STATISTICAL DESCRIPTION OF NOISE PROPAGATION
IN A BUILT-UP AREA

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The paper presents three diffusion models of sound propagation through a space with obstacles proposed by Kurze, Kuttruff and Yeow. The models contain simple expressions for the sound attenuation what makes them attractive to use.

Here, the models are applied to a segment of a built-up area. Considerable differences appearing between the values of the sound excess attenuation confirm the fact that the diffuse model can be used only as a tool for very rough field investigation.

One has to be very careful while intepreting the results obtained, bearing in mind all the limitations of the models applied and the vague way of estimation of the model parameters.

1. Introduction

The noise abatement problem can be most efficiently solved at the stage of planning. To this end, a tool for prediction of the sound pressure level is needed. Three kinds of treatment are involved:

- development of empirical formulae,
- scale modeling,
- mathematical model.

The procedure of noise prediction in urban area usually includes a source model and a propagation model.

For the source model construction [13], [25], [26], [28], [30], [31], [38], [41] the way of developing empirical formuale is mainly used. The simplified mathematical source models contain also the averaged values of the parameters established by field measurements.

For construction of the propagation model all three kinds of treatments are applied. The emprical model [27], founded on a large number of measurements, offers a general description of noise propagation in a built-up area. The generality of description is achieved at the expense of accuracy, since an urban system is described only by a single parameter: the ratio of the ground area of buildings to the site area.

The scale modeling with, for example, the scale factor 1:30 raises a lot of problems of the site modeling with proper frequency dependence [6]. Investigation with such a scale factor requires special instrumentation and is not easily adjustable to changing the sites.

More advisable is the application of scale models for a small segment of a built-up area [8], [33], [35] or investigation of some special effects such as, for example, transmission between rooms, through windows, by semi-reverberent space in a built-up area [7]. For such cases the scale factor $1:d > 0.1$, does not create so many problems in modeling.

The most flexible and widely applicable are the mathematical models. The principal problem is, how should we describe the acoustical field in a space with reflecting obstacles. This is the case when noise propagates in industrial halls, and from a highway into the urban area. One of the possibilities is a detailed simulation of the wave path. Another possibility is offered by the statistical description of acoustical energy distribution.

The detailed noise propagation models for a built-up area, with several obstacles, involve multiple reactions with obstacles [4], [9], [32], [34], [39], [40]. As elementary interactions, the specular reflections and diffractions at the edges are included. Despite the relative complexity of the model, which comes from incorporation of diffraction [36], [37], the model can provide any required accuracy. The models using ray tracing [1], [29], [42] are less accurate.

For a built-up area there is a custom to treat separately rush streets, applying for them the wave-guides propagation models [5], [17], [20], [24]. For an area relatively distant from a highway, statistical description is advised [16], [23], [43], [44].

In industrial halls, by modeling the acoustic energy distribution as phonon random walk in a fitted space, statistical description is applied [11], [12], [18], [19].

The same statistical description as that used for industrial halls can be applied for a built-up area. In the extreme, this description results in application of the diffusion equation [2], [14], [15], [16], [43].

Here, a special attention is paid to the description of noise propagation as a diffusion process.

2. Noise propagation as a statistical process

When sound propagates through an irregular structure, where many of interactions take place, it can be described in a statistical way. The propagation process is observed as a macroscopic one but, it appears as a consequence of a large number of microscopic ones.

In statistical description, the sound wave is regarded as a stream of particles (phonons). Travelling through a structure, the particles are scattered. In the case of an urban enclosure, the structure contains the ground with sited buildings. Interaction of phonons with a building as an obstacle is characterized by the building scattering cross-section (Q).

In a two-dimensional space, an obstacle is modeled as a cylinder. The obstacle circumference (C) is made equal to the circumference of a cylinder $(C = 2\pi r_c)$ for which $Q_c = 2r_c = 2\pi r_c/\pi$ [21]. Thus the building scattering cross-section is

$$Q = C/\pi \text{ [m]}. \tag{1}$$

In a three-dimensional space, an obstacle is represented by a sphere for which $Q_s = \pi r_s^2$ [22]. The obstacle surface area (S) is equalized to the surface area of the sphere $(S = 4\pi r_s^2)$, and thus

$$Q = S/4 \text{ [m}^2]. \tag{2}$$

The probability density distribution for travelling a path r without collision is assumed to be exponential and it is related to scattering cross-section (Q) of the obstacles and their space density (n) by equation [22]

$$w(r) = \exp(-Qnr), \tag{3}$$

where the density of obstacles is expressed in m^{-2} or in m^{-3} for a two-dimensional space and a three-dimensional space, respectively.

