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MEASUREMENTS OF THE NONLINEARITY PARAMETER B/A OF SEAWATER

E. KOZACZKA

Naval Academy,
(81-919 Gdynia, Smidowicza 71, Poland)
and
Technical University of Koszalin,
(75-620 Koszalin, Racławicka 13-17, Poland)

G. GRELOWSKA

Naval Academy,
(81-919 Gdynia, Smidowicza 71, Poland)

The application of modern hydroacoustic systems making use of the phenomenon of the nonlinear wave propagation is the object of interest in the sphere of basic scientific research as well as solving construction problems. The problem of particular importance for practical application is the knowledge of nonlinear properties of the medium in which propagate the finite amplitude waves. This is why the research in the sphere of space distribution of the nonlinear parameter of the medium becomes so important. The work contains the description of principal relations which make it possible to delineate the parameter of the nonlinearity of the medium B/A. Apart from that method has been proposed to delineate that parameter on the basis of the results obtained from maritime research. For that purpose a measurement set up has been constructed which makes it possible to obtain the measurement of nonlinearity parameter *in situ*. Results are also presented of the maritime research in the shape of distribution of the parameter B/A for the region of the Southern Baltic.

1. Introduction

The nonlinearity parameter B/A, defined as a ratio of coefficients in a Taylor series expansion of the isentropic equation of state [17], is a material constant which characterizes a phenomenon of distortion of a finite amplitude wave during its propagation in fluids.

There are two principal methods for determining the nonlinearity parameter B/A: the finite amplitude method and the thermodynamic method. The thermodynamic method is based on measurements of changes in thermodynamic parameters of medium caused by finite amplitude wave [7, 9]. The value of the nonlinearity parameter B/A is usually obtained using the following formula introduced by R.T. BEYER [3]:

$$\frac{B}{A} = 2\varrho_0 c_0 \left(\frac{\delta c}{\delta p}\right)_{T,\varrho=\varrho_0} + \frac{2c_0\alpha T}{C_p}\left(\frac{\delta c}{\delta T}\right)_{p,\varrho=\varrho_0}, \tag{1}$$

where ϱ_0 — the equilibrium density of medium, c_0 — the infinitesimal amplitude sound wave speed, T — the absolute temperature, C_p — the specific heat capacity at constant pressure, $\alpha = (1/V)(\delta V/\delta T)_p$ — the volume coefficient of thermal expansion, $(\delta c/\delta p)_T$ — change of the sound speed with pressure at constant temperature, $(\delta c/\delta T)_p$ — change of the sound speed with temperature at constant pressure.

In the finite amplitude method the nonlinearity parameter B/A is determined by means of measurement of nonlinear wave distortion. That method is based on the investigation of changes in spectrum due to distortion of the propagating sinusoidal wave [6, 10, 15].

The nonlinearity parameter B/A of seawater is obtained mainly for high-salinity seawater [8]. Low-salinity of the Baltic Sea are the fundamental reason of our interest in the nonlinear properties of the low-salinity water.

The results obtained by means of the thermodynamic method for several points of the South Baltic Sea were presented in papers [14, 15]. This method is based on the thermodynamic equation of the medium and measurements of temperature and salinity as a function of depth (static pressure).

The acoustical method based on measurements of distortion of the finite amplitude wave was applied too. The results received in preliminary measurements encouraged us to advance the acoustical method. A special device for measurements of the nonlinearity parameter B/A *in situ* was designed and used in practice.

2. Method of measurements

The method of determination of the nonlinearity parameter B/A of seawater is an adaptation of the one proposed by W.N. COBB [6] for sea conditions [13]. The main idea of that method is using a receiver of the same area as a transmitter and evaluating the parameter B/A by means of measurement of the second harmonic component of the wave. The value of the nonlinearity parameter B/A is obtained using the formula [6]:

$$\frac{B}{A} = 2\left(\frac{|\langle p_2(z)\rangle|}{p_0^2}\frac{\varrho_0 c_0^3}{\pi f|I_1 - I_2|} - 1\right), \tag{2}$$

where $|\langle p_2(z)\rangle|$ denotes the amplitude of the second harmonic of the pressure averaged over the receiver's area measured at the distance z from the transmitter, p_0 — pressure at the transmitter surface, f — frequency. The term I_1 is a factor describing the influence of attenuation on the amplitude of the second harmonic pressure component:

$$I_1 = \frac{e^{2\alpha_1 z} - e^{\alpha_2 z}}{\alpha_2 - 2\alpha_1}, \tag{3}$$

where α_1 and α_2 are the fundamental wave and the second harmonic component attenuation coefficients, respectively.

