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VERTICAL DISTRIBUTION OF ROAD TRAFFIC NOISE

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Road noise generated by freely flowing traffic is assessed by the time-average sound level, L_{AT}. The noise interaction with the ground surface is the only wave phenomenon that modifies geometrical spreading. A simplified model of the ground effect is applied. The model is founded on the Weyl-Van der Pol solution. Prediction of the time-average sound level, at a perpendicular distance less than 100 m and an altitude not exceeding a few tens of meters, requires two simultaneous measurements of L_{AT} at the site of interest.

1. Introduction

There are many models for road traffic noise calculation. Although these models generate precise predictions, they require a tremendous expense in computing time. Thus, there is a need for methods that are practical for everyday use, despite limited accuracy (see e.g. Refs. [2, 9]). In this paper we propose such a method based on two simultaneous measurements at the site of interest.

Above a plane ground surface, without buildings or any other interfering structures, sound propagation depends upon geometrical spreading, ground effect, air absorption, refraction, and scattering by atmospheric turbulence [1]. Close to the road, within the range of 100 m, geometrical spreading is influenced mainly by ground effect. To quantify this influence we use the concept of the time-average sound level, L_{AT}, which is widely used as a measure of road traffic noise,

$$L_{AT} = 10\log\left\{\frac{\langle p_A^2\rangle}{p_0^2}\right\}, \qquad p_0 = 20\,\mu\text{Pa}, \tag{1.1}$$

where the time-average A-weighted squared sound pressure is,

$$\langle p_A^2 \rangle = \frac{1}{T} \int_0^T p_A^2(t)\, dt. \tag{1.2}$$

Because road traffic noise consists of noise events that do not interact with each other, we write,

$$p_A^2(t) = \sum_{i=1}^{N} p_{Ai}^2(t), \tag{1.3}$$

where $p_{Ai}^2(t)$ corresponds to the noise generated by a single vehicle (Fig. 1), and N denotes the number of vehicles passing the receiver during the time period T. Combing Eqs. (1.2), (1.3) yields,

$$\langle p_A^2 \rangle = \frac{1}{T} \sum_{i=1}^{N} E_i\,, \tag{1.4}$$

where

$$E_i = \int_{-\infty}^{\infty} p_{Ai}^2(t)\, dt, \tag{1.5}$$

denotes the sound exposure.

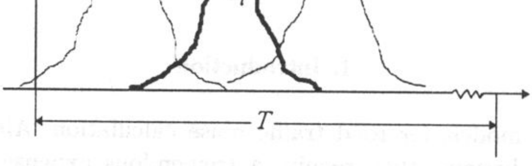

Fig. 1. Road traffic noise consists of noise events, i.e., passage of vehicles.

In Sec. 2 an approximated model of ground effect and the equation for p_A^2 is derived. Random nature of the sound exposure, E_i, is studied in Sec. 3. Then, a two parameter equation for the time-average sound level, L_{AT}, is derived and its application for noise prediction is demonstrated (Sec. 4).

2. Ground effect

A road vehicle can be modeled by a point source [5]. Assuming near grazing propagation (Fig. 2),

$$H \ll d, \qquad H_0 \ll d, \tag{2.1}$$

and neglecting the width of the road so that the ground surface is uniform, i.e., without impedance jumps, the A-weighted squared sound pressure can be written as [7],

$$p_A^2 = \frac{W_A \varrho c}{4\pi d^2} \cdot G(d, H, H_0), \tag{2.2}$$

where: W_A is the A-weighted sound power, ϱc is the characteristic impedance of air, and d is the horizontal distance between the source, S, and receiver, O. There are "exact" equations for the ground factor, G; among others, one based on the Weyl–Van der Pol solution is used [1]. The ground factor meets the principle of reciprocity,

$$G(H, H_0) = G(H_0, H), \tag{2.3}$$

and decreases with the second power of the horizontal distance,

$$\lim_{d\to\infty} G \propto d^{-2}. \tag{2.4}$$

To find a simple model of ground effect, we make use of the result obtained by Li *et al.* [6]: the ground factor, G, is a function of the grazing angle (Fig. 2),

$$\Psi = \cot^{-1}\left(\frac{d}{H + H_0}\right). \tag{2.5}$$

Taking into account Eqs. (2.3)–(2.5) we obtain the following approximation [8],

$$\widetilde{G} = \beta \cdot \left[1 + \gamma \cdot \frac{d^2}{(H + H_0)^2}\right]^{-1}. \tag{2.6}$$

Far away from the source, $d \to \infty$, the condition defined by Eq. (2.4) is met. Close to the source, $d \to 0$, the above formula yields, $\widetilde{G} \to \beta$, so parameter β may be interpreted as a measure of noise reflection from the road surface beneath the source (e.g., for $\beta = 2$ the sound level increases by 3 dB).

