ARCHIVES OF ACOUSTICS
24, 1, 57–74 (1999)

LOSSES IN A PIEZOELECTRIC CERAMIC IN HIGH FIELDS

M. SZALEWSKI

Institute of Fundamental Technological Research
Polish Academy of Sciences
(00-049 Warszawa, Świętokrzyska 21)

The paper concerns the problem of measurements of electrical and mechanical losses in a piezoelectric ceramic driven by high electric field and vibrating with high vibration velocity. Deformations and discontinuities of resonance curves appear in this range of the fields. Therefore the application of typical methods of measurements of quality factors is difficult or even impossible. The author proposes to apply the measurements of voltage ratios in a piezoelectric transformer instead of the measurements of resonator quality factors. In the paper the relations connecting the voltage ratios in the piezoelectric transformer with the values of electrical and mechanical losses as well as experimental results are presented. The formulae have been derived using the KLM equivalent circuit for two limits of the transformer load resistance. The obtained results are compared with the results obtained using one of the earlier known measurement method in the range of its applicability.

1. Introduction

Electrical and mechanical losses in a piezoelectric ceramic and methods of their measurements were the subject of many theoretical and experimental works. However the problem of measurements of losses in the range of high electrical and mechanical fields has not been solved. This problem is important because piezoelectric ceramics are widely used in high-power devices, such as piezoelectric actuators, ultrasonic motors, piezoelectric transformers and sending transducers in hydroacoustic devices. In recent years one can observe the development of high-power piezoelectric ceramic devices and the growing number of their applications. Experimental investigations of multicomponent ceramics have proved that the ceramic compositions with excellent electromechanical properties under a relatively low vibration level did not necessarily guarantee good operation under a relatively high vibration level [38]. Therefore investigations of properties of a piezoelectric ceramic in high fields are still the subject of many researches, e.g. [5, 28, 40].

In a high electric field, when a vibration velocity is high, resonance curves deform at first and next discontinuities of resonance curves appear. These effects make difficult or even impossible to determine losses applying classical methods of measurements of a quality factor. We have tried to solve this problem applying measurements of a voltage ratio in a piezoelectric transformer instead of measurements of a resonator quality factor. Relations between voltage ratios in a piezoelectric transformer and mechanical and electrical losses are presented in Sec. 4. These equations have been derived using the KLM equivalent circuit. Experimental results and their comparison with the results obtained using one of the earlier known methods (described in Sec. 3), in the range of its applicability, are presented in Sec. 5.

2. Origin of losses in a piezoelectric ceramic

As yet there is no complete doubtless description of the mechanism of electrical and mechanical losses in a piezoelectric ceramic. Many theories exist attributing a leading part to different effects. According to [27] mechanical losses are caused by the scattering at grain boundaries, by internal friction in ceramic grains and domains and by micro- or macrocrackings. According to [4] mechanical losses are mainly due to the motional resistivity and electrical losses are mainly due to bulk grain and grain boundary effects, domain walls effects are very important in the frequency range above 10^3 Hz. According to [11, 12] both types of losses are induced by 90° ferroelectric domain walls moving under the influence of electrical fields or mechanical stresses. The physical nature of electrical losses is explained by the damping of moving 90° domain walls. For frequencies up to 10^8 Hz the origin of damping of the wall motion is ascribed to point defects within the 90° domain wall. At higher frequencies damping results from the reflection of thermal-lattice waves impinging on a moving 90° domain wall. The ratio of electrical and mechanical losses depends on material properties like spontaneous polarization, spontaneous striction, elastic compliance, dielectric constant [12]. The analysis of effects induced by the domains motion allows also to explain the strong increase of electrical losses at frequencies above 10^8 Hz and their decrease at very high frequencies [11]. The description of the phenomena accompanying the domains motion in a polycrystalline ceramic is difficult because many factors influence these processes — external electric fields and mechanical stresses, temperature, grain size, grain boundaries, internal stresses, defects, microcracks, ceramic composition, ageing, ferroelectric fatigue, chemical inhomogeneities, porosity. The existing theories both microscopic and phenomenological are too complicated to apply them in practice. Besides as a rule quantities that cannot be measured in real polycrystalline materials exist in these theories [8, 43].

Changes of losses caused by compositional modifications of ceramics confirm that losses are mainly due to the domain walls motion. Electrical and mechanical losses increase in $Pb(Zr,Ti)O_3$ ceramics with donor additives. Donor additives induce the increase of domain walls mobility, then even small electric fields or mechanical stresses can displace domain walls. Acceptor additives reduce domain walls mobility and electrical and mechanical losses in result [19].

Electrical and mechanical losses increase in result of the increase of driving electric field, vibration velocity, temperature. Ageing causes the decrease of losses, also this effect is due to the domain walls motion [26].

3. Measurements of losses in high fields

Various methods of the measurements of losses are known. In many cases only a mechanical quality factor Q_m is measured. One assumes that electrical losses are negligibly small in relation to mechanical losses. This assumption is satisfied when a transducer is excited by low electrical field to resonant vibrations and its acoustical load is small [6, 9]. The mechanical quality factor can be determined from the width of transducer resonance curve on the 3-dB level, from the characteristic frequencies of the transducer admittance circle or from the measurements of the absolute admittance as a function of frequency [27, 30]. Electrical losses predominate when a transducer is driven by high electric field at frequencies much below its resonance [42]. They can be measured using Schering bridge [7]. However for high-coupling piezoelectric ceramics of a low mechanical Q the measured electrical losses contain an excess portion resulting from the mechanical losses. These additional losses are caused by a piezoelectrically induced quasi-static strain due to low-frequency electric field [34]. The authors of [23] have presented a method of the determination of total losses (sum of electrical and mechanical losses) from measurements of a real part of an electric impedance of a transducer. The measurements were done only for low electric fields, the measured losses were mainly mechanical.

The above-mentioned methods can be applied only in the linear range of the transducer work. In high electric fields, when the vibration velocity is high, the resonance curves deform — decrease of the resonance frequency, asymmetry. The stiffness of the ceramic decreases. The ceramic is not obeying Hooke's law, the material may be classified as a soft spring [25, 36, 42]. When the electric field continues to increase the curves become discontinuous, so-called jump phenomenon appears. The jump phenomenon appears at different frequencies for a upgoing or downgoing frequency sweep during measurements (hysteresis effect). This phenomenon was thoroughly investigated experimentally and theoretically for quartz resonators [10], for a piezoelectric ceramic it was described for the first time by K. NEGISHI [29]. This nonlinear behaviour of the piezoelectric ceramic can be described theoretically extending up to the second order the constitutive piezoelectric equations [13]. The discontinuities are also observed during the measurements of transducer admittance circles or absolute admittance as a function of frequency [39]. The determination of the quality factor from the width of the resonance curve on the 3-dB level gives large error in the case of the asymmmetric curves. For the curves with the discontinuities it makes no sense.

The range of the fields applicable during the measurements is limited by irreversible effects inducing durable changes of ceramic properties. Large mechanical stresses, high electric fields and thermal effects connected with them can cause the ferroelectric fatigue, partial depoling of the ceramic, changes in ageing effects [42].

