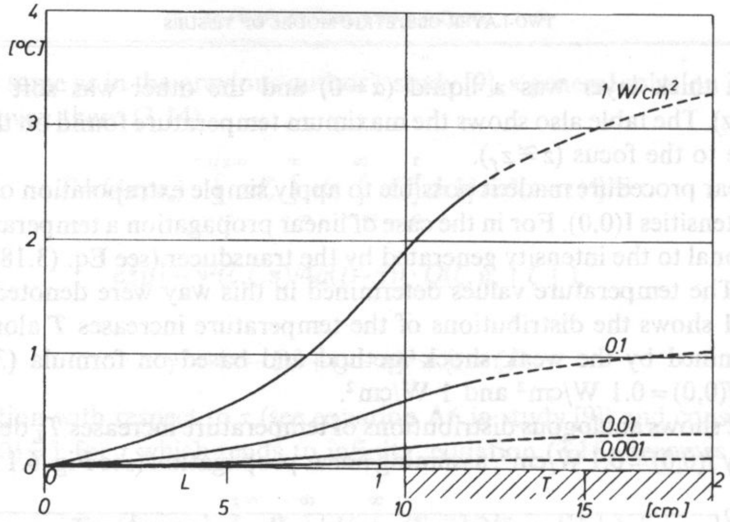

Fig. 3. Distribution of the shock parameter σ along the ultrasound beam axis for the source intensity $I(0,0) = 1$, 0.1, 0.01 and 0.001 and 1 W/cm². The solid represents the applicability range of shock parameter calculations. T — tissue and L — liquid.

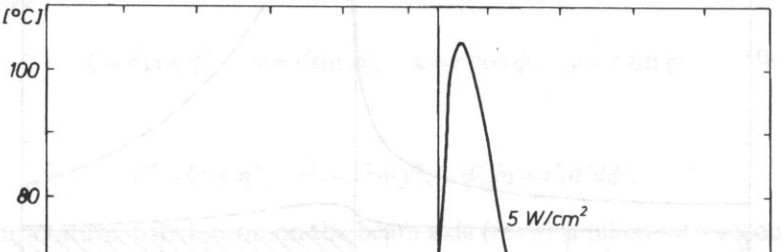

Fig. 4. Distribution of the temperature increase determined using the weak shock method for the intensities $I(0,0) = 0.1$, 1, 3 and 5 W/cm². The solid represents the applicability range of the weak shock theory. T — tissue and L — liquid.

Figure 3 shows the distribution of the shock parameter along the beam axis, calculated from expression (3.12).

Figure 4 shows comparison of the distributions of temperature increases determined using the weak shock method for the intensities I(0,0) = 0.1, 1, 3 and 5 W/cm².

4. Discussion and conclusion

The present analysis shows that in the case of ultrasonic diagnostic probes which generates the highest potential temperature increases, when working at a continuous wave, temperature increases determined with the assumption of linear and nonlinear propagation are the same for source intensities lower than 1 W/cm² (see Table). This result was obtained assuming a two-layer tissue model which is applied in obstetrics. Moreover, the weak shock theory and the calculation procedure for linear propagation were used [9].

For the source intensity of 1 W/cm² the shock phenomenon can be seen. The temperature increase is then 8% higher than that for linear propagation. The considerations enter the range of nonlinearity. The shock parameter σ is higher than unity (Fig. 3). After the study by Carstensen *et al.* [3] the shock effects decidedly the losses of the propagating wave.

For intensities lower than 1 W/cm², σ is lower than unity and the shock has no effect on wave attenuation. The distribution of the shock parameter determined here along the beam is valid only in the range of the first lossless liquid layer. In tissue for z higher than z_l there is considerable absorption, as a result of which the harmonic frequencies are strongly attenuated, and, therefore, the shock decays. Then the curve $\sigma = \sigma(z)$ becomes invalid.

The temperature distributions on the beam axis determined using the weak shock theory (Fig. 1) and the linear procedure (Fig. 2) are very close to each other. It seems to confirm the validity of both methods.

It is interesting to note the fact that the weak shock theory is a correct description of nonlinear losses as long as the propagation path is shorter than some critical value. In the case of a plane wave this value is equal to the inverse of the linear absoprtion coefficient of the medium [2]. In the present case, at 3 MHz, the critical value of the propagation path is $1/\alpha = 1/15\,\mathrm{m} = 6.7$ cm, in the focal area the Gaussian beam can be approximated by a plane wave. Therefore, the weak shock theory applied here is certain to describe correctly the phenomena which occur in tissues at the depth range three times lower, from 10 to 12 cm, where the maximum temperature can be found.

The final conclusion may be that the boundary value of the intensity of the ultrasound beam described here at which it is necessary to take into account the phenomena of nonlinear propagation, is 1 W/cm², measured on the transducer which radiates the beam.

What is important for clinical applications is the result that the intensity $I(z=0, r=0)$ should be lower than 0.1 W/cm² for the temperature increase of the probe in question to be less than 1.66°C. In this case, the temperature increases are unlikely to grow as a result of nonlinear propagation.

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