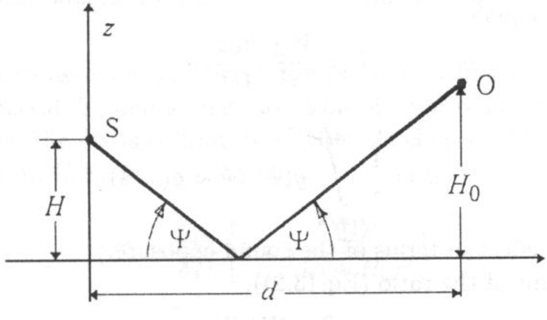

Fig. 2. Source, S, receiver, O, geometry is determined by the heights, H and H_0, horizontal distance, d, and the grazing angle, Ψ.

Introducing the perpendicular distance, D, and the angle Φ (Fig. 3), we get (Eqs. (2.2), (2.6)),

$$p_A^2 = \frac{\beta W_A \varrho c \cdot \cos^2 \Phi}{4\pi D^2} \cdot g(\Phi), \tag{2.7}$$

where

$$g(\Phi) = \left[1 + \gamma \cdot \frac{D^2}{(H+H_0)^2 \cos^2 \Phi}\right]^{-1}. \tag{2.8}$$

The *ground coefficient*, γ, characterizes the noise variations due to ground effect within the vertical plane ($y = D$, $z = H_0$).

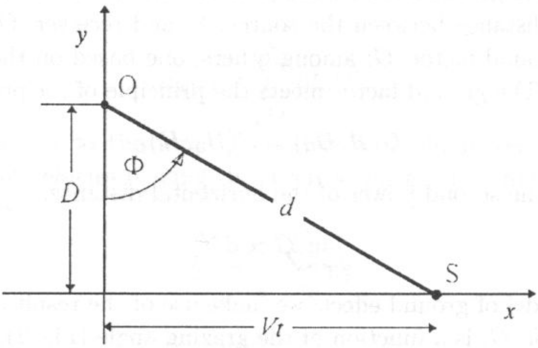

Fig. 3. Source, S, and receiver, O, in the horizontal plane, $x - y$, with the perpendicular distance, D, and the angle, Φ.

3. Sound exposure

If a source is moving with a steady speed, V, along the x axis, at the perpendicular distance, D, from the receiver, then the integral Eq. (1.5) takes the form,

$$E = \frac{D}{V} \int_{-\pi/2}^{\pi/2} \frac{p_A^2(\Phi)}{\cos^2 \Phi}\, d\Phi, \tag{3.1}$$

and sound exposure equals,

$$E = \frac{W_A}{V} \frac{\beta \varrho c}{4D} \cdot J, \tag{3.2}$$

where (Eq. (2.8))

$$J = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} g(\Phi)\, d\Phi \approx g(\pi/4) \tag{3.3}$$

accounts for ground effect in terms of the sound exposure.

Note, that the unit of the ratio (Eq. (3.2)),

$$S = W_A / V, \tag{3.4}$$

is *Joule* per *meter*. Thus, S is called the *linear density of energy*. This energy is spend during the passage of a vehicle. Making use of the reference power, $W_0 = 10^{-12}$ Watts, we introduce the *reference linear density of energy*,

$$S_0 = W_0 / V_0\,, \tag{3.5}$$

where $V_0 = 1\,\text{m/s}$.

Passing vehicles yield random values of E. The reasons for the random values are the variations of the sound power, W_A, velocity, V, height, H, distance to the receiver, D, and ground parameters, β and γ. Therefore, for the i-th vehicle we can write (Eqs. (2.8), (3.2)–(3.4)),

$$E_i = \frac{S_i\beta_i \cdot \varrho c}{4D_i}\left[1+\gamma_i\frac{2D_i^2}{(H_i+H_0)^2}\right]^{-1}. \tag{3.6}$$

Consequently, the time-average A-weighted squared sound pressure is (Eq. (1.4)),

$$\langle p_A^2\rangle = \frac{N}{T}\langle E\rangle , \tag{3.7}$$

where

$$\langle E\rangle = \frac{1}{N}\sum_{i=1}^{N} E_i , \tag{3.8}$$

denotes the average sound exposure.

