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PROBLEM OF EXCESS ATTENUATION IN ACOUSTIC MEASUREMENTS OF GAS BUBBLE CONCENTRATION IN THE SEA

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This study is aimed at describing the possible errors in the acoustie estimation of gas buble concentration in the sea made by the assumption of the first order scattering approximation, i.e. by neglecting the interaction between individual bubbles enclosed in the water column. An experimental method of counting the bubbles is sketched, results are presented and two different ways of taking into account the excess attenuation in bubbly medium are used for correcting the bubble number, the results being compared. The problem of bubble swarms in the vicinity of the transmitter is also considered.

1. Introduction

Hydroacoustic methods based on the phenomenon of backscattering of the acoustic energy in the sea water are widely applied in the oceanic investigations, especially in estimating plankton and fish biomass. It is usually assumed that the number of scattering objects is small enough to enable the neglecting of the interaction between scatterers, what is connected with the neglecting of the excess attenuation and multiple scattering. In the majority of cases such assumption is valid, but this problem should be considered in each individual case. Usually the determination of gas bubble populations in the sea water is also based on the first order scattering approximation and this simplification can produce some significant errors.

2. Sound backscattering as a method of counting bubbles

One of the methods of determination of gas bubble density in the sea water is based on the interrelation between the measurable volume backscattering strength and the number of resonating bubbles [1], [3]. Sound pulse coming upon the single bubble makes it radiate the secondary spherical wave which returns to the receiver as the backscattered sound. Because bubbles are randomly distributed in the water column, scattering is assumed to be incoherent — the total intensity of backscattered sound is a sum of intensities originating from the individual bubbles. If the number of bubbles in the medium is small (i.e. the average distance between them is large in comparison with their size), it can be assumed that backscattered wave hits immediately the receiver without being scattered and attenuated by other bubbles. Then the sonar equation for volume backscattering has a well-known simple form [1]:

$$I_{bs} = \frac{I_0}{(z-z_0)^2} S_{bs} \frac{c\tau\psi_D}{2}, \tag{1}$$

where I_{bs} — intensity of the backscattered signal (echo intensity), I_0 — intensity of the emitted signal on the acoustic axis at a distance 1 m from the source, $c\tau$ — spacial length of a pulse, ψ_D — integrated beam width factor, z_0 — depth of transducer, z — actually penetrated depth, S_{bs} — volume backscattering coefficient.

This formula is traditionally used for the determination of the number n of scattering objects, provided that the backscattering cross section σ_{bs} of the individual scatterer is known, $S_{bs} = n\sigma_{bs}$.

If, on the other hand, the number of bubbles is so large that multiple scattering can not be neglected, the sonar equation must include the attenuation factor:

$$I_{bs} = \frac{I_0}{(z-z_0)^2} S_{bs} \frac{c\tau\psi_D}{2} \exp\left(-2\int_{z_0}^{z} S_e(z)\,dz\right), \tag{2}$$

where S_e is the volume extinction (scattering + absorption) coefficient.

In bubbly medium the backscattering coefficient S_{bs} and the extinction coefficient S_e are expressed in the following way [1]:

$$S_{bs} = \frac{\pi}{2} \frac{n(a_R, z)\, a_R^3(z)}{\delta_R}, \tag{3}$$

$$S_e = \frac{2\pi^2 n(a_R, z)\, a_R^3(z)}{\delta_{rR}} = \frac{2\pi^2 n(a_R, z)\, a_R^2(z)}{k}, \tag{4}$$

where a_R — resonant bubble radius (for air bubbles in water a_R [μm] $= 3280(1 + 0.1z)^{1/2}/f$[kHz]), f — incident sound frequency, δ_R — resonant damping constant of individual bubble, $\delta_{rR} = ka_R$ — resonant damping constant due to reradiation only, k — wave number, $n(a_R, z)$ — number of resonant bubbles of radius between a_R and $a_R + 1$ μm in 1 m³ of water at the depth z.

It can be seen (Eq. 1) that the measurement of the echo intensity allows to determine the backscattering coefficient S_{bs} as a function of depth. Subsequently, knowing the values of S_{bs}, we can deduce the values of $n(a_R, z)$ calculating previously values of $a_R(z)$ and $\delta_R(z)$ (Eq. 3). Damping constant δ can be easily evaluated; it depends on the frequency of incident sound, bubble radius, depth and some physico-chemical gas and water constants [1].