Now, the free path is defined by

$$\lambda = \int_0^\infty w(r)\,dr = \frac{1}{Qn}. \tag{4}$$

When a certain physical quantity (F) obeys the continuity equation

$$\frac{\partial F}{\partial t} = -\nabla \mathbf{J}, \tag{5}$$

and its flow $\mathbf{J}$ fulfills the equation

$$\mathbf{J} = -D\nabla * F, \tag{6}$$

it is governed by the diffusion equation

$$\frac{\partial F}{\partial t} = -D\nabla^2 F, \tag{7}$$

where D is a constant (see Eq. (11)).

The Eq. (6) is modified depending on the particular nature of diffusion. As a consequence, in Eqs. (5), (6) appear the terms describing attenuation and action of the additional sources.

As macroscopically observed, the diffusion process results from elementary events at a microscopic level. For example, when diffusion is observed in gases under normal conditions, then the number of colisions in 1 cm^3 per sec is of the order of 10^9, while the molecule diameters range about 10^{-8} cm, and the free path length is about 10^{-5} cm.

When as the quantity under consideration the sound intensity (I) is used, then the distance travelled by phonons in an urban structure have to be large as compared to the mean free path (Eq. (4)), so that the number of interactions with buildings is large. This means that the chosen segment of an urban structure has to be large enough, to contain a large number of buildings which are irregularly scattered. The last condition of irregular scattering of the buildings is usually not satisfied in a built-up area.

2.1. The Kurze model

KURZE [14] applied the diffusion equation to the three-dimensional space with obstacles (scatters). His aim was to develop the noise propagation model in industrial halls. Assuming propagation in half-space with reflecting ground, the model can be applied to an urban enclosure. For a point source stationary radiation with the power output P, the sound intensity of the scattered part of the field obeys the equation

$$D\nabla^2 I_s - \chi I_s = (1-\alpha')\frac{Pc}{2\pi r^2}\frac{\exp(r/\lambda_z)}{\lambda_z},\tag{8}$$

where c is the sound speed.

Since the model is three-dimensional, we have

$$Q_z = S/4, \quad \text{for} \quad \sqrt{\pi S} > \lambda_w,\tag{9}$$

where λ_w is the sound wave length. The mean free path is defined by

$$\lambda_z = \kappa 4V/S, \quad \text{for} \quad \kappa < 1,\tag{10}$$

where V is the total volume of the space considered, S is the total surface of the scatters. The coefficient κ is chosen arbitrary to achieve good agreement with the experimental results. It reflects the fact that the investigated field is not sufficiently homogeneous and isotropic [10].

The scattered field is formed by phonons which suffer at least one collision. In Eq. (8), the first term on the left-hand side describes the spatial distribution of phonons, with the diffusion constant

$$D = c\lambda_z/3.\tag{11}$$

The second term describes the energy absorbed during collision, with

$$\chi/D = 3\alpha'/\lambda_z^2,\tag{12}$$

$$\alpha' = -\ln(1-\alpha_s),\tag{13}$$

where α_s is the obstacle absorption coefficient.

The source factor on the right-hand side of Eq. (8) describes the rate of conversion of the sound energy from the direct field to the scattered one.

After solving the diffusion equation (Eq. (8)) with

$$\alpha' < 1/3, \quad \text{and} \quad r/\lambda_z \gg 1,\tag{14}$$

the scattered part of the field (I_s) is obtained. The total sound intensity for a far field condition is

$$I(\lambda_z) = I_d(\lambda_z) + I_s(\lambda_z) =$$
$$= (P/2\pi r^2)\{\exp(-r/\lambda_z) + 3(r/\lambda_z)\exp[-(3\alpha')^{1/2}(r/\lambda_z)]\}, \tag{15}$$

where, according to Eq. 3, the direct part of the field is

$$I_d(\lambda_z) = (P/2\pi r^2)\exp(-r/\lambda_z), \tag{16}$$

which is formed by the phonons that suffer an exponential decay as a result of conversion of the direcrt sound energy into the scattered field.