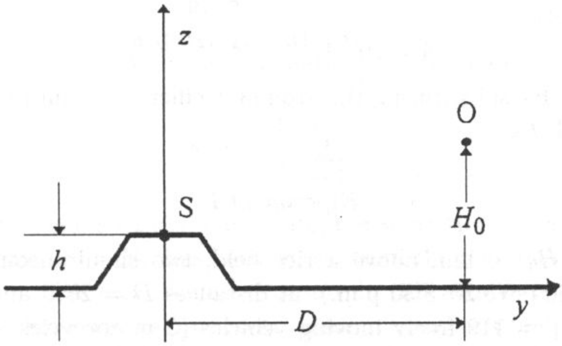

Fig. 4. Vertical plane with the receiver, $O(y = D, z = H_0)$, and the source, $S(y = 0, z = h)$.

Now we make two assumptions. First: the ground coefficient, γ, remains constant during the time interval, T, and second: location of each vehicle's track is determined by the height, h, and the distance from the center, D (Fig. 4). Thus, the average sound exposure is (Eqs. (3.6), (3.8)),

$$\langle E\rangle = \frac{\widetilde{S}\varrho c}{4D}\left[1+\gamma\frac{2D^2}{(h+H_0)^2}\right]^{-1}, \tag{3.9}$$

where

$$\widetilde{S} = \frac{1}{N}\sum_{i=1}^{N} S_i\beta_i , \tag{3.10}$$

expresses the average of the linear density of energy S, which is modified by the ground reflection, β. Because of variations of mix of vehicles and weather conditions, the values of $\widetilde{S}$ and γ might change during the day and night (see the next section).

4. Time-average sound level

To obtain the time-average sound level we apply Eqs. (1.1), (3.5), (3.7) and (3.9),

$$L_{AT} = L_S + 10\log\left\{\frac{Nt_0}{T}\frac{l_0}{4D}\right\} - 10\log\left\{1+\gamma\frac{2D^2}{(h+H_0)^2}\right\}, \tag{4.1}$$

where $l_0 = 1\,\mathrm{m}$ and $t_0 = 1\,\mathrm{s}$. The *energy density level* (Eqs. (3.5), (3.10)),

$$L_S = 10\log\left\{\widetilde{S}/S_0\right\}, \tag{4.2}$$

and the ground coefficient, γ, are unknown. To determine them, two simultaneous measurements of L_{AT}, at the perpendicular distances, D and $2D$, at the same height, H_0, are needed. Considering Eq. (4.1) as a theoretical prediction with two adjustable parameters, we obtain,

$$\gamma = \frac{1}{2}\frac{(h+H_0)^2}{D^2}\cdot\frac{A-2}{8-A}, \tag{4.3}$$

where the parameter,

$$A = 10^{[L_{AT}(D)-L_{AT}(2D)]/10}, \tag{4.4}$$

is fully determined. By substituting the ground coefficient, γ, into Eq. (4.1) we find the energy density level, L_S.

Experiment I.

At the height, $H_0 = 1\,\mathrm{m}$, above a rice field, two simultaneous measurements of L_{AT} during $T_1 = 600\,\mathrm{s}$ (5:20–5:30 p.m.), at distances $D = 25\,\mathrm{m}$ and $2D = 50\,\mathrm{m}$, have been performed. $N_1 = 119$ freely moving vehicles (3 motorcycles, 88 automobiles, 28 trucks) passed the microphones. There was cloudy and windless weather. Substituting the results of measurements, $L_{AT}(D) = 61.4\,\mathrm{dB}$ and $L_{AT}(2D) = 56.7\,\mathrm{dB}$ into Eq. (4.4) gives $A = 2.95$, and then, for $h = 0$ (horizontal road) and $H_0 = 1\,\mathrm{m}$, we obtain: $\gamma_1 = 0.00015$ and $L_{S1} = 89.2\,\mathrm{dB}$.

When the numerical values of N_1/T_1, L_{S1}, and γ_1 are available, one can predict the time-average sound level at any distance from the road, $y = D$, and the height above the ground, $z = H_0$ (Eq. (4.1)),

$$L_{AT}(y,z) = L_{S1} + 10\log\left\{\frac{N_1t_0}{T_1}\cdot\frac{l_0}{4y}\right\} - 10\log\left\{1+\gamma_1\frac{y^2}{(h+z)^2}\right\}. \tag{4.5}$$

The graphs of $L_{AT} = \mathrm{const}$ are given by,

$$z = y\cdot\left[\frac{\gamma_1 y}{y_0 - y}\right]^{1/2} - h, \tag{4.6}$$

where

$$y_0 = l_0\frac{N_1t_0}{4T_1}10^{(L_{S1}-L_{AT})/10}. \tag{4.7}$$

Experiment II.