In recent years the authors from Japan published the series of papers (e.g. [14–16, 37–39]) concerning the applications of the quality factor measurements in the resonance and antiresonance to the determination of the electrical and mechanical losses in wide range of input voltages. For example they obtained for length extensional vibrations of a thin, long and narrow plate poled along the thickness direction [14, 15, 18]:

$$\operatorname{tg}\delta_m = \frac{1}{Q_B}, \tag{3.1}$$

$$\operatorname{tg}\delta_e = \gamma\left(\frac{1}{Q_A} - \frac{\omega_B}{\omega_A}\frac{1}{Q_B}\right), \tag{3.2}$$

$$\gamma = \frac{1-k_{31}^2}{k_{31}^2}\frac{\pi^2}{8}, \tag{3.3}$$

where: Q_A – the quality factor measured at the electrical resonance angular frequency ω_A under mechanically free conditions (or at mechanical resonance angular frequency ω_A giving maximum vibration velocity under electrically short-circuit conditions), Q_B – the quality factor measured at the electrical antiresonance angular frequency ω_B under mechanically free conditions (or at mechanical resonance angular frequency ω_B giving maximum vibration velocity under electrically open-circuit conditions), k_{31} – electromechanical coupling coefficient.

Similar relations were obtained for thickness vibrations of a plate [20].

The authors of the above-mentioned method determined the quality factors using the frequency perturbation method [16, 33]. In this way they could apply their method in the range of the occuring of the resonance curve deformations. However it does not solve the problem of the loss measurements in the range of the occuring of the resonance curves discontinuities. We have tried to solve this problem applying the measurements of voltage ratios in a piezoelectric transformer instead of the measurements of a resonator quality factor.

4. Application of a piezoelectric transformer to loss measurements

Various designs of piezoelectric transformers are known. They have various shapes of individual parts [24]. For the loss measurements we applied a ring-shaped ceramic piezoelectric transformer – Fig. 1. The thickness and width of a ring were small in comparison with its radius. Such a shape has an important advantage - the stress and strain distribution is uniform in the whole ring [3, 42]. The piezoelectric ceramic was poled along the thickness direction. One pair of vacuum evaporated silver electrodes constituted the input of the transformer, the second pair – its output (Fig. 1).

The foundations of the theory of piezoelectric transformers were done by C.A. Rosen [35]. Applying Mason's equivalent circuit Rosen presented an equivalent circuit of a piezoelectric transformer and he derived the equations relating a voltage amplification and components of the equivalent circuit. Rosen's analysis cannot be applied in the case discussed in this paper. Firstly – Rosen did not consider electrical losses assuming that they were negligibly small. Secondly – he analysed the parallepiped – shaped transformer with two mechanically free ends.

Piezoelectric transformers have been already applied for the measurements of various material constants. In [41] piezoelectric constants and an electromechanical coupling coefficient of ceramics operating in the range of high mechanical strains were measured.

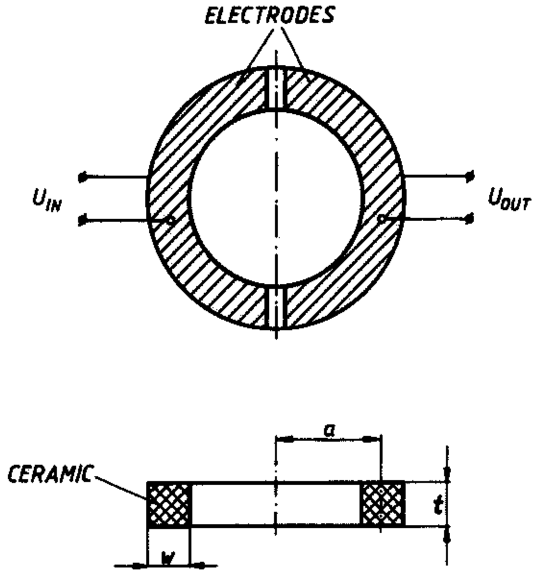


Fig. 1. Ring-shaped piezoelectric transformer.

Rosen's theoretical analysis [35] was applied. In [32] piezoelectric and elastic constants of piezoelectric polymers and composites were measured. In both cases electrical losses were not considered.

The equivalent circuit of the ring-shaped piezoelectric transformer can be derived if each half of the ring is replaced by the KLM equivalent circuit [21, 22] – Fig. 2. We assume that both halves of the ring are identic and we neglect the gaps between the electrodes. In the applied transformers the gap width was equal 1 mm – about 1% of the mean circumference of the ring. In a very strict analysis the gaps between electrodes may be taken into consideration [17] and inserted into the KLM circuit [22].

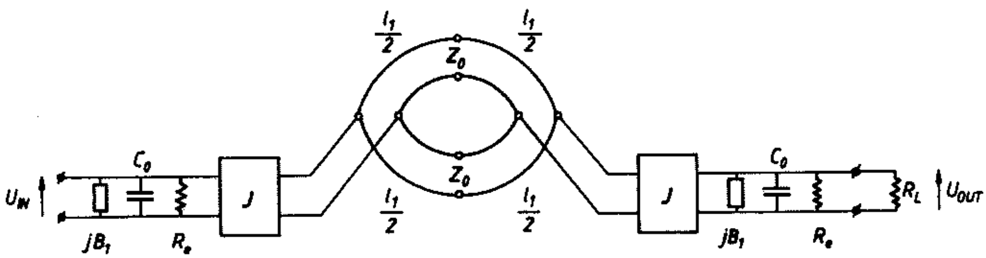


Fig. 2. KLM equivalent circuit of a ring-shaped piezoelectric transformer.

For the ring at the resonance frequency the values of the elements of the circuit presented in Fig. 2 are as follows:

$$l_1 = \frac{\lambda}{2} = \pi a, \tag{4.1}$$

$$B_1 = \frac{1}{Z_0} M^2 \sin\left(\frac{l_1 \omega}{v}\right) = 0, \tag{4.2}$$

$$J = |B_2|, \tag{4.3}$$

$$B_2 = -\frac{\phi}{Z_0} \sin\left(\frac{l_1 \omega}{2v}\right) = -\frac{\phi}{Z_0}, \tag{4.4}$$

$$\phi = \frac{\pi w d_{31}}{s_{11}^E}, \tag{4.5}$$

$$C_0 = \frac{\pi a w \varepsilon_{33}^T}{t} \left(1 - k_{31}^2\right), \tag{4.6}$$

$$Z_0 = \pi w t \sqrt{\frac{\varrho}{s_{11}^E}} \tag{4.7}$$

a, w, t – ring dimensions (cf. Fig. 1), R_L – load resistance, λ – wavelength, Z_0 – characteristic impedance, v – acoustic wave velocity, ω – angular frequency, J – inverter parameter, d_{31} – piezoelectric constant, s_{11}^E – elastic compliance, ε_{33}^T – permittivity, k_{31} – piezoelectric coupling coefficient, ϱ – density. The definitions of the material constant and the subscripts and superscripts are as in [3], B_1, B_2, J, M are defined as in [21, 22].

Electrical losses are represented by the resistance R_e [3], mechanical losses – by the complex propagation constant of the transmission line (central part of the KLM equivalent circuit – Fig. 2):

$$\gamma = \alpha + j\beta, \qquad \alpha = \frac{\pi}{\lambda Q_m}, \qquad \beta = \frac{2\pi}{\lambda}. \tag{4.8}$$

Certain simplifications are necessary for the derivation of the equations describing the voltage ratio in the piezoelectric transformer. We assumed that $\alpha l_1 = \pi/(2Q_m)$ was small (true for typical values of Q_m of PZT-type ceramics) and that the transformer was driven by the source of ac voltage of constant amplitude. Rosen [35] ascertained that such assumptions were admissible in the analysis of piezoelectric ceramic transformers. We assumed also, as we have already mentioned, that both halves of the transformer were identic and we neglected the gaps between the electrodes. The simplifications allow to determine the influence of physical properties of the ceramic on the operation of the transformer and to express the voltage ratio as a function of the values that can be measured.