Because of diffraction, sources for which the quantity ka is much greater than 1 (a is the radius of the piston) have a fine structure in the nearfield. The term I_2 introduces a correcting element. It takes into account the diffraction spreading and phase cancellation over the receiver.

$$I_2 = Q\left(\int\limits_{z/2}^{z} e^{2\alpha\Theta}\left[(\Theta^2+4a^2)^{1/2}-\Theta\right]^{-1/2} d\Theta - \frac{1}{8a^2}\int\limits_{z/2}^{z} e^{2\alpha\Theta}\left[(\Theta^2+4a^2)^{1/2}-\Theta\right]^{3/2} d\Theta\right), \tag{4}$$

where

$$Q = \frac{8e^{i\pi/4}}{(\pi k)^{1/2}} e^{-2(\alpha_2-\alpha_1)z} \tag{5}$$

and

$$\alpha = \alpha_2 - 2\alpha_1 . \tag{6}$$

An acoustic part of the device for measurements of the nonlinearity parameter B/A of seawater consists of a piston circular source of 46 mm diameter driven at its central frequency equal to 1 MHz corresponding to $ka = 96$ (k is a wave number, a is the transmitter radius). For pseudo CW measurements a sinusoidal tone burst of about 50 cycles is applied to the device. The pressure is measured using a receiver of the same area as the transmitter placed coaxial with the source of the wave. Its receiving characteristic covers the frequency range from up to 5 MHz. The distance between transducers can be changed up to 251 mm.

The distortion of an electrical signal obtained from the receiver depends on the nonlinearity parameter B/A of water. The value of the amplitude of the second harmonic component is extracted in two independent ways, by means of the analog filtering and applying the Fourier analysis. In the second method the digital storage oscilloscope is used as a digitizer. An example of changes in a wave spectrum with the distance between the transmitter and the receiver is presented in Fig. 1.

The assumed arrangement of the device causes that the receiver is placed in the nearfield area of the transmitter. It means that measurements are done in the area of pressure field of fine structure. The changes in pressure distribution along the axis of the system are shown in Fig. 2. They are obtained theoretically by means of linear approximation [18] and using parabolic approximation [1]. In the Fig. 3 a, b are shown transverse pressure distributions of the first and the second pressure harmonic components at the distance of 251 mm from the source of the wave determined theoretically by means of parabolic approximation and experimentally using needle 1 mm-diam PVdF hydrophone.