The same measurements used in Experiment I were repeated 1 hour later during $T_2 = 600\,\mathrm{s}$ (6:20–6:30 p.m.). The weather conditions were the same. This time noise was produced by $N_2 = 88$ vehicles (1 motorcycle, 74 automobiles, 13 trucks). Making use of the results of the measurements, $L_{AT}(D) = 57.5\,\mathrm{dB}$ and $L_{AT}(2D) = 53.6\,\mathrm{dB}$, we get (Eqs. (4.1), (4.3)–(4.4)): $\gamma_2 = 0.000066$ and $L_{S2} = 86.2\,\mathrm{dB}$.

(L_{S1}, γ_1) and (L_{S2}, γ_2) characterize traffic noise during the "short" time intervals, T_1 (5:20–5:30 p.m.) and T_2 (6:20–6:30 p.m.), respectively. What are the noise characteristics for longer time interval, T (e.g. 5:00–7:00 p.m.)? In other words, how to predict the time-average sound level for a long time interval, L_{AT}, when the values of (L_{S1}, γ_1) and (L_{S2}, γ_2) are available?

To answer this question, suppose n is the number of vehicles passing the receiver during the long time interval, T, so that the time-average A-weighted sound pressure is (Eqs. (3.7), (3.8)),

$$\langle p_A^2 \rangle_T = \frac{n}{T} \cdot \langle E \rangle_T \,, \tag{4.8}$$

where the mean sound exposure in the population of n noise events is defined by,

$$\langle E \rangle_T = \frac{1}{n} \cdot \sum_{i=1}^{n} E_i \,. \tag{4.9}$$

By using two measurements carried out during the short time intervals, T_1 and T_2, the sample of noise events, $N_1 + N_2$, has been drawn from the population of n noise events, where $N_1 + N_2 \ll n$. The mean sound exposure in this sample is defined by Eq. (3.8) with $N = N_1 + N_2$. Since the sample mean is an unbiased estimator of the population mean [3], one can write,

$$\langle E \rangle_T = \frac{1}{N_1 + N_2} \sum_{i=1}^{N_1+N_2} E_i \,. \tag{4.10}$$

The above average can be rewritten as (Eq. (4.8)),

$$\langle E \rangle_T = a_1 \cdot \langle E \rangle_1 + a_2 \cdot \langle E \rangle_2 \,, \tag{4.11}$$

where

$$\langle E \rangle_1 = \frac{\widetilde{S}_1 \varrho c}{4D} \left[1 + \gamma_1 \frac{2D^2}{(h + H_0)^2} \right]^{-1} , \tag{4.12}$$

$$\langle E \rangle_2 = \frac{\widetilde{S}_2 \varrho c}{4D} \left[1 + \gamma_2 \frac{2D^2}{(h + H_0)^2} \right]^{-1} , \tag{4.13}$$

and the relative numbers of vehicles are,

$$a_1 = N_1/(N_1 + N_2), \qquad a_2 = N_2/(N_1 + N_2). \tag{4.14}$$

Finally, for $y = D$ and $z = H_0$, we arrive at the time-average sound level (Eqs. (1.1), (3.4), (3.5), (4.8), (4.11)),

$$L_{AT} = 10\log\left\{\frac{nt_0}{T}\cdot\frac{l_0}{4y}\right\} + 10\log\left\{\frac{a_1\cdot 10^{L_{S1}/10}}{1+\gamma_1\dfrac{2y^2}{(h+z)^2}} + \frac{a_2\cdot 10^{L_{S2}/10}}{1+\gamma_2\dfrac{2y^2}{(h+z)^2}}\right\}, \tag{4.15}$$

where n denotes the number of vehicles passing the receiver during a long time interval, T. The contours $L_{AT} = \text{const}$ can be found from,

$$z = y\cdot[f(y, L_{AT})]^{1/2} - h, \tag{4.16}$$

where

$$f = \frac{\gamma_1(y-y_2)+\gamma_2(y-y_1)}{y_1+y_2-y} + \frac{\sqrt{[\gamma_1(y-y_2)+\gamma_2(y-y_1)]^2 + 4\gamma_1\gamma_2 y(y_1+y_2-y)}}{y_1+y_2-y}, \tag{4.17}$$

with

$$y_1 = l_0\frac{a_1 n t_0}{4T}10^{(L_{S1}-L_{AT})/10} \qquad \text{and} \qquad y_2 = l_0\frac{a_2 n t_0}{4T}10^{(L_{S2}-L_{AT})/10}. \tag{4.18}$$

Figure 5 shows the contours of $L_{AT} = 50$, 55, and 60 dB calculated for the output data of the above experiments: $a_1 = 0.57$, $\gamma_1 = 0.00015$, $L_{S1} = 89.2$ dB and $a_2 = 0.43$, $\gamma_2 = 0.000066$, $L_{S2} = 82.2$ dB, with the traffic flow during a long time interval, $n/T = (N_1+N_2)/(T_1+T_2) = 0.17$ [veh./s] and the road's height, $h = 0$.