Voltage ratios in piezoelectric transformers can be calculated applying relations known from the electrical circuit theory and the theory of transmission lines (see Appendix) to the KLM circuit presented in Fig. 2. Under above-mentioned assumptions we have obtained for two limits of the load resistance R_L:

for $R_L \to 0$

$$A_0 = \frac{U_{\mathrm{OUT}}}{U_{\mathrm{IN}}} = \frac{\phi^2 Q_m R_L}{\pi Z_0 + \phi^2 Q_m R_L}; \tag{4.9}$$

for $R_L \to \infty$

$$A_\infty = \frac{U_{\mathrm{OUT}}}{U_{\mathrm{IN}}} = \frac{\phi^2 Q_m X_e R_e}{\phi^2 Q_m X_e^2 + \pi Z_0 R_e}. \tag{4.10}$$

Thus

$$\operatorname{tg}\delta_m = \frac{\phi^2 R_L(1-A_0)}{\pi Z_0 A_0} \tag{4.11}$$

and

$$\begin{aligned} \operatorname{tg}\delta_e &= \left(1-k_{31}^2\right)\frac{\phi^2 Q_m X_e - A_\infty \pi Z_0}{\phi^2 Q_m X_e A_\infty}\,, \\ X_e &= \frac{1}{\omega C_0}\,. \end{aligned} \tag{4.12}$$

For the determination of the losses it is not sufficient to measure the voltage ratios because in the range of high electric fields the piezoelectric, dielectric and elastic constants change as a function of the field intensity. This effects are due to the domain structure of the ceramics. The domain walls motion changes values of the elastic compliance, the dielectric and piezoelectric constants and the electromechanical coupling coefficient [1, 19, 42].

5. Experimental results

Figure 3 presents the examples of the measured resonance curves of the ceramic rings, Fig. 3 a – input voltage $U_{\mathrm{IN}} = 1\,\mathrm{V}$, Fig. 3 b – $U_{\mathrm{IN}} = 40\,\mathrm{V}$. In Fig. 3 b nonlinear effects described in Sec. 3 (jump effects with hysteresis) are visible. The resonance curve in Fig. 3 a is already slightly asymmetric in spite of low input voltage. The ring was made of hard PZT-type ceramic. Figure 4 presents the output voltage of this transformer as a function of its input voltage. One can see that the output voltage does not increase proportionally to the increase of the input voltage even for small values of U_{IN}.

The values of the ceramic parameters necessary for the calculation of losses were measured. Figure 5 presents the examples of the measured values of s_{11}^E, k_{31}, d_{31} and ε_{33}^T as a function of the input electric field for the same ring as above.The changes of the ceramic parameters begin in the same range of the input electric field as the nonlinear effects visible in the $U_{\mathrm{OUT}} = f(U_{\mathrm{IN}})$ curve. These results are similar to the results presented earlier, e.g. in [37, 41]. The density ϱ of this ceramic was equal $7.5 \cdot 10^3\,\mathrm{kg/m^3}$.

Figure 6 presents the examples of the calculated (using the equations given in Sec. 4) dependences of the electrical and mechanical losses on the input electric field. Results presented in Fig. 6 a were obtained for the ceramic ring with the parameters presented in Fig. 5.

We measured also the losses in the ring-shaped transformers made of the ceramic of low quality factor (soft PZT-type ceramic). For these rings the discontinuities of the resonance curves did not appear in the whole measurement range. Therefore it was possible to measure the losses applying Hirose's method [14] described in Sec. 3 and to compare the results obtained using two methods. Figure 7 presents the quality factor Q_m as a function of input electric field for two kinds of ceramics: I – the ceramics of high quality factors (the results of their measurements are presented in Fig. 6), II – the ceramics of low quality factors used for the comparative measurements. The quality factors Q_m were determined as the reciprocals of $\operatorname{tg}\delta_m$ calculated using Eq. (4.11). The values of

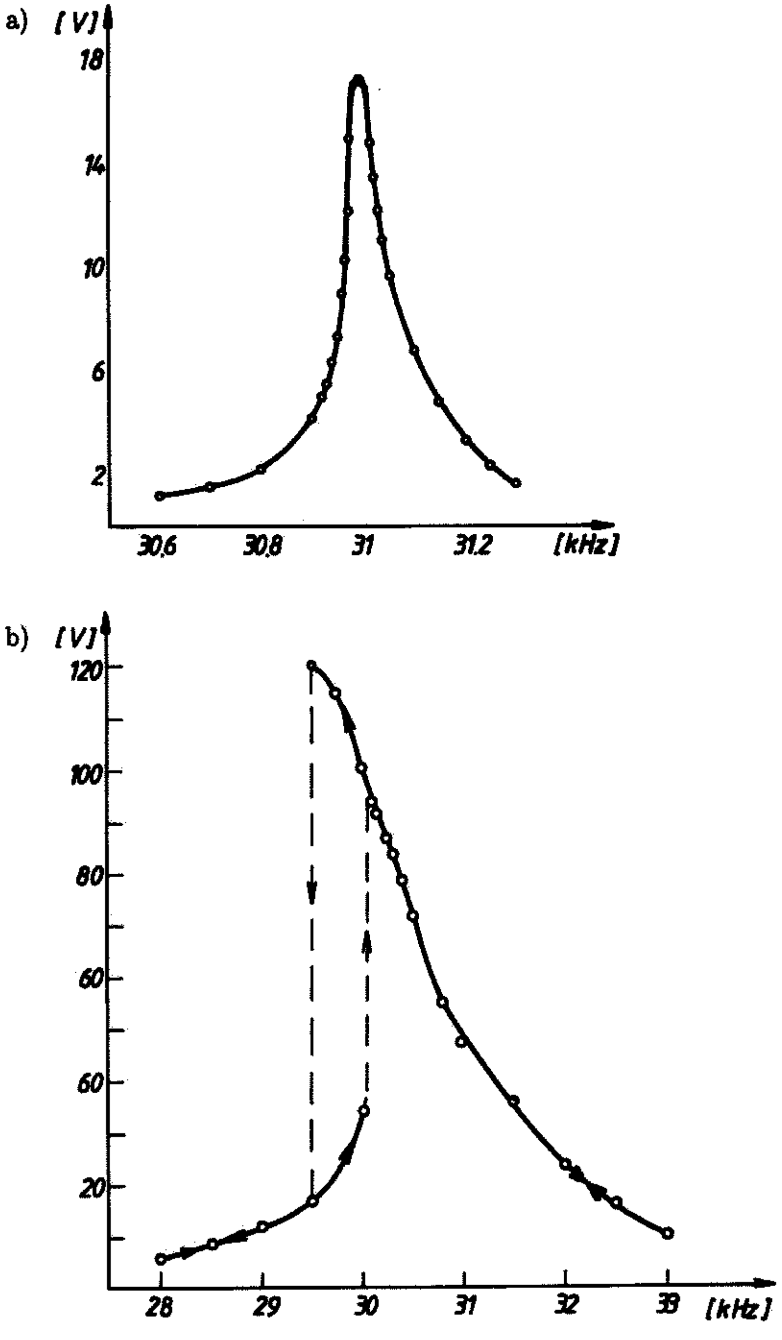


Fig. 3. Resonance curves of the ceramic ring transformer, external diameter $D_{ext} = 38$ mm, internal diameter $D_{int} = 30$ mm, thickness $t = 1$ mm, hard PZT-type ceramic, a) input voltage $U_{IN} = 1$ V, b) $U_{IN} = 40$ V. The arrows show the direction of frequency change during the measurements.