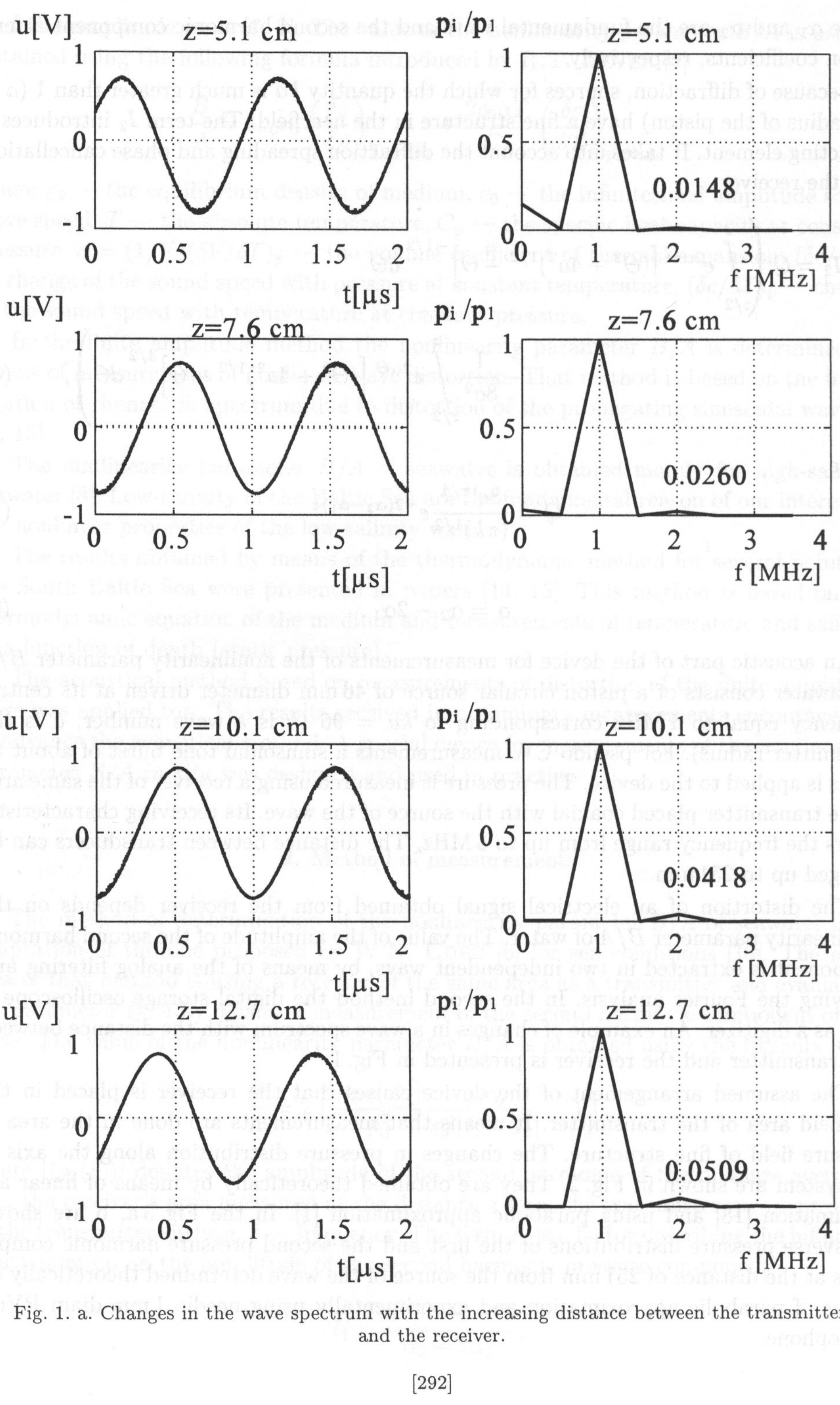

Fig. 1. a. Changes in the wave spectrum with the increasing distance between the transmitter and the receiver.

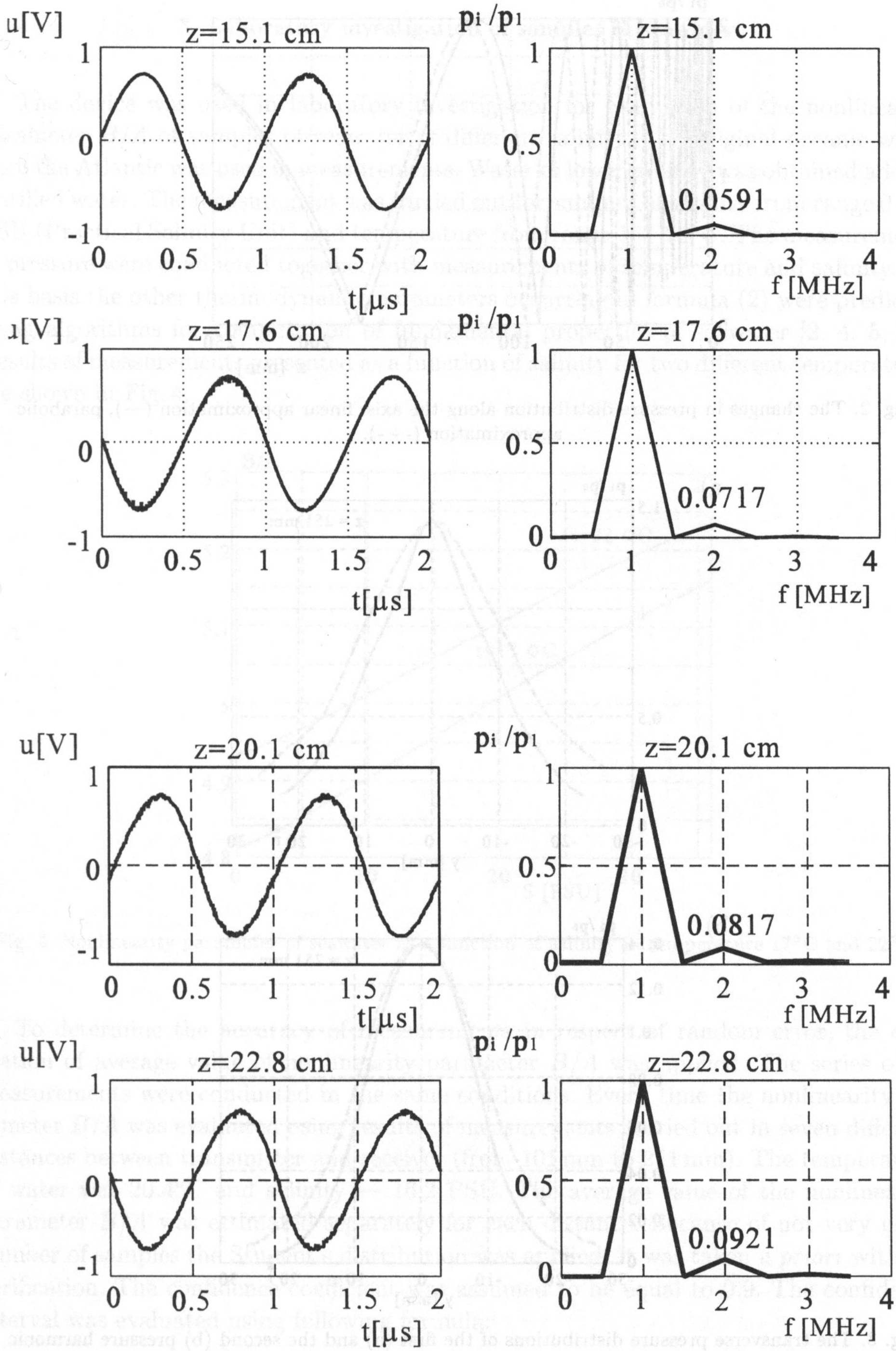

Fig. 1. b. Changes in the wave spectrum with the increasing distance between the transmitter and the receiver.