3. Results of experiment

Our measurements of gas bubble concentration were carried out in the subsurface layer of the coastal zone of the Baltic Sea [3]. By sounding at frequencies 80, 63, 50 and 40 kHz we could infer vertical profiles of the number of bubbles with radii 40 – 100 μm. Evaluation of gas bubble density was based on the equations (1) and (3) according to the assumption of single scattering. The data recorded in the shallowest layer $[z_0, z_1]$ were useless because of ringing in the trasmitting-receiving transducer; the data from the greater depths ($z > z_N$) were also useless, because the noise level was reached (at different depth z_N for different frequencies), so only the midrange data from the interval $[z_1, z_N]$ were considered. Fig. 1 shows the obtained number of bubbles resonating at

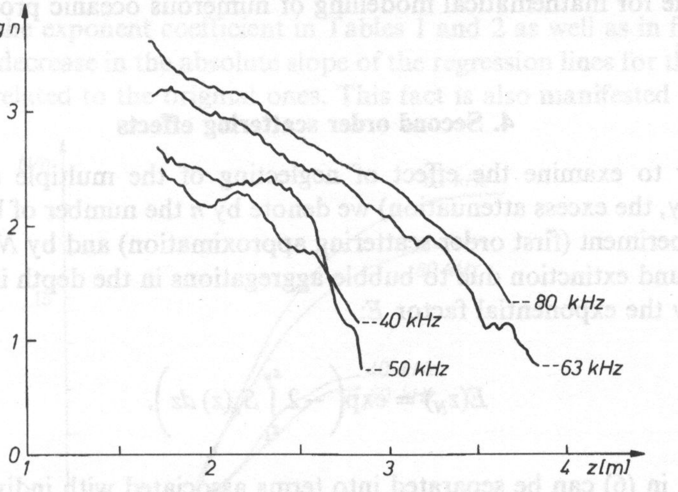

Fig. 1. Vertical profiles of the concentration of gas bubbles resonating at various frequencies used in experiment

different frequencies as a function of depth. It displays the increase of bubble density when the bubble size decreases (bubble radius is inversely proportional to the incident sound frequency). Taking values $n(z)$ for the given frequency and chosen depth levels one can find the dependence of bubble number on the depth of their occurrence $n(z) = A\exp(Bz)$. The coefficients A and B depend on the productivity of bubble source and on the intensity of turbulent mixing in the subsurface water layer connected with the dynamical meteorological conditions. By the linear regression method we have found the following values of the coefficients A and B.

Table 1

f [kHz]	A	B
40	$2.4 \cdot 10^4$	-2.40
50	$1.2 \cdot 10^5$	-2.95
63	$2.0 \cdot 10^5$	-2.57
80	$2.1 \cdot 10^5$	-2.34

These regression lines are depicted as curves 1 in Fig. 5.

Choosing the values of $n(f,z)$ for particular frequencies and selected depth levels, it is possible to determine a functional dependence of the number of bubbles both on the depth and on the bubble size. Approximation by a linear two-variables regression method gave the following relationship:

$$n(a,z) = 2.35 \cdot 10^{11} a^{-3.67} \exp(-2.28z), \tag{5}$$

where the radius a was expressed in micrometres, the depth z in metres and $n(a,z)$ was the number of bubbles of radii $[a, a+1\ \mu\mathrm{m}]$ in 1 m³ of water. Forumula of such type is indispensable for mathematical modelling of numerous oceanic processes involving gas bubbles.

4. Second order scattering effects

In order to examine the effect of neglecting of the multiple scattering (and, consequently, the excess attenuation) we denote by n the number of bubbles inferred from the experiment (first order scattering approximation) and by N the **real** bubble number. Sound extinction due to bubble aggregations in the depth interval $[z_1, z_N]$ is described by the exponential factor E:

$$E(z_N) = \exp\left(-2 \int_{z_1}^{z_N} S_e(z)\, dz\right). \tag{6}$$

The integral in (6) can be separated into terms associated with individual sublayers determined by the sampling frequency f_s ($f_s = 45$ kHz, $\Delta t = f_s^{-1}$, $\Delta z = c\Delta t/2 = 1.62$ cm). If we assume that S_e is constant in each individual sublayer, the integration can be replaced by the summation. By inserting the expression (4) for S_e we obtain:

$$\int_{z_1}^{z_N} S_e(z)\, dz = \int_{z_1}^{z_2} S_e(z)\, dz + \ldots + \int_{z_{N-1}}^{z_N} S_e(z)\, dz = \frac{2\pi^2}{k} \Delta z \sum_{j=1}^{N-1} N(z_j)\, a_R^2(z_j). \tag{7}$$