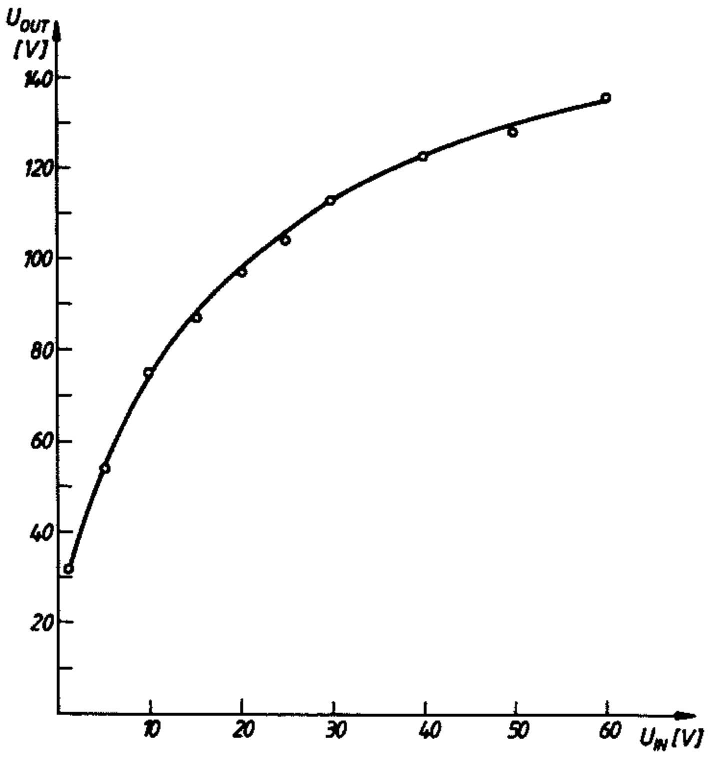
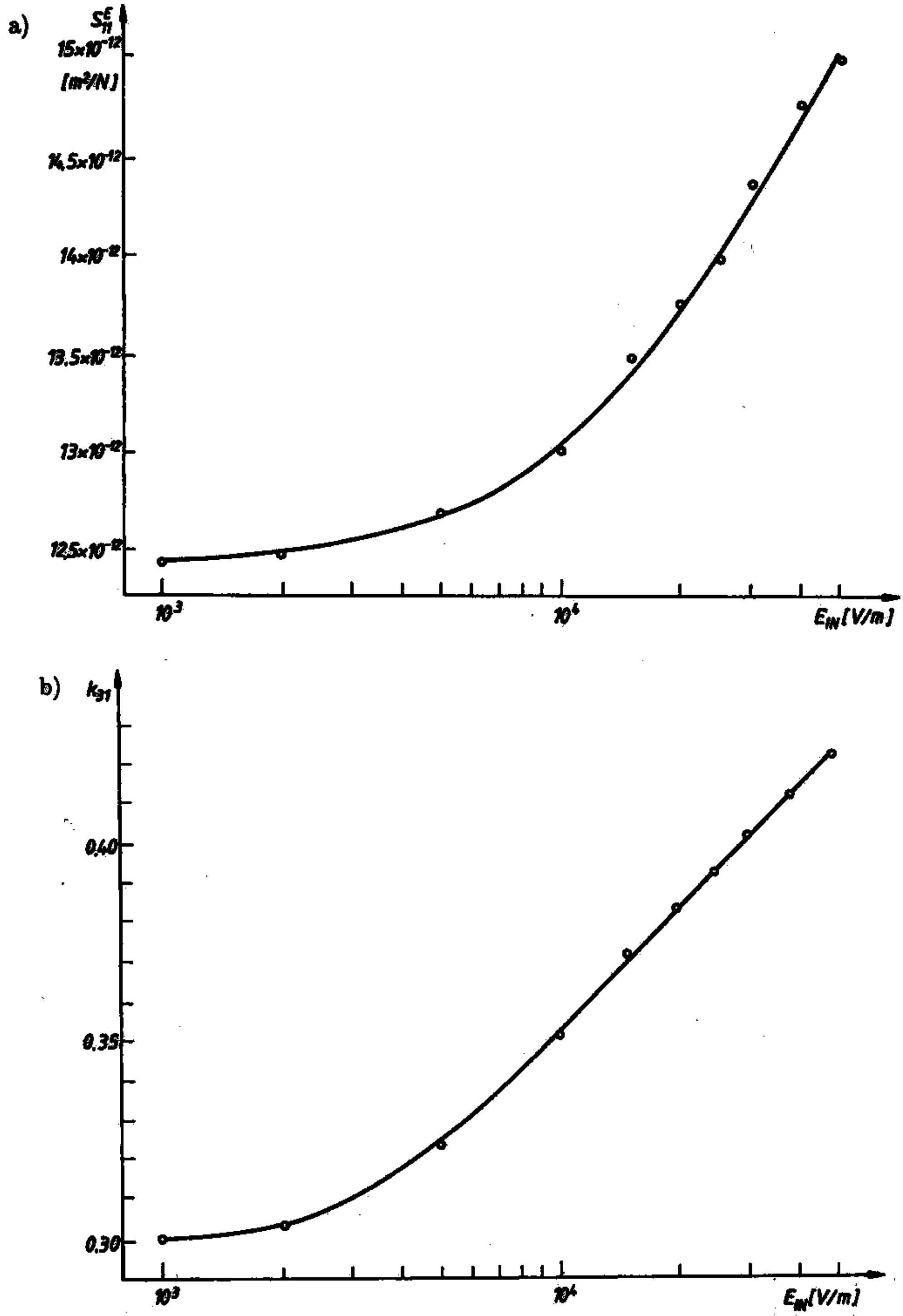


Fig. 4. The output voltage of the ring piezoelectric transformer versus its input voltage, dimensions of the ring as in Fig. 3.

the ceramic parameters were measured as previously. The density of the ceramics II was $\varrho = 7.4 \cdot 10^3\,\mathrm{kg/m^3}$. Figure 8 presents the values of losses determined applying two methods. One can see that the results agree qualitatively. The values of the losses measured using Hirose's method are lower. It may result from the fact that the quality factors Q_A and Q_B have been determined from the widths of resonance curves on the 3 dB-level instead of the perturbation method described in [33]. The curves were deformed, specially at higher input electric fields. The deformations may cause the apparent increase of the measured quality factors. For low input electric fields the differences between the obtained results are small.

During all measurements the rings were air-cooled using a propeller-fan. The losses cause the heating of the ceramic specially in the range of high input electric fields. The temperature increase causes the decrease of quality factors and changes of ceramic parameters [39]. The ageing and ferroelectric fatigue causes changes of the losses with time [26, 31]. The effect of heat generation impedes the measurements of the ceramic properties in high fields. It is possible to avoid this effect applying special, fully computerized, measuring circuits enabling to perform measurement in less than 500 ms [39].