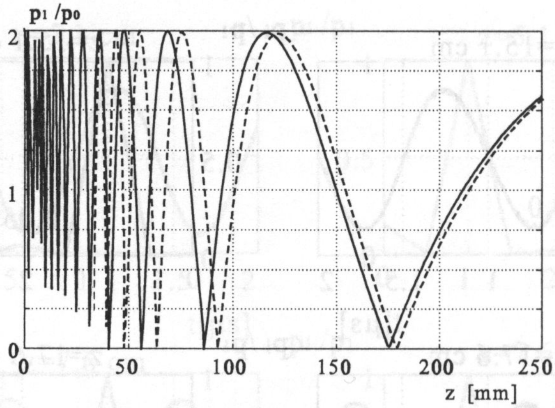

Fig. 2. The changes in pressure distribution along the axis: linear approximation (—), parabolic approximation (- - -).

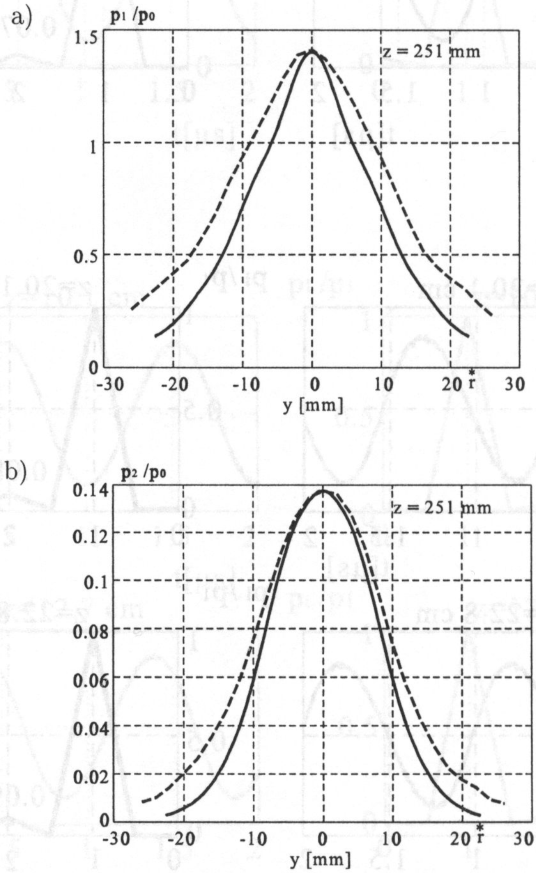

Fig. 3. The transverse pressure distributions of the first (a) and the second (b) pressure harmonic components at the distance of 251 mm from the source (—) parabolic approximation, (- - -) results of measurement.

3. Laboratory investigation of samples of seawater

The device was used in laboratory investigation for evaluation of the nonlinearity parameter B/A of samples of seawater of different salinity. The original oceanic water from the Atlantic was used in measurements. Water of lower salinity was obtained adding distilled water. The measurement was carried out for samples of salinity from range 0 – 36 PSU (Practical Salinity Unit) and temperature from range 15 – 25°C. The measurements of pressure were conducted together with measurements of temperature and salinity. On this basis the other thermodynamic parameters occurring in formula (2) were predicted using algorithms for computation of fundamental properties of seawater [2, 4, 5, 16]. Results of measurements presented as a function of salinity for two different temperatures are shown in Fig. 4.

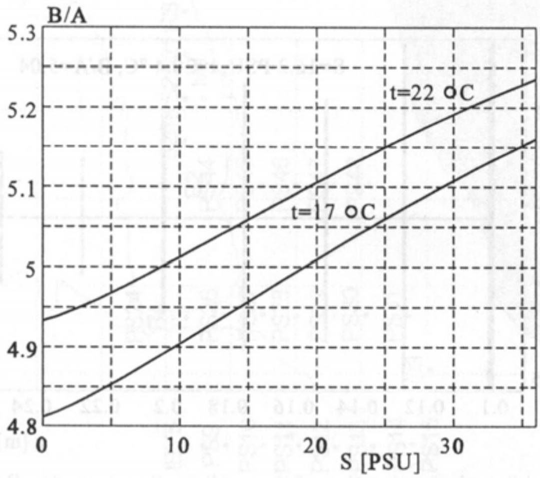

Fig. 4. Nonlinearity parameter of seawater as a function of salinity in temperature 17°C and 22°C.