At the beginning we know nothing about the attenuation and we infer the number of bubbles $n(z_1)$ using the first order scattering approximation; then we use $N(z_1) = n(z_1)$ to calculate the attenuation factor $E(z_1)$ in the first sublayer, which influences the determination of the bubble number in the second one. In the next stage we correct the measured concentration $n(z_2)$ by the attenuation factor from the preceding minilayer according to the general formula

$$N(z_i) = E^{-1}(z_{i-1})\, n(z_i), \tag{8}$$

and we calculate the attenuation in the second sublayer, i.e. the correction factor for the next minilayer. Such adaptive processing is conducted for each next sublayer up to the depth z_N. Corrected bubble concentrations can be described by the curves $N(z) = A_1 \exp(B_1 z)$ with the following coefficients:

Table 2

f [kHz]	A_1	B_1
40	$1.5 \cdot 10^4$	-2.13
50	$8.4 \cdot 10^4$	-2.71
63	$1.6 \cdot 10^5$	-2.34
80	$1.7 \cdot 10^5$	-2.13

The formula analogous to (5) takes the form

$$N(a,z) = 3.26 \cdot 10^{11} a^{-3.82} \exp(-2.03z) \tag{9}$$

Comparison of the exponent coefficient in Tables 1 and 2 as well as in formulae (5) and (9) shows a decrease in the absolute slope of the regression lines for the corrected concentrations related to the original ones. This fact is also manifested in Fig. 5.

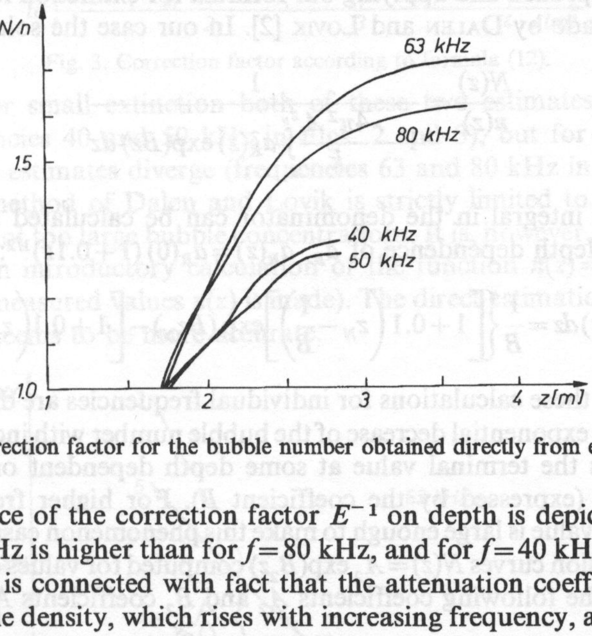

Fig. 2. Correction factor for the bubble number obtained directly from experiment.

The dependence of the correction factor E^{-1} on depth is depicted in Fig. 2. Its value for $f = 63$ kHz is higher than for $f = 80$ kHz, and for $f = 40$ kHz it is higher than for $f = 50$ kHz. It is connected with fact that the attenuation coefficient S_e depends both on the bubble density, which rises with increasing frequency, and on the bubble size, which falls (Eq. 4). Bubble concentrations for the frequency pairs mentioned above differ less than the third power of the respective bubble radius and this is the reason for the observed effect.

Another approach to the problem of neglecting of the multiple scattering is presented by DALEN and LOVIK [2]. They simplify this problem by assuming $S_e = N\sigma_e$, what gives the attenuation factor in the form

$$\exp\left(-2\int_0^z N\sigma_e\,dz\right),$$

where σ_e — extinction cross-section of the individual resonant bubble. Formula (8) leads to the following equation:

$$N(z)=n(z)\exp\left(2\int\limits_0^z N\sigma_e dz\right). \tag{10}$$

Differentiating (10) with respect to z and multiplying both sides of this expression by n leads to the Bernoulli equation. Its general solution shows that the real number of bubbles N depends on the functional form of n deduced from the mesaurement. Applying the formula $n(z)=A\exp(Bz)$ and assuming that σ_e is constant provides the particular solution:

$$\frac{N(z)}{n(z)}=\frac{1}{1-2\sigma_e A\int\limits_0^z \exp(Bz)\,dz}=\frac{1}{1-\frac{2\sigma_e A}{B}[\exp(Bz)-1]}. \tag{11}$$

Following this approach and applying our formula for extinction factor (6) we avoid simplifications made by DALEN and LOVIK [2]. In our case the solution is:

$$\frac{N(z)}{n(z)}=\frac{1}{1-\frac{4\pi^2 A}{k}\int\limits_{z_1}^{z_N} a_R^2(z)\exp(Bz)\,dz} \tag{12}$$

The value of the integral in the denominator can be calculated analytically using additionally the depth dependence of a_R, $a_R(z)=a_R(0)(1+0.1z)^{1/2}$:

$$\int\limits_{z_1}^{z_N}(1+0.1z)\exp(Bz)dz=\frac{1}{B}\left\{\left[1+0.1\left(z_N-\frac{1}{B}\right)\right]\exp(Bz_N)-\left[1+0.1\left(z_1-\frac{1}{B}\right)\right]\exp(Bz_1)\right\}.$$

The results of these calculations for individual frequencies are displayed in Fig. 3.