2.2. The Kuttruff model

KUTTRUFF [16] describes the penetration of phonons through a space with obstacles using the phonon distribution function $f(\mathbf{r}, \mathbf{c}, t)$, which is defined in the phase space, and satisfies the continuity equation. For isotropic scattering the function obeys the Maxwell − Boltzman equation

$$\frac{\partial f}{\partial t} = -\mathbf{c} * \nabla f - \frac{c}{\lambda_t}f + \frac{1-\alpha}{4\pi\alpha}\int f d\mathbf{c}. \tag{17}$$

The changes in the distribution function are caused not only by the existence of its gradient (the first term on the right-hand side in Eq. (17)). The second term and the third term on the right-hand side in Eq. (17) describe scattering of phonons from their original direction and rescattering to the original direction, respectively. For a stationary process of propagation in a two-dimensional space (over the ground plane), the equation reduces to

$$\frac{\partial f}{\partial r}\cos\varphi + \frac{1}{r}\frac{\partial f}{\partial \varphi}\sin\varphi + \frac{1}{\lambda_t}f = \frac{1-\alpha}{2\pi\alpha}\int_0^{2\pi} f(r,\varphi)\,d\varphi. \tag{18}$$

In the Kuttruff two-dimensional model, the scattering cross-section (Eq. (1)) is calculated on the ground area, containing the plot of buildings, and can be called the visual building width:

$$Q_t = 2(a+b)/\pi, \tag{19}$$

where a, b are the lengths of the building sides.

The mean free path is

$$\lambda_t = 1/(nQ_t), \tag{20}$$

where

$$n = \frac{\text{buildings number}}{\text{ground area}}\ [1/m^2], \tag{21}$$

is the average density of buildings.

By solving (Eq. (18)) for the phonon density,

$$\rho(r) = \int_0^{2\pi} f(r,\varphi)\, d\varphi = A_0 K_0(kr), \tag{22}$$

$$k = [\alpha(2-\alpha)]^{1/2}/\lambda_t, \tag{23}$$

$$A_0 = Nk^2\lambda_t/4\pi\alpha c, \tag{24}$$

where $K_0(kr)$ is the Hankel function of zero order, and N is the number of phonons per second emitted by a point source, the sound intensity of the scattered part of the field is found

$$I_s = c\,\rho(r)\,e, \tag{25}$$

with the individual phonon energy

$$e = (1/N)\,(P_s/h). \tag{26}$$

The quantity (P_s/h) is the mean sound energy, in the strip of the width h equal to the average height of the buildings, rescattered back by obstacles, and

$$P_s = \mu(h/\lambda_t)\,(1-\alpha)\,P, \tag{27}$$

where $\mu(h/\lambda_t)$ is the special function approximated by the expression

$$\mu(h/\lambda_t) = (h/\lambda_t)\,[0.423 - \ln(h/\lambda_t)], \quad \text{for} \quad h \ll \lambda_t. \tag{28}$$

Thus, the scattered sound intensity is equal to

$$I_s(\lambda_t) = K_0(kr)\,(2-\alpha)P_s/2\pi h\lambda_t. \tag{29}$$

The total sound intensity is the sum of the direct and scattered parts.

The direct part is also calculated as the average value in the strip of width h:

$$I_d(\lambda_t) = (P/2\pi r^2)\arctan(h/r)\exp(-r/\lambda_t). \tag{30}$$

The attenuation coefficient α appearing in Eq. (29), contains the attenuation at reflection surfaces, the air attenuation, and the effect of leaving the propagation plane by phonons after collision (what is characteristic for two-dimensional problem). For a loosely built-up area with the low buildings it is assessed to be:

$$\alpha = 0.5. \tag{31}$$

With this value of the absorption coefficient, for far-field conditions, where:

$$K_0(x) = (\pi/2x)^{1/2}\exp(-x), \tag{32}$$

the total sound intensity is:

$$I(\lambda_t) = I_s(\lambda_t) + I_d(\lambda_t) = (P/2\pi r^2)\{\exp(-r/\lambda_t) +$$

$$+ [0.423 - \ln(h/\lambda_t)]\,(r/\lambda_t)^{3/2}\exp[-0.87\,(r/\lambda_t)]\}. \tag{33}$$