To determine the accuracy of measurements in respect of random error, the estimation of average value of nonlinearity parameter B/A was applied. The series of 20 measurements were conducted in the same conditions. Every time the nonlinearity parameter B/A was evaluated using results of measurements carried out in seven different distances between transmitter and receiver (from 101 mm to 251 mm). The temperature of water was 20.4°C and salinity — 16.2 PSU. The average value of the nonlinearity parameter B/A was estimated separately for each distance. Because of not very great number of samples the Student's distribution was applied. It was taken *a priori* without verification. The confidence coefficient was assumed to be equal to 0.9. The confidence interval was evaluated using following formula:

$$p\left(\frac{\overline{B}}{A} - t_\alpha S < \frac{B}{A} < \frac{\overline{B}}{A} + t_\alpha S\right) = 1 - \alpha, \tag{7}$$

where $\overline{B/A}$ denotes the average value of the nonlinearity parameter at fixed distance:

$$\frac{\overline{B}}{A} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{B}{A}\right)_i \tag{8}$$

and S denotes the error meansquare of measurement:

$$S = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{B}{A} - \frac{\overline{B}}{A}\right)^2}. \tag{9}$$

For a sample size $n = 20$ the value of parameter t_α equals 1.752 [11]. The confidence interval determined for each of measurement distances are shown in Fig. 5. The average value of the parameter B/A equals 5.04. The results of estimation indicate the higher accuracy of measurements at greater distances between transducers.

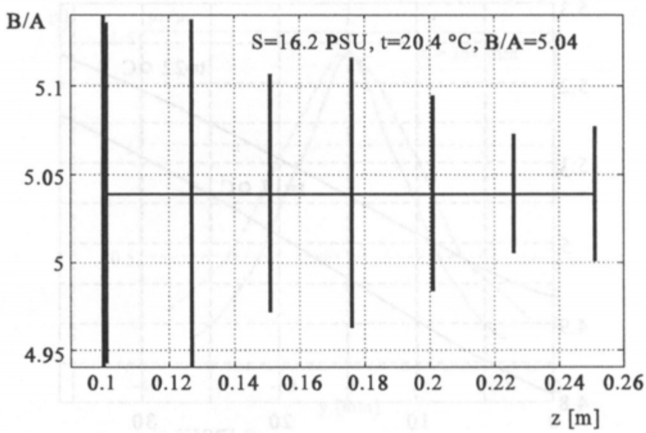

Fig. 5. The confidence interval of estimation of the nonlinearity parameter B/A at various distances between transmitter and receiver obtained for the confidence coefficient assumed to be equal to 0.9.

4. Investigation of the nonlinear parameter B/A in chosen points of the South Baltic Sea

The device verified in laboratory investigations was next used in measurement carried out *in situ* in the Baltic Sea. The investigations were done in April 1994 in stations marked in Fig. 6. The measurements of the nonlinearity parameter B/A were conducted together with the measurements of changes in pressure and salinity as a function of the depth using the STD sounder produced by Guideline.

For each station the vertical distribution of the nonlinearity parameter B/A was determined based on results of measurements of the second harmonic component of the pressure. The receiver was most often placed at the distance greater then 200 mm from transmitter (226 mm or 251 mm). Sometimes measurements at the same station were carried out twice applying two different distances between transducers.