The measurements were done in the frequency range 30 – 44 kHz according to the ring dimensions determining its resonance frequency.


a)
S_{11}^{E}
15×10^{-12}
[m²/N]
14.5×10^{-12}
14×10^{-12}
13.5×10^{-12}
13×10^{-12}
12.5×10^{-12}
10^3
10^4
E_{IN} [V/m]
b)
k_{31}
0,40
0,35
0,30
10^3
10^4
E_{IN} [V/m]

[Fig. 5a, b]

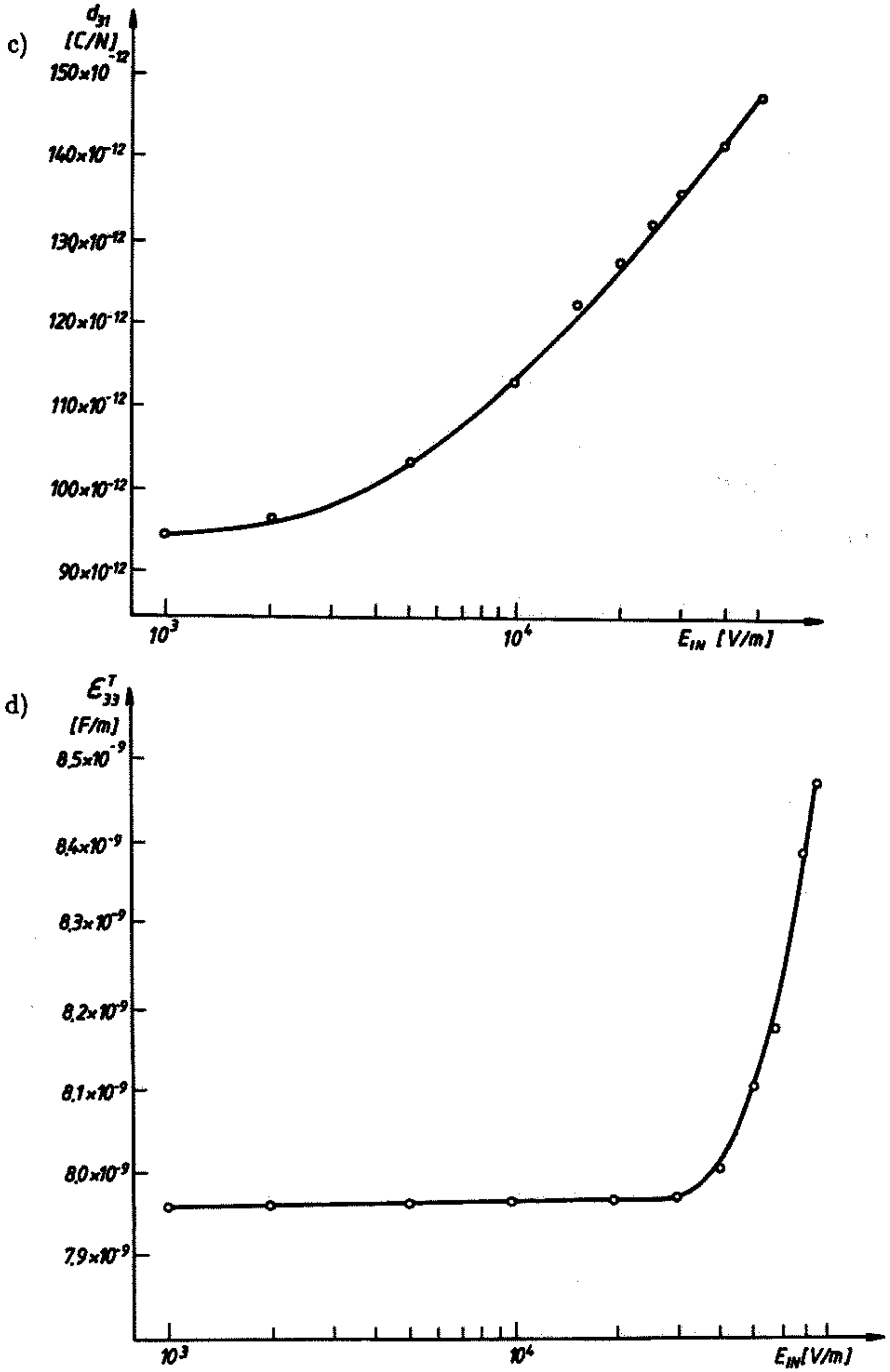


Fig. 5. Examples of measured dependences of the ceramic parameters (hard PZT-type ceramic) on the input electric field, the ceramic ring as in Figs. 3 and 4, a) elastic compliance s_{11}^{E}, b) electromechanical coupling coefficient k_{31}, c) piezoelectric constant d_{31}, d) permittivity.