Because of the exponential decrease of the bubble number with increasing depth, the ratio N/n reaches the terminal value at some depth dependent on the intensity of turbulent mixing (expressed by the coefficient B). For higher frequencies used in experiment the z_N value is large enough to make this phenomenon easily noticeable (Fig. 3). A linear regression curves $N(z)=A_2\exp(B_2 z)$ computed for values obtained from last estimation have the following coefficients A_2 and B_2 coefficients A_2 and B_2:

Table 3

f [kHz]	A_2	B_2
40	$1.6\cdot10^4$	-2.14
50	$9.0\cdot10^4$	-2.74
63	$1.7\cdot10^5$	-2.34
80	$1.7\cdot10^5$	-2.13

A two-variables regression gives

$$N(a,z) = 3\cdot10^{11}a^{-3.80}\exp(-2.03z). \tag{13}$$

Since the function (12) has the form $1/(1-x)$, its physical sense is limited to the region $x<1$. It is noteworthy that for $x\ll1$ the function $\exp(x)$, i.e. N/n in formula (8), and the function $1/(1-x)$, i.e. N/n in formula (12), have the same first order Taylor expansion:

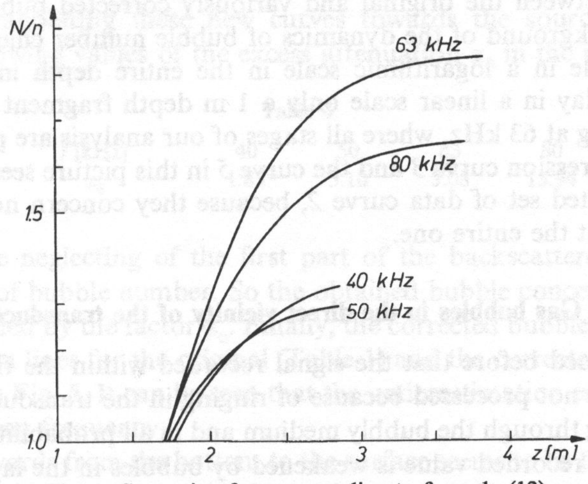

Fig. 3. Correction factor according to formula (12).

$1+x$. In fact, for small extinction both of these two estimates are very similar (compare frequencies 40 and 50 kHz in Figs. 2 and 3), but for greater values of attenuation these estimates diverge (frequencies 63 and 80 kHz in Figs. 2 and 3). It means that the method of Dalen and Lovik is strictly limited to the case of small attenuation, i.e. not too large bubble concentrations. It is, however, very simple in use (provided that an introductory calculation of the function $n(z)=A\exp(Bz)$ on the basis of directly measured values $n(z)$ is made). The direct estimation method is more laborious but it seems to be more accurate.

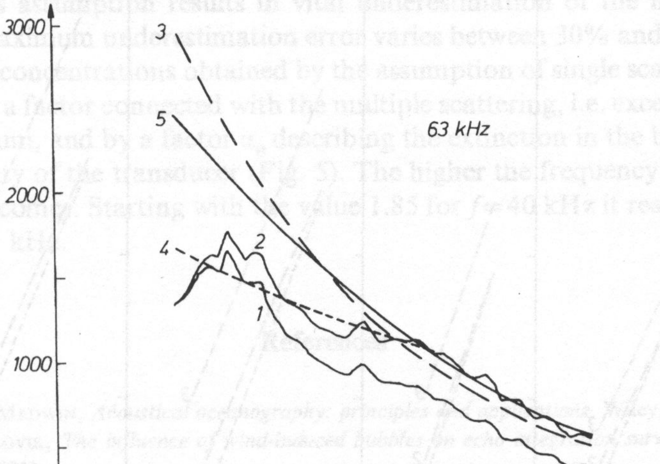

Fig. 4. Number of bubbles resonating at frequency 63 kHz in a chosen small interval of depth, (*1*) — obtained from experiment, (*2*) — after direct correction (Eqs. 6–8) and its (*3*) — total regression and (*4*) local regression, (*5*) — corrected according to Eq. (12).