2.3. The Yeow model

YEOW [43] analyzed an urban space as a room, with perfectly absorbing walls, with dimensions which grow up to infinity. The room contains obstacles, e.g. buildings. They are box-shaped, $(l_0 \times b_0) \times h_0$, and randomly distributed on the ground. After estimation of the reverberation time for such an enclosure, acoustical energy distribution is described by the function

$$(E/E_0) = (r_0/r)^2 \exp[-\sigma_1(r-r_0)]. \tag{34}$$

The Eq. (34) gives acoustical energy E at the point P, at distance r from a stationary point source located on the ground, where there is no direct wave. It is estimated in relation to energy E_0 at distance r_0.

The basic assumptions of the approach are the following: the sound fields are, at least locally perfectly diffuse, reflections are specular, distribution of absorption is uniform.

The decay factor in Eq. (34) for a three-dimensional space is

$$\sigma_1 = 4(1/\lambda_{f1})(1-f)(\bar{\alpha} + m\lambda_{f1}), \tag{35}$$

where m is the air absorption coefficient and $\bar{\alpha}$ is the effective absorption coefficient.

$$\bar{\alpha} = [(1-f)(1+\alpha_g) + \alpha_0 fg_1]/[fg_1 + 2(1-f)]. \tag{36}$$

The free path is defined by the relation

$$\lambda_{f1} = 4V/S = 4h_0(1-f)/[fg_1 + 2(1-f)], \tag{37}$$

where V is the enclosed volume, and S is the total bounding surface area.

The packing function

$$f = nQ_f, \tag{38}$$

represents the buildings plan area per unit ground area, where n (Eq. (21)) is the building density, and the building scattering cross-section

$$Q_f = l_0 b_0, \tag{39}$$

is the building base surface area. The factor

$$g_1 = 2h_0(l_0 + b_0)/(l_0 b_0), \tag{40}$$

is the ratio of a building surface area to its cross-section (Eq. (39)).

The building absorption coefficient is put assumed to be

$$\alpha_0 = \begin{cases} 0.1, & fg_1 \gg (1-f) \quad \text{(high buildings)}, \\ 0.5, & fg_1 \ll (1-f) \quad \text{(low buildings)}. \end{cases} \tag{41}$$

The decay factor (Eq. (35)) is

$$\sigma_1 = (4/\lambda_{f1})(1-f)(\bar{\alpha} + 0.001\lambda_{f1}), \tag{42}$$

with the average absorption coefficient

$$\bar{a} = [(1+\alpha_g)(1-f)+\alpha_0 fg_1]/[fg_1+2(1-f)], \tag{43}$$

where the building absorption coefficient (α_0), according to Eq. (41), is assumed to be

$$\alpha_0 = 0.1, \quad \text{or} \quad \alpha_0 = 0.5. \tag{44}$$

The ground absorption coefficient is

$$\alpha_g = 0.1, \tag{45}$$

and the air absorption coefficient equals

$$m = 0.001. \tag{46}$$

The sound intensity, according to Eq. (34), is expressed by the function

$$I(\sigma_1) = (P/2\pi r^2)\exp(-\sigma_1 r), \tag{47}$$

where it is assumed that at the distance r_0, near the source, the free field conditions are satisfied.

For the two-dimensional case of propagation in the horizontal plane, energy distribution is governed by the formula

$$(E/E_0) = (r_0/r)\exp[-\sigma_2(r-r_0)], \tag{48}$$

where:

$$\sigma_2 = (\pi/\lambda_{f2})(1-f)(\alpha_0+m\lambda_{f2}), \tag{49}$$

$$\lambda_{f2} = \pi(1-f)/fg_2, \tag{50}$$

$$g_2 = 2(l_0+b_0)/(l_0 b_0). \tag{51}$$

The sound intensity for the two-dimensional case is

$$I(\sigma_2) = (P/2\pi r^2)r\exp(-\sigma_2 r). \tag{52}$$

3. Example of the urban structure segment

As a segment of an urban structure, a built-up area with independent houses, modeled by shoe-boxes (10 m × 10 m × 10 m) (Fig. 1a), is assumed. The number of buildings equals

$$N = 15. \tag{53}$$

There exist two possibilities of assuming the ground area. The ground surface can be assumed in the form of a segment of a circle (Fig. 1b),

$$G_c = 15\,943 \text{ m}^2, \tag{54}$$

or in a rectangular form,

$$G_r = 14\,400 \text{ m}^2. \tag{55}$$

For the chosen segment the three models described above will now be applied.