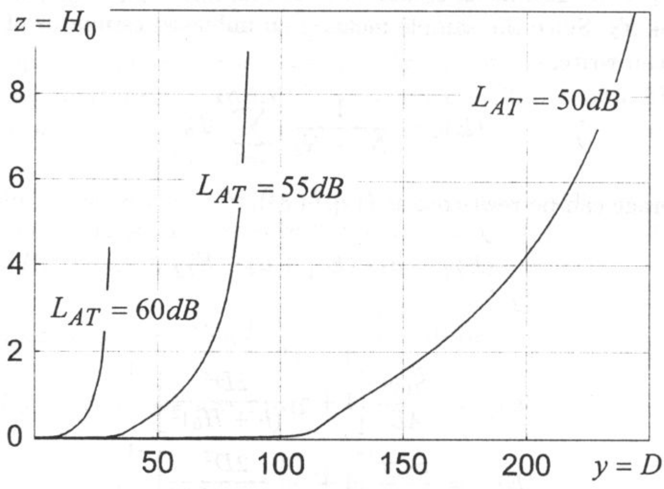

Fig. 5. Contours of $L_{AT} = \text{const}$ calculated from Eq. (4.16).

If m measurements of L_{AT} were made during the short time intervals $T_1, T_2, \ldots, T_i, \ldots, T_m$ (Fig. 6), with the relative numbers of vehicles, $a_1, a_2, \ldots, a_i, \ldots, a_m$ (Eq. (4.14)),

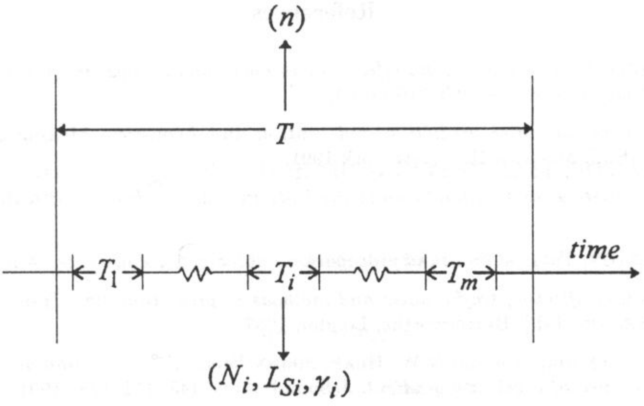

Fig. 6. During a short time interval, T_i, passages of N_i vehicles generate noise which is characterized by L_{Si} and γ_i.

respectively, then Eqs. (4.1), (4.3), and (4.4) yield the noise characteristics, (L_{S1}, γ_1), (L_{S2}, γ_2) ... (L_{Si}, γ_i) ... (L_{Sm}, γ_m), and the time-average sound level for a long time interval, $T > T_1 + T_2 + \ldots T_i + \ldots T_m$, can be calculated from,

$$L_{AT} = 10 \log \left\{ \frac{nt_0}{T} \cdot \frac{l_0}{4y} \right\} + 10 \log \left\{ \sum_{i=1}^{m} a_i \cdot 10^{L_{Si}/10} \cdot \left[1 + \gamma_i \frac{2y^2}{(h+z)^2} \right]^{-1} \right\}. \tag{4.19}$$

5. Conclusions

The model of road traffic noise has been developed under the following conditions:

• vehicles are moving with steady speeds along a straight road,

• the road width and receiver height ($z = H_0$) are significantly less than the perpendicular distance between the road center and the receiver ($y = D$),

• geometrical spreading is modified only by ground effect.

Two adjustable parameters of the model, i.e., energy density level, L_S, and the ground coefficient, γ, can be estimated from two measurements *in situ* of the time-average sound level, $L_{AT}(D)$ and $L_{AT}(2D)$. Making use of Eqs. (4.6), (4.16), and (4.19), one can calculate, $L_{AT}(y, z)$ at the distance, $y = D$, less than 100 m and at the height, $z = H_0$ not exceeding a few meters (Eq. (2.1)).

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