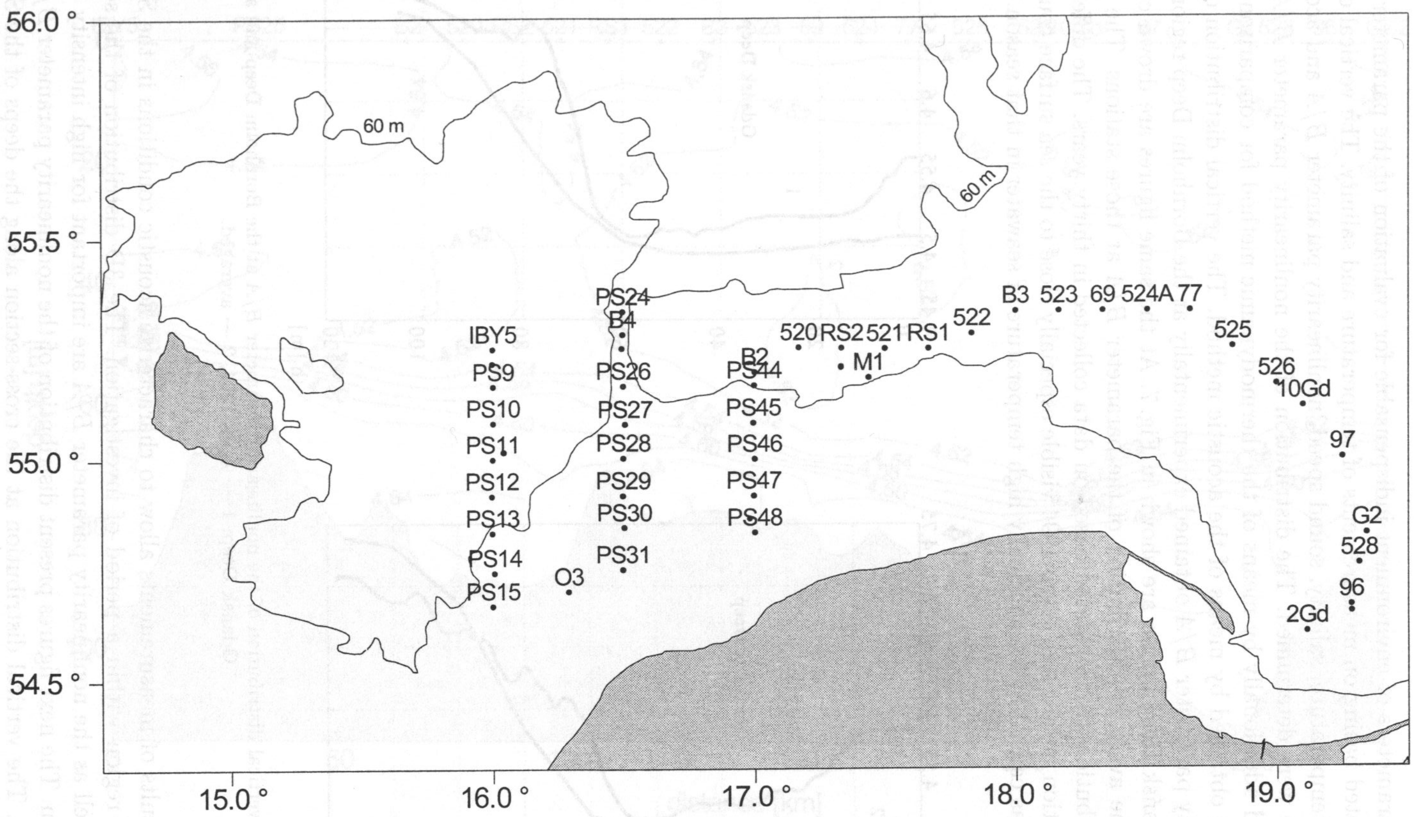

Fig. 6. Localization of the measurement stations.

The parameters of environment indispensable for evaluation of the parameter B/A are calculated basing on measurements of temperature and salinity. The vertical distribution of temperature, salinity, sound speed, nonlinearity parameter B/A and acoustic impedance were determined. The distribution of the nonlinearity parameter B/A was determined additionally by means of the thermodynamic method for comparison with the results obtained by means of the acoustic method. The vertical distribution of the nonlinearity parameter B/A obtained experimentally at the Bornholm Deep region and at the Gdańsk Deep region are shown in Fig. 7. At the same figures are drown curves showing the averaged distribution of the parameter B/A at those stations. The averaged distribution are calculated based on data collected in thirty years. The difference in distribution obtained in April 1994 visible especially close to the sea surface could be explained as caused by exceptionally high temperature of seawater in that season.

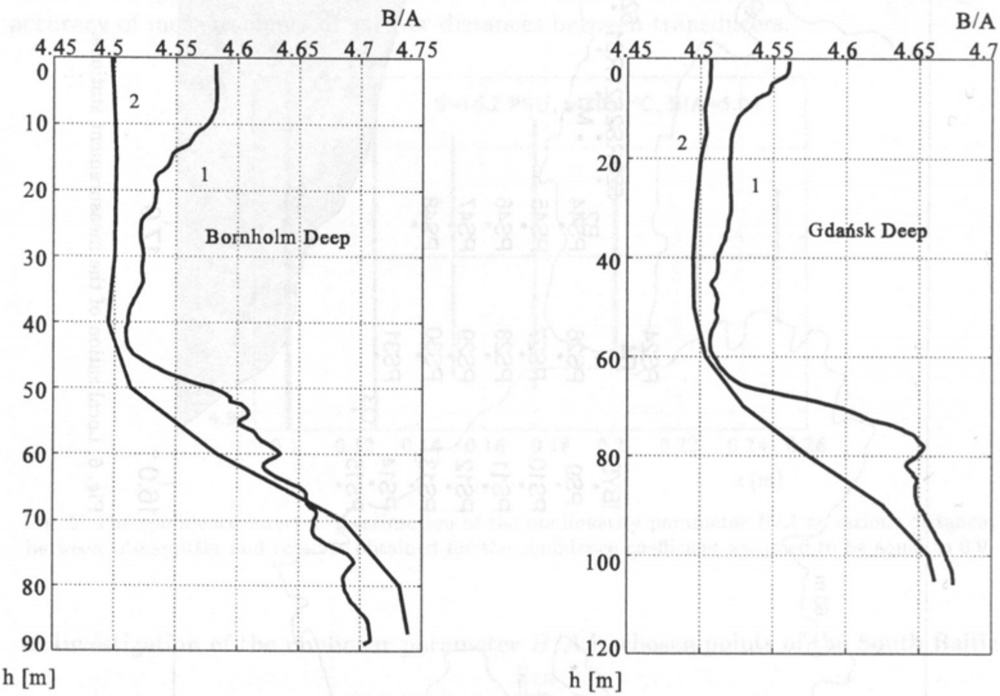

Fig. 7. The vertical distribution of the nonlinearity parameter B/A at the Bornholm Deep and at the Gdańsk Deep: 1 — April 1994, 2 — averaged.