3.1. Application of the Kurze model

In the Kurze model the analyzed space is closed in the disc segment of the base equal to the segment of the circle in Fig. 1b, and its thickness is equal to the building height. Thus,
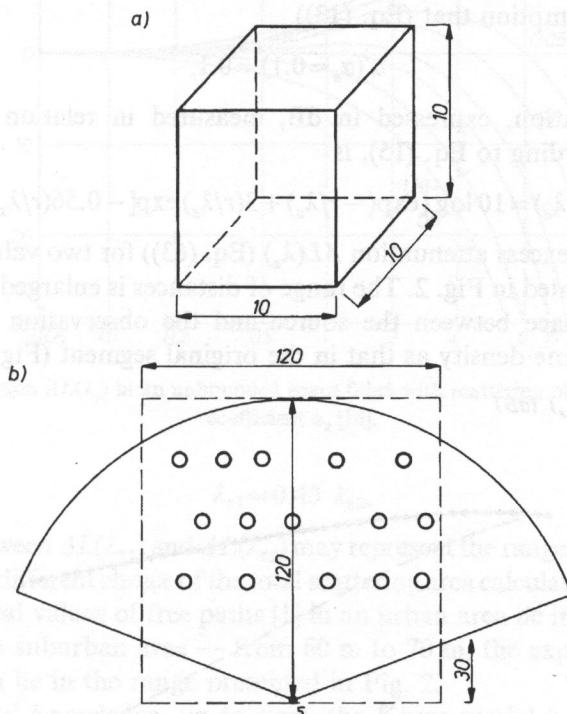

Fig. 1. Urban space segment, built-up by independent houses (a), with building density calculated for rectangular and circular shape of segment (b). All dimensions in meters.

$$V = 159\,430 \text{ m}^3. \tag{56}$$

The total area of the scatters includes the ground surface (Eq. (53)) plus the surfaces of the building walls and roofs

$$S_1 = 20\,443 \text{ m}^2, \tag{57}$$

and according to Eq. (10)

$$\lambda_{z1} = 31.20 \text{ m}, \tag{58}$$

where it is assumed that

$$\kappa = 1. \tag{59}$$

When the model without the ground is considered then the total area of the scatters is equal to

$$S_2 = 9\,000 \text{ m}^2, \tag{60}$$

and

$$\lambda_{z2} = 72.38 \text{ m}. \tag{61}$$

Under the assumption that (Eq. (13))

$$\alpha'(\alpha_s = 0.1) = 0.1, \tag{62}$$

the excess attenuation, expressed in dB, measured in relation to the free field propagation, according to Eq. (15), is

$$\Delta L(\lambda_z) = 10 \log \{ \exp(-r/\lambda_z) + 3(r/\lambda_z) \exp[-0.56(r/\lambda_z)] \}. \tag{63}$$

The calculated excess attenuation $\Delta L(\lambda_z)$ (Eq. (63)) for two values of λ_z (Eq. (58), Eq. (61)) are presented in Fig. 2. The range of distances is enlarged up to 500 m. This means that the space between the source and the observation point is filled by obstacles of the same density as that in the original segment (Fig. 1).
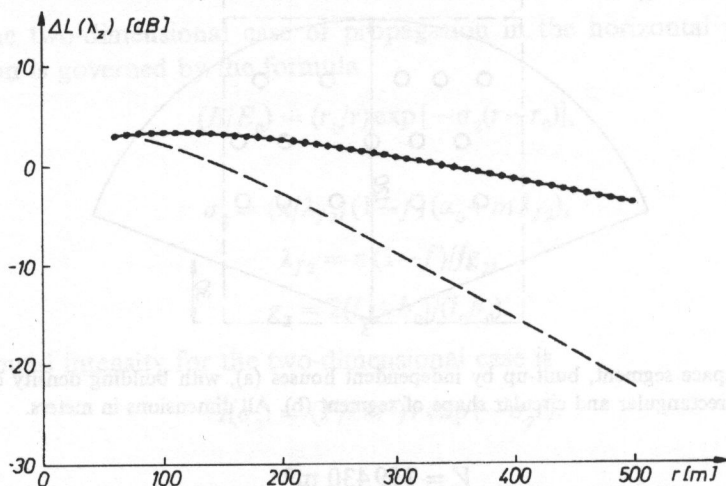

Fig. 2. Excess attenuation $\Delta L(\lambda_z)$ (Eq. (63)) calculated according to the Kurze model: $(- - -)$ $\Delta L(\lambda_{z1} = 31.20$ m) with ground, and $(-\cdot-\cdot-)$ $\Delta L(\lambda_{z2} = 72.38$ m) without ground.