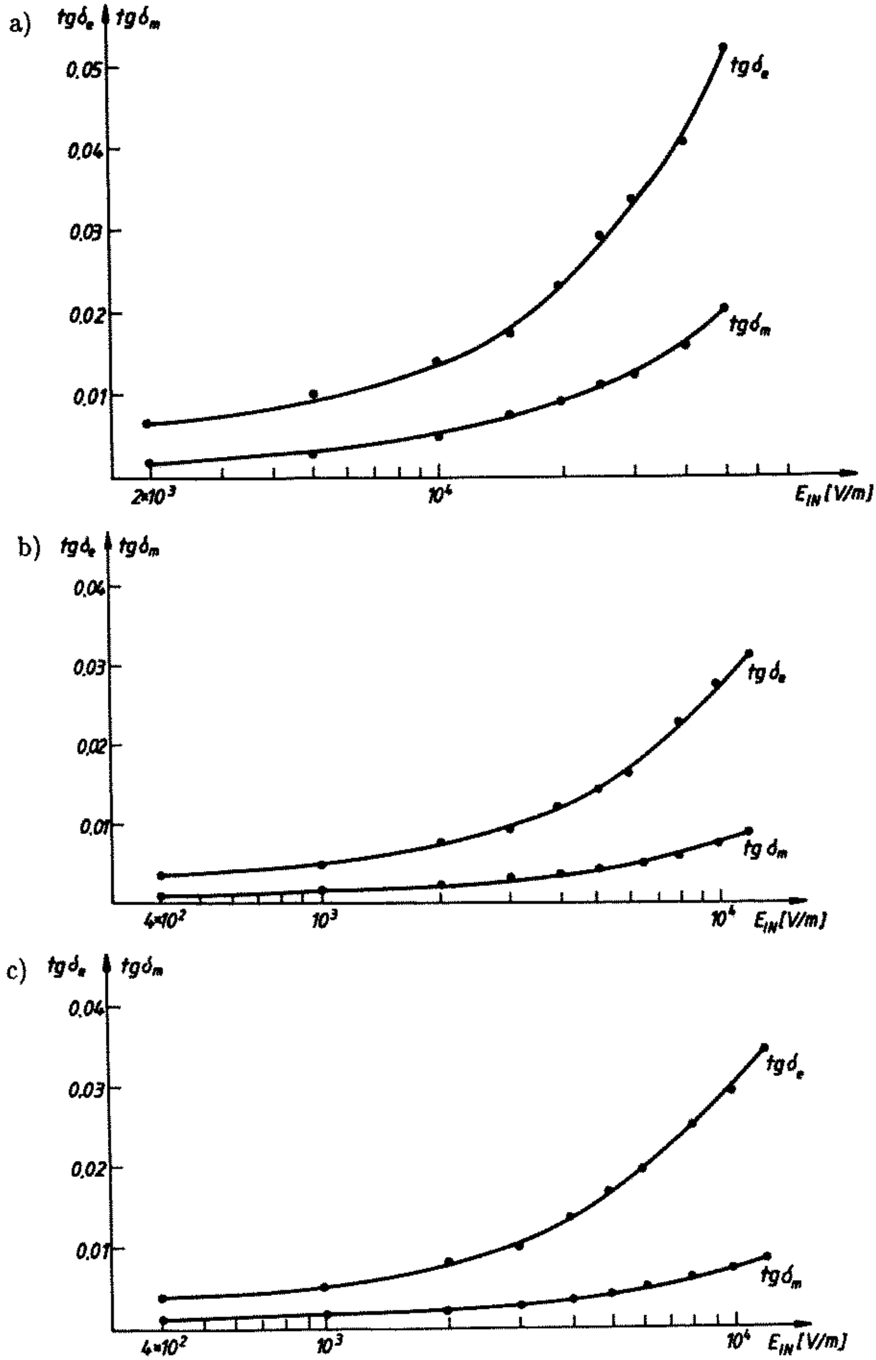


Fig. 6. Electrical (tg δ_e) and mechanical (tg δ_m) losses as a function of input electric field, calculated using Eqs. (4.11) and (4.12) for three piezoelectric ceramic transformers, a) $D_{\text{ext}} = 38$ mm, $D_{\text{int}} = 30$ mm, $t = 1$ mm, hard PZT-type ceramic, as in Figs. 3–5, b) $D_{\text{ext}} = 38$ mm, $D_{\text{int}} = 28$ mm, $t = 5$ mm, hard PZT-type ceramic, c) $D_{\text{ext}} = 38$ mm, $D_{\text{int}} = 24$ mm, $t = 5$ mm, hard PZT-type ceramic.

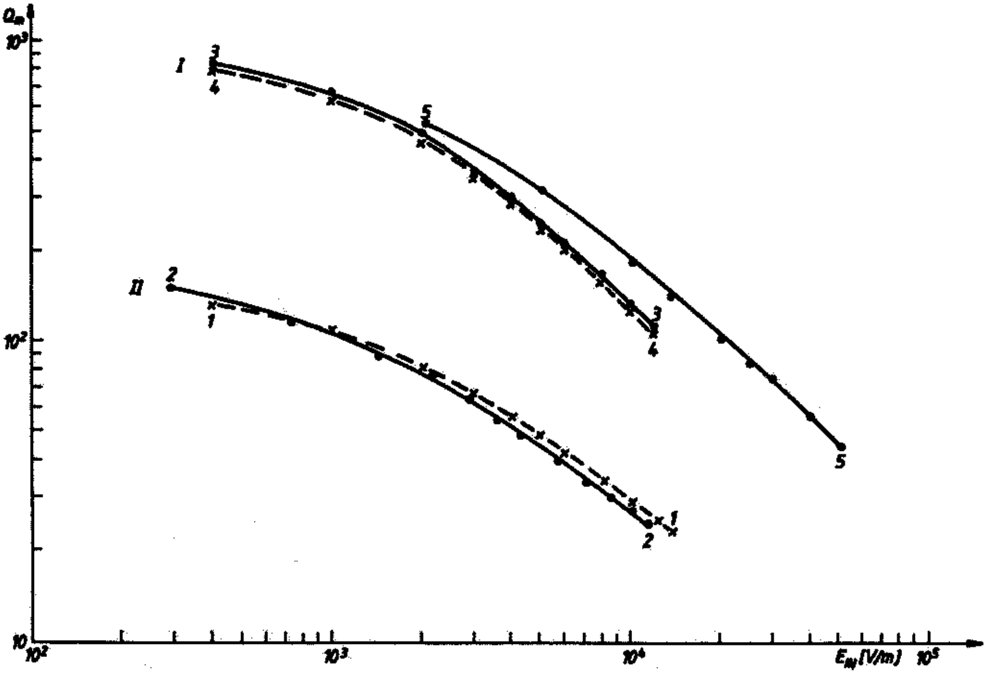
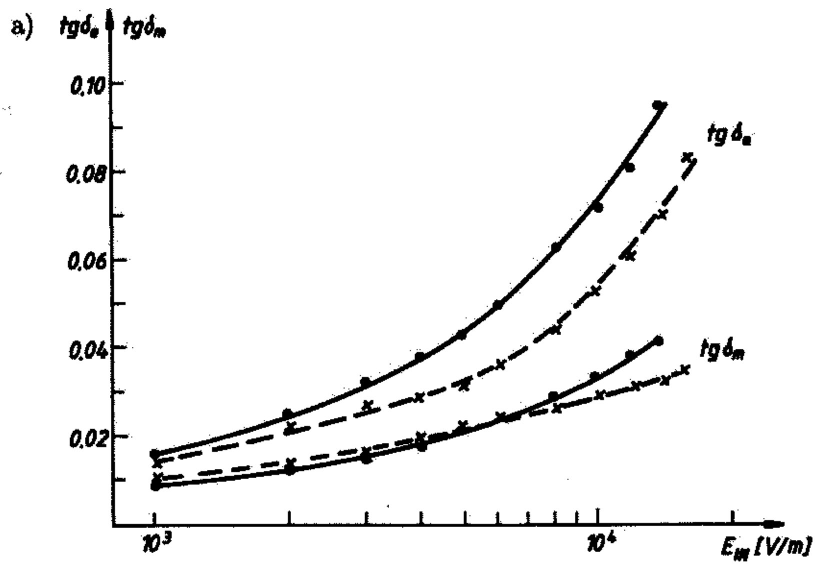


Fig. 7. Mechanical quality factors Q_m of several investigated ceramics as a function of input electric field, *1* – $D_{ext} = 30$ mm, $D_{int} = 16$ mm, $t = 5$ mm, soft PZT-type ceramic, *2* – $D_{ext} = 30$ mm, $D_{int} = 16$ mm, $t = 7$ mm, soft PZT-type ceramic, *3* – ceramic ring as in Fig. 6 c, *4* – ceramic ring as in Fig. 6 b, *5* – ceramic ring as in Fig. 6 a.



[Fig. 8a]

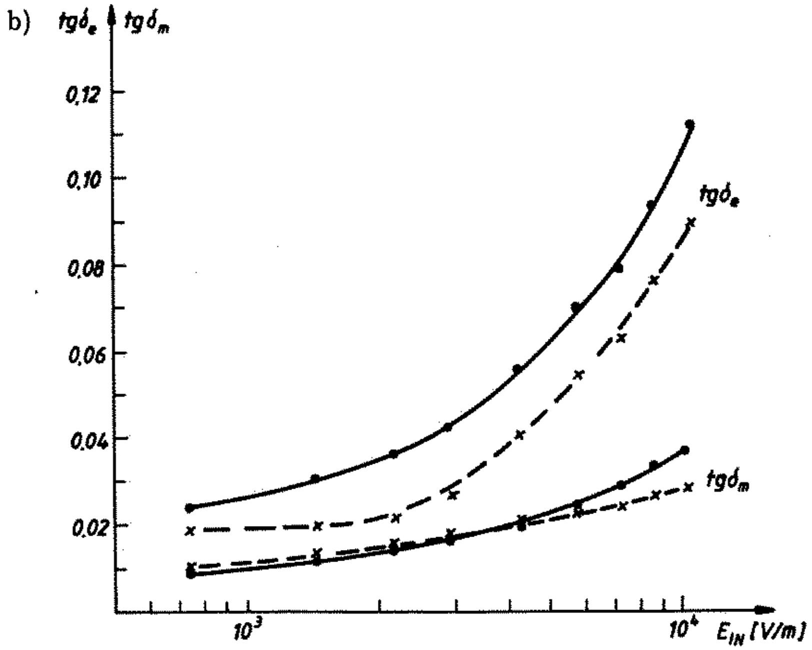


Fig. 8. Comparison of the results obtained for two piezoelectric ceramic transformers applying two methods: solid line – piezoelectric transformer method presented in the paper, dashed line – Hirose's method [14] described in Sec. 3, a) ceramic ring no. 1 in Fig. 7, b) ceramic ring no. 2 in Fig. 7.