Differences between the original and variously corrected bubble concentration curves on the background of the dynamics of bubble number changeability are too slight to be visible in a logarithmic scale in the entire depth interval. So, as an example, we display in a linear scale only a 1 m depth fragment of the density of bubbles resonating at 63 kHz, where all stages of our analysis are presented (Fig. 4). Both the total regression curve *3* and the curve *5* in this picture seem not to be fitted well to the corrected set of data curve *2*, because they concern not only this small depth interval, but the entire one.

5. Gas bubbles in the direct vicinity of the transducer

It was mentioned before that the signal recorded within the first few hundreds microseconds was not processed because of ringing in the transducer. Nevertheless, this signal is going through the bubbly medium and in all probability it is attenuated, so in fact the first recorded value is weakened by bubbles in the layer nearest to the transmitter. Magnitude of this extinction is unknown but it should be significant because of bubble abundance in this area. It can be evaluated by extrapolation of the function $N(z)=A_1 \exp(B_1 z)$ up to the source depth and calculation of the sound attenuation in the interval $[z_0, z_1]$. We can see from Fig. 4, however, that total regression line in the shallowest layer runs remarkably higher than the experimental curve, so an additional regression computation for the first 0.5 m is needed. In fact such local regression gives more realistic values of bubble concentration in this region

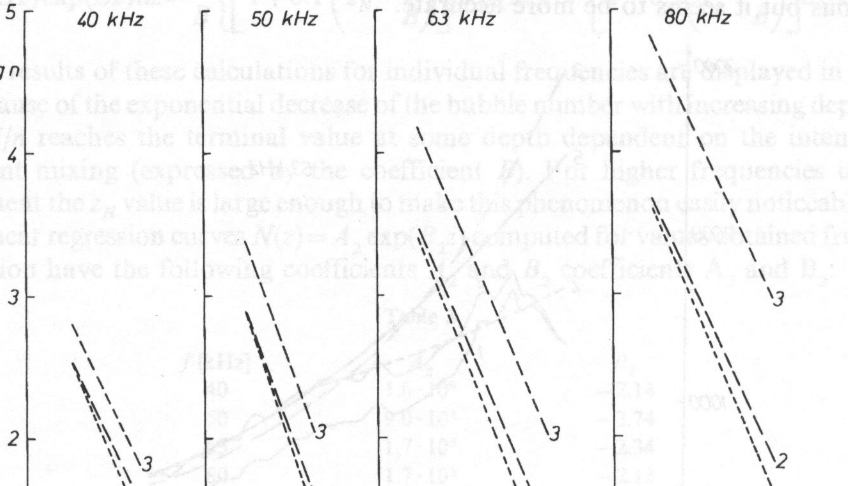

Fig. 5. Regression lines for concentrations of bubbles resonating at frequencies used in experiment: (*1*) obtained directly from the first-order scattering experiment, (*2*) corrected according to Eqs. (6–(8) and (*3*) finally corrected by a factor α_0 describing the extinction in the direct vicinity of the transducer.

(Fig. 4). By extrapolating these new curves towards the source depth we have obtained the following values of the excess attenuation α_0 in the interval $[z_0, z_1]$:

Table 4

f [kHz]	40	50	63	80
α_0	1.85	3.10	5.08	13.94

It means that the neglecting of the first part of the backscattered signal leads to underestimation of bubble number. So the obtained bubble concentration functions should be multiplied by the factor α_0. Finally, the corrected bubble densities together with the regression lines for the original (Table 1) and the corrected (Table 2) sets of data are shown in Fig. 5. It can be seen that the underestimation error increases with increasing sounding frequency.

Sounding upwards from the bottom to the surface seems to be the best solution to this problem. It allows us to avoid the problem of rather dense layer of bubbles occuring in the direct vicinity of the transmitter that is not considered because of the technical break caused by switching the working mode from transmitting to receiving.

6. Summary

To sum up, we have obtained some experimental results concerning gas bubble populations in the coastal zone of the Baltic Sea. The problem of using the first order approximation (neglecting of the multiple scattering) has been considered, and it was shown that this assumption results in vital underestimation of the number of gas bubbles. The maximum underestimation error varies between 30% and 73% (Fig. 2).

Gas bubble concentrations obtained by the assumption of single scattering should be corrected by a factor connected with the multiple scattering, i.e. excess attenuation in bubbly medium, and by a factor α_0 describing the extinction in the bubble layer in the direct vicinity of the transducer (Fig. 5). The higher the frequency is, the greater the factor α_0 becomes. Starting with the value 1.85 for $f=40$ kHz it reaches the value 13.94 for $f=80$ kHz.

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