Figure 2 shows that the larger is the free path, the smaller will be the excess attenuation.

From the chart (Fig. 3) reprinted after KURZE [14] it can be observed that, in a certain range of distances, the excess attenuation is positive. It means that the total sound intensity decrease is smaller than in the free space conditions. This is the general phenomenon, when for a certain obstacle density, due to multiple reflections, the sound intensity increase is observed. In the other range, the dominant effect is screening by obstacles, therefore the sound intensity decays faster than in the free space conditions.

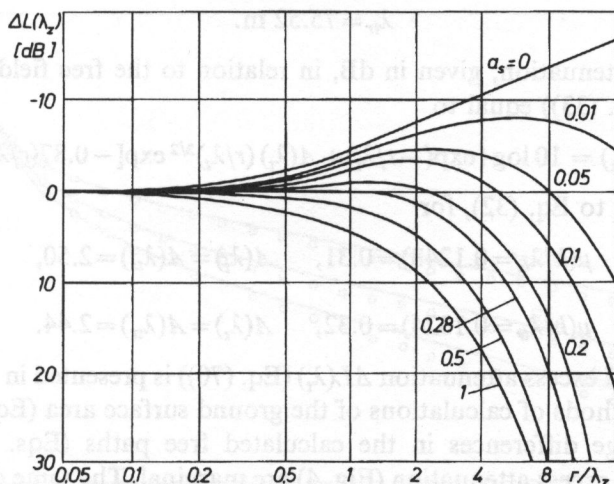Since in the free path definition (Eq. (10)) the parameter κ is used, and

Fig. 3. Excess attenuation $\Delta L(\lambda_z)$ in an unbounded space filled with scattering obstacles with absorption coefficient α_s [14].

$$\lambda_{z1}=0.43 \ \lambda_{z2}, \tag{64}$$

the differences between $\Delta L(\lambda_{z1})$ and $\Delta L(\lambda_{z2})$ may represent the range of confidence of the Kurze model with different choice of the total scattering area calculation (Eqs. (57), (60)).

Since the typical values of free paths [1] in an urban area lie in the range from 40 m to 50 m, and in suburban area — from 60 m to 70 m, the expected values of the excess attenuation lie in the range presented in Fig. 2.

To the authors' knowledge, up to now, the Kurze model has no experimental confirmation in the literature concerning urban areas, since Kurze has prepared it for industrial halls.

3.2. Application of the Kuttruff model

When the Kuttruff model is applied, the average building density (Eq. (21)) can be calculated using either the ground surface area of the circular segment (Eq. (54))

$$n_c=0.00098 \ 1/m^2, \tag{65}$$

or the rectangular segment (Eq. (55))

$$n_r=0.00104 \ 1/m^2. \tag{66}$$

According to Eq. (19),

$$Q_t=12.73 \ m, \tag{67}$$

and according to Eq. (20),

$$\lambda_{tc}=80.16 \ m, \tag{68}$$

$$\lambda_{tr} = 75.52 \text{ m.} \tag{69}$$

The excess attenuation, given in dB, in relation to the free field propagation is (according to Eq. (33)) equal to

$$\Delta L(\lambda_t) = 10 \log \{\exp(-r/\lambda_t) + A(\lambda_t)(r/\lambda_t)^{3/2} \exp[-0.87(r/\lambda_t)]\}, \tag{70}$$

where, according to Eq. (32), for

$$\mu(h/\lambda_{tc} = 0.1248) = 0.31, \qquad A(\lambda_t) = A(\lambda_{tc}) = 2.50, \tag{71}$$

and for

$$\mu(h/\lambda_{tr} = 0.1324) = 0.32, \qquad A(\lambda_t) = A(\lambda_{tr}) = 2.44. \tag{72}$$