6. Conclusions

The results obtained applying the method proposed by the author are qualitatively consistent with the results obtained using the different methods and with the theoretical descriptions of the loss effects. The method presented in the paper does not require to measure the quality factor. The measurements of the quality factor are difficult or even impossible for the ceramics driven by high fields. In above described method the losses are determined from the measurements of the voltage ratios for two kinds of the loading of the piezoelectric transformer. However, the additional measurements of the material constants as a function of the input electric field are necessary. The described method has also certain disadvantage i.e., the sample in the form of the ring transformer is necessary for the loss measurements.

The equations determining the losses have been derived using the simplificating assumptions similarly as in all cited methods of loss analysis. Otherwise the derived equations could not be applied in practice. The simplifications are necessary to obtain the relations permitting to determine the influence of physical parameters of investigated ceramic on the transformer operation and to express the voltage ratios as a function of the mesurable quantities. Further experiments with transformers made of various ceramics with different properties are necessary to estimate quantitatively the accuracy of the described method.

Appendix. Application of KLM equivalent circuit for the calculation of losses

The KLM equivalent circuit is presented in Fig. 2. We calculate the ratios of the output voltage U_{OUT} and the input voltage U_{IN} for two limits of the transformer loading: $R_L \to 0$ and $R_L \to \infty$.

The output current of the inverter J:

$$I = -jB_2U, \tag{A.1}$$

where: U – output voltage of the inverter, B_2 – inverter parameter. The ideal admittance inverter J has also the property that when an admittance Y is connected to one port, the input admittance at the other port is J^2/Y, where $J = |B_2|$ [22].

The voltage at the output of the transmission line:

$$U_2 = \frac{I_1}{\frac{1}{Z_2}\mathrm{ch}\gamma x + \frac{1}{Z_0}\mathrm{sh}\gamma x} \tag{A.2}$$

and

$$U_2 = \frac{U_1}{\mathrm{ch}\gamma x + \frac{Z_0}{Z_2}\mathrm{sh}\gamma x}, \tag{A.3}$$

where: I_1, U_1 – current and voltage at the input of the transmission line, Z_2 – loading impedance at the end of the transmission line, Z_0 – characteristic impedance of the transmission line, $\gamma = \alpha + j\beta$ – propagation constant of the transmission line described by Eq. (4.8), x – length of the transmission line.

The input impedance of the transmission line:

$$Z_{IN} = \frac{Z_2 + Z_0 \mathrm{th}\,\gamma x}{1 + \frac{Z_2}{Z_0}\mathrm{th}\,\gamma x}. \tag{A.4}$$

If αx is small (cf. Sec. 4) then:

$$U_2 = \frac{I_1}{\frac{1}{Z_2}(\cos\beta x + j\alpha x \sin\beta x) + \frac{1}{Z_0}(\alpha x \cos\beta x + j\sin\beta x)}. \tag{A.5}$$

In our case Eqs.(A.2) – (A.5) can be considerably simplified because $l_1 = \lambda/2$ in the equivalent circuit in Fig. 2 and the characteristic impedances are identic in all sections of the transmission line. The values of the elements of the equivalent circuit are described by Eqs. (4.1)–(4.7).

Similarly as Rosen we assume that the transformer is driven by the source of ac voltage of constant amplitude. This simplificating assumption allows to neglect the influence of the internal resistance of the voltage source [35].

Using standard relations for the electric circuits and transmission lines [2, 35] we obtain for $R_L \to \infty$:

$$A_\infty = \frac{U_{OUT}}{U_{IN}} = \frac{\phi^2 Q_m \left(R_e^2 + X_e^2\right)^{1/2} X_e R_e}{\pi Z_0 \left(R_e^2 + X_e^2\right) + \phi^2 Q_m R_e X_e^2}. \tag{A.6}$$

Since $R_e^2 \gg X_e^2$ then Eq. (A.6) can be simplified:

$$A_\infty = \frac{\phi^2 Q_m X_e R_e}{\pi Z_0 R_e + \phi^2 Q_m X_e^2} \tag{A.7}$$

then

$$R_e = \frac{\phi^2 Q_m X_e^2 A_\infty}{\phi^2 Q_m X_e - A_\infty \pi Z_0} \tag{A.8}$$

as [3]:

$$\operatorname{tg}\delta_e = \frac{(1-k_{31}^2)X_e}{R_e} \tag{A.9}$$

therefore:

$$\operatorname{tg}\delta_e = \frac{(1-k_{31}^2)(\phi^2 Q_m X_e - \pi A_\infty Z_0)}{\phi^2 Q_m X_e A_\infty} \tag{A.10}$$

similarly for $R_L \to 0$ we obtain:

$$A_0 = \frac{U_{\mathrm{OUT}}}{U_{\mathrm{IN}}} = \frac{\phi^2 Q_m R_L}{\pi Z_0 + \phi^2 Q_m R_L} \tag{A.11}$$

then Q_m necessary to calculate (A.10) is:

$$Q_m = \frac{\pi Z_0 A_0}{\phi^2 R_L (1 - A_0)} \tag{A.12}$$

and

$$\operatorname{tg}\delta_m = \frac{1}{Q_m} = \frac{\phi^2 R_L (1 - A_0)}{\pi Z_0 A_0}. \tag{A.13}$$

The measurements were done for two values of the transformer loading: a) corresponding to $R_L \to 0$, $R_L = 11\,\Omega \ll R_e$ and b) corresponding to $R_L \to \infty$, in this case the transformer output was loaded by the internal resistance of the voltmeter $R_i \gg R_e$.

The same results of the calculations may be obtained using Mason's equivalent circuit, but the calculations are more time-consuming.

References

[1] G. Arlt, *Domain contributions to piezoelectricity in ceramics*, 1990 IEEE Ultrasonics Symposium Proceedings, 733–742.

[2] G.I. Atabiekow, *Theory of linear electric circuits* [in Russian], Sowietskoje Radio, Moskwa 1960.

[3] D.A. Berlincourt, D.R. Curran and H. Jaffe, *Piezoelectric and piezomagnetic materials*, Physical Acoustics, Vol. IA, W.P. Mason [Ed.], Academic Press, New York 1964, Ch. 3.

[4] R. Briot, P. Gonnard and M. Troccaz, *Comparison of PZT ceramics based on the frequency dependence of conductance and a grain boundary-domain walls model*, Ferroelectrics, **127**, 155–160 (1992).

[5] J.L. Butler, K.D. Rolt and F.A. Tito, *Piezoelectric ceramic mechanical and electrical stress study*, J. Acoust. Soc. Am., **96**, 3, 1914–1917 (1994).

[6] P. Champ and P. Gonnard, *Characterization of high power piezoelectric ceramics under high mechanical driving levels*, 1er Congres Français d'Acoustique 1990, 621–624.

[7] A. Chełkowski, *Physics of dielectrics* [in Polish], PWN, Warszawa 1993, Ch. 5.

[8] D. DAMJANOVIC, M. DEMARTIN, H.S. SCHULMAN, M. TESTORF and N. SETTER, *Instabilities in the piezoelectric properties of ferroelectric ceramics*, Sensors and Actuators, **A53**, 353–360 (1996).