The results of measurements allow to characterize acoustic conditions in the South Baltic Sea region within a period of investigation. The 3D distribution of the sound speed as well as the nonlinearity parameter B/A are important for high intensity wave propagation. The next figures present distribution of the nonlinearity parameter B/A in April 1994. The vertical distribution at the cross-section along the deeps of the South Baltic is shown in Fig. 8 while the horizontal distribution at the sea surface and at depth of 20 m are presented in Fig. 9.

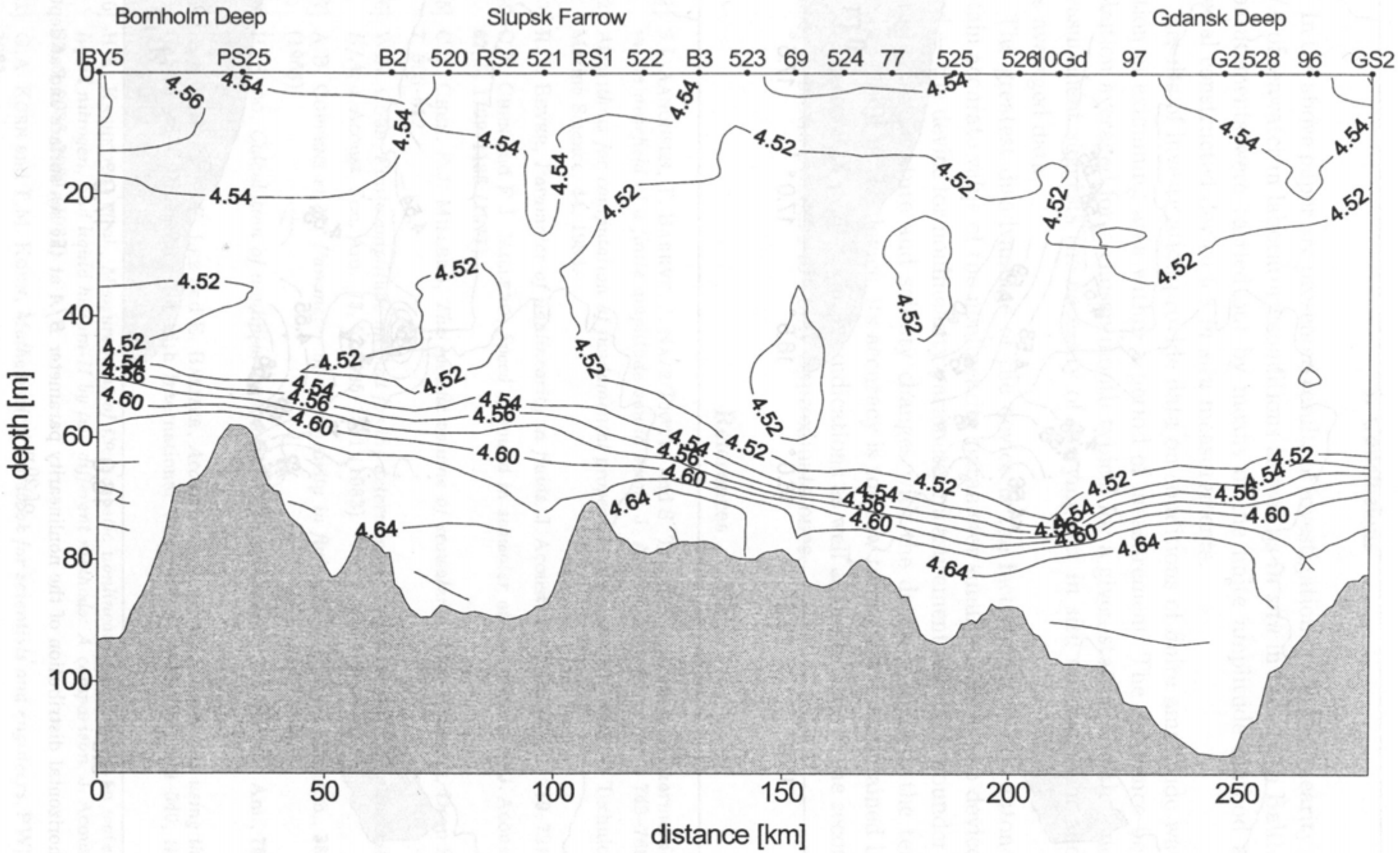

Fig. 8. The vertical distribution of the nonlinearity parameter B/A in the cross-section along the deeps of the South Baltic Sea.

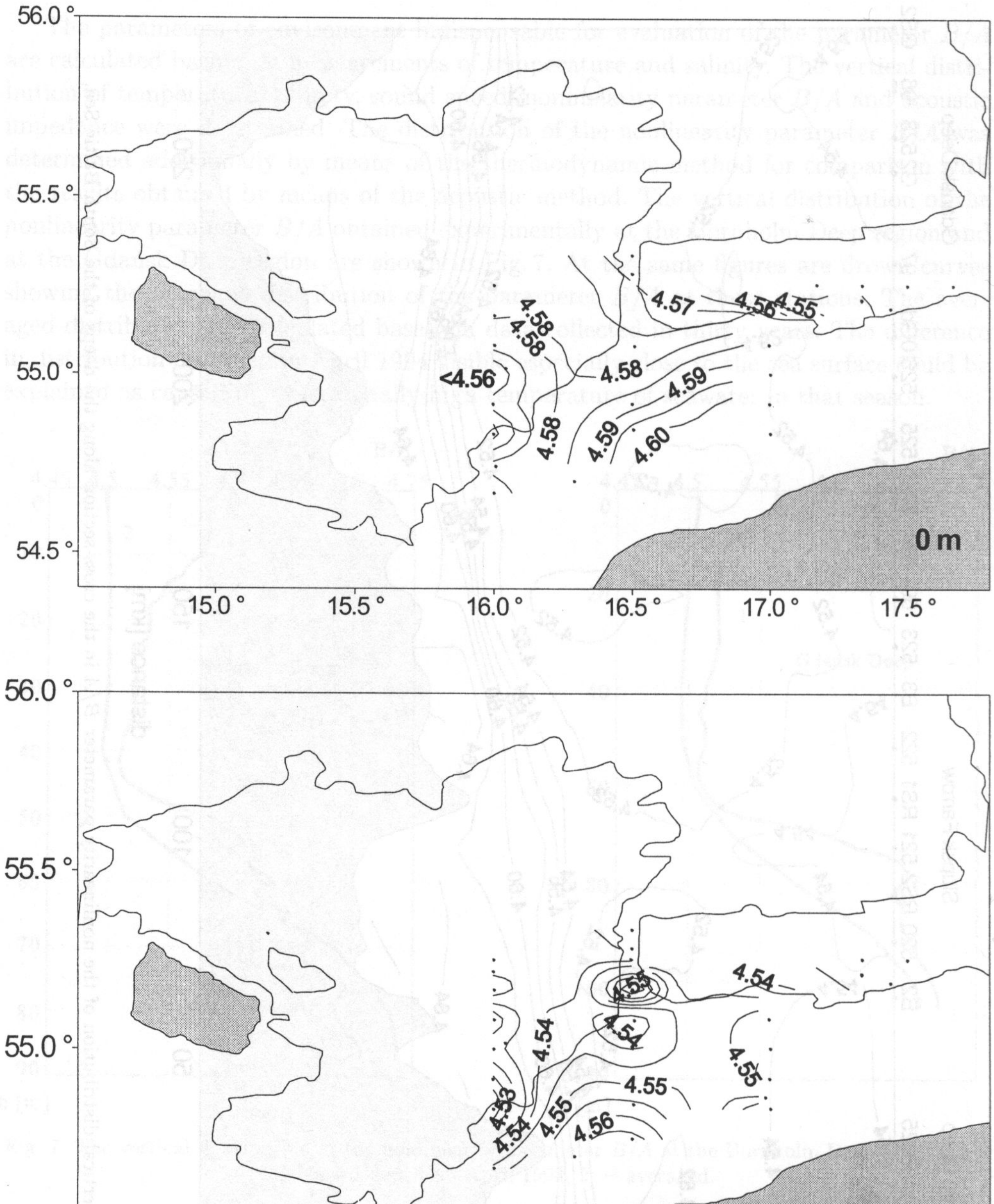

Fig. 9. The horizontal distribution of the nonlinearity parameter B/A at the sea surface and at depth of 20 m.

5. Conclusions

In the above paper are presented results of investigation of the nonlinearity parameter B/A of seawater in laboratory conditions as well as *in situ* in the South Baltic Sea. The measurements were carried out by means of the finite amplitude method applying a special constructed device for *in situ* measurements.

Results of investigation provide data on conditions of finite amplitude waves propagation in examining area within a period of measurement. The difference between distribution averaged for a chosen month typical for a given station and the one based on measurement suggests the necessity of carrying out *in situ* measurement and verifying the averaged data.

The greatest disadvantage of the device is the fact that it is not autonomous. To obtain accurate value of the nonlinearity parameter simultaneously two devices must be used — the device for nonlinearity parameter measurement and STD sounder to sample data on temperature and salinity changes with the depth. Although the temperature sensor is built in the device, its accuracy is too low to use the data obtained in this way in parameter B/A evaluation. Its indications as well as indications of the second built-in sensor of static pressure are only approximate ones.

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