[9] L. EYRAUD, P. GONNARD and M. TROCCAZ, *Piezoceramics and their characterization for high power ultrasound*, Ultrasonics International 1989 Conference Proceedings, 295–300.

[10] J.J. GAGNEPAIN and R. BESSON, *Nonlinear effects in piezoelectric quartz crystals*, Physical Acoustics, Vol. XI, W.P. MASON and R.N. THURSTON [Eds.], Academic Press, New York 1975, Ch. 5.

[11] J.O. GENTNER, P. GERTHSEN, N.A. SCHMIDT and R.E. SEND, *Dielectric losses in ferroelectric ceramics produced by domain-wall motion*, J. Appl. Phys., **49**, 8, 4485–4489 (1978).

[12] P. GERTHSEN, K.H. HÄRDTL and N.A. SCHMIDT, *Correlation of mechanical and electrical losses in ferroelectric ceramics*, J. Appl. Phys., **51**, 2, 1131–1134 (1980).

[13] D. GUYOMAR, N. AURELLE and L. EYRAUD, *Piezoelectric ceramics nonlinear behavior. Application to Langevin transducer*, J. Phys. III, **7**, 6, 1197–1208 (1997).

[14] S. HIROSE, M. AOYAGI and Y. TOMIKAWA, *Dielectric loss in a piezoelectric ceramic transducer under high power generation; increase of dielectric loss and its influence on transducer efficiency*, Jpn. J. Appl. Phys., Part 1, **32**, 5B, 2418–2421 (1993).

[15] S. HIROSE, *New method for measuring mechanical vibration loss and dielectric loss of piezoelectric transducer under high-power excitation*, Jpn. J. Appl. Phys., Part 1, **33**, 5B, 2945–2948 (1994).

[16] S. HIROSE, M. AOYAGI, Y. TOMIKAWA, S. TAKAHASHI and K. UCHINO, *High power characteristics at antiresonance frequency of piezoelectric transducers*, Ultrasonics, **34**, 2–5, 213–217 (1996).

[17] R. HOLLAND, *The equivalent circuit of an N-electrode piezoelectric bar*, Proc. IEEE, **54**, 7, 968–975 (1966).

[18] T. IKEDA, *Fundamentals of piezoelectricity*, Oxford University Press, 1990, Ch. 6.

[19] B. JAFFE, W.R. COOK and H. JAFFE, *Piezoelectric ceramics*, Academic Press, New York 1971, Ch. 7.

[20] U. KIEFER, TH. FINKLER and K.-D. BECKER, *Piezoelektrische Ultraschallwandler modelliert. Simulation mechanischer und elektrischer Verluste*, Materialprüfung, **36**, 5, 192–195 (1994).

[21] R. KRIMHOLTZ, D.A. LEEDOM and G.L. MATTHAEI, *New equivalent circuits for elementary piezoelectric transducers*, Electron. Lett., **6**, 13, 398–399 (1970).

[22] D.A. LEEDOM, R. KRIMHOLTZ and G.L. MATTHAEI, *Equivalent circuits for transducers having arbitrary even- or odd-symmetry piezoelectric excitation*, IEEE Trans., **SU-18**, 3, 128–141 (1971).

[23] M. LETHIECQ, L.P. TRAN-HUU-HUE, F. PATAT and L. POURCELOT, *Measurement of losses in five piezoelectric ceramics*, IEEE Trans., **UFFC-40**, 3, 232–236 (1993).

[24] W.W. ŁAWRINIENKO, *Piezoelectric transformers* [in Russian], Energia, Moskwa 1975.

[25] I. MALECKI, *Physical foundations of technical acoustics*, Pergamon Press, Oxford 1969, Ch. 14.

[26] W.P. MASON, *Aging of the properties of barium titanate and related ferroelectric ceramics*, J. Acoust. Soc. Amer., **27**, 1, 73–85 (1955).

[27] F.R. MONTERO DE ESPINOZA, J.L. SAN EMETERIO and P.T. SANZ, *Summary of the measurements methods of Q_m for piezoelectric materials*, Ferroelectrics, **128**, 1–4, 61–66 (1992).

[28] K. NAGATA, J. THONGMENG and K. KATO, *Evaluation of the reliability of piezoelectric ceramic transformers*, Jpn. J. Appl. Phys., Part 1, **36**, 9B, 6103–6105 (1997).

[29] K. NEGISHI, *Jump phenomenon in resonance curve of ferroelectric ceramics*, J. Phys. Soc. Jpn., **15**, 3, 534 (1960).

[30] W. PAJEWSKI and M. SZALEWSKI, *Methods of the measurements of the quality factor of piezoelectric resonators with high electrical and mechanical losses*, Arch. Acoust., **20**, 4, 373–385 (1995).

[31] W. PAN, S. SUN and P. FUIERER, *Effects of ferroelectric switching on the dielectric and ferroelectric properties in lead zirconate titanate ceramics and their modelling*, J. Appl. Phys., **74**, 2, 1256–1264 (1993).

[32] D.L. Rastorgujew, *Measurements of electromechanical characteristics of piezoelectric materials using piezoelectric transformer* [in Russian], Pribory i Technika Eksperimenta, 1, 155–158 (1997).

[33] S. Saito, R. Aoyagi and H. Shimizu, *A method for automatic measurement of ultrasonic transducer constants*, J. Acoust. Soc. Jpn., **33**, 10, 540–548 (1977).

[34] S. Saito and H. Shimizu, *Influence of elastic loss on measured dielectric-loss-angle of high-coupling piezoelectric material*, Jpn. J. Appl. Phys., **24**, suppl. 3, 145–147 (1985).

[35] *Solid state magnetic and dielectric devices*, H.W. Katz [Ed.], John Wiley and Sons, New York 1959, ch. 5.

[36] J.J. Stoker, *Nonlinear vibrations in mechanical and electrical systems*, Interscience Publishers, New York 1950, ch. 2 and 4.

[37] S. Takahashi, S. Hirose, K. Uchino and K.Y. Oh, *Electromechanical characteristics of lead-zirconate-titanate ceramics under vibration-level change*, Proc. 9th IEEE Int. Symp. Appl. Ferroel., 377–382 (1994).

[38] S. Takahashi, Y. Sasaki, S. Hirose and K. Uchino, *Electromechanical properties of* $PbZrO_3$-$PbTiO_3$-$Pb(Mn_{1/3}Sb_{2/3})O_3$ *ceramics under vibration level change*, Mat. Res. Soc. Symp. Proc., **360**, 305–310 (1995).

[39] S. Tashiro, M. Ikehiro and H. Igarashi, *Influence of temperature rise and vibration level on electromechanical properties of high-power piezoelectric ceramics*, Jpn. J. Appl. Phys., Part 1, **36**, 5B, 3004–3009 (1997).

[40] K. Uchino, *Materials issues in design and performance of piezoelectric actuators: an overview*, Acta Mater., **46**, 11, 3745–3753 (1998).

[41] Vo Duy Dan, *Investigation of piezoelectric ceramic in the range of high mechanical strains* [in Polish], Doctor Thesis, IFTR, Warsaw 1985.

[42] R.S. Woollett and C.L. Le Blanc, *Ferroelectric nonlinearities in transducer ceramics*, IEEE Trans., SU-**20**, 1, 24–31 (1973).

[43] X.D. Zhang and C.A. Rogers, *A macroscopic phenonenological formulation for coupled electromechanical effects in piezoelectricity*, J. Intell. Mater. Syst. Struct., **4**, 3, 307–316 (1993).