The calculated excess attenuation $\Delta L(\lambda_t)$ (Eq. (70)) is presented in Fig. 4. Since the two different methods of calculations of the ground surface area (Eqs. (54), (55)) do not produce large differences in the calculated free paths (Eqs. (68), (60)), the differences in the excess attenuation (Fig. 4) are marginal. The same effect of positive excess attenuation as that occurring in the Kurze model is observed. This effect was analyzed by KUTTRUFF [15] in the reverberation chamber.
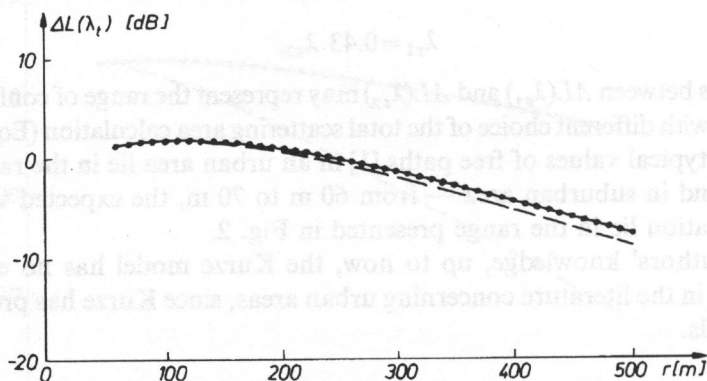

Fig. 4. Excess attenuation $\Delta L(\lambda_t)$ (Eq. (70)) calculated according to the Kuttruff model: (—•—•—) $\Delta L(\lambda_{tc} = 80.16$ m) for circular ground segment, and (— — —) $\Delta L(\lambda_{tr} = 75.52$ m) for rectangular ground segment.

In order to verify his model [16], Kuttruff presents the results of the Monte Carlo experiment (Fig. 5). The plot of the function:

$$G = L - L_p + 20 \log(\lambda_t/1\text{m}) \tag{73}$$

is shown, where $L - L_p$ is the total sound level decrease with zero source level output. The agreement is found to be satisfactory.

In the Kuttruff model the attenuation coefficient (Eq. (31)) has to contain attenuation at the building surfaces, the air absorption, and the effect of leaving the propagation plane. BULLEN made a comparison [3] of the Kuttruff model with his own three-dimensional model [1]. He assumed the random walk of phonons and specular
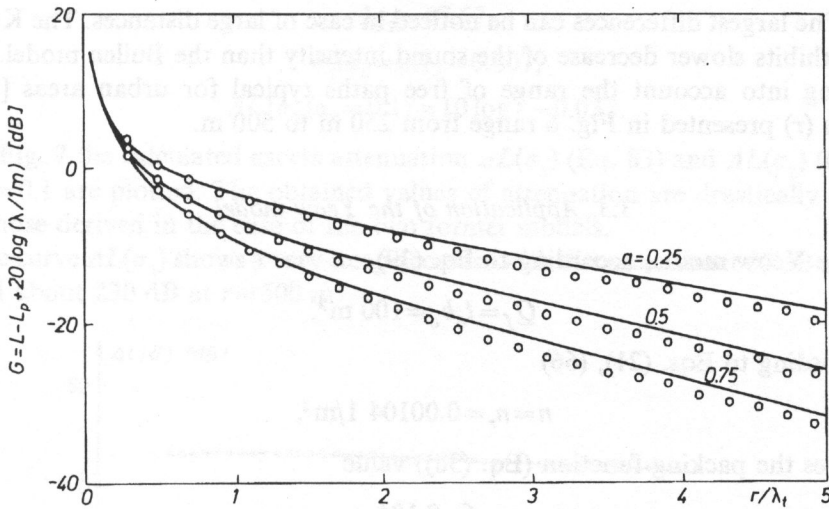
Fig. 5. Comparison of the Kuttruff model results with the Monte Carlo experiment: $h/\lambda_t = 0.1$, $L - L_p$ is total sound level decrease with zero source level output [16].

reflection from the building surfaces, placed in an urban structure, providing the phonon free path of a given value. This gives the attenuation factor as a function of the number of reflections, similarly to the geometrical acoustics.

In Fig. 6 is presented the same function (Eq. (73)) as that in Fig. 5; first it was calculated according to the Kuttruff model, and then — according to the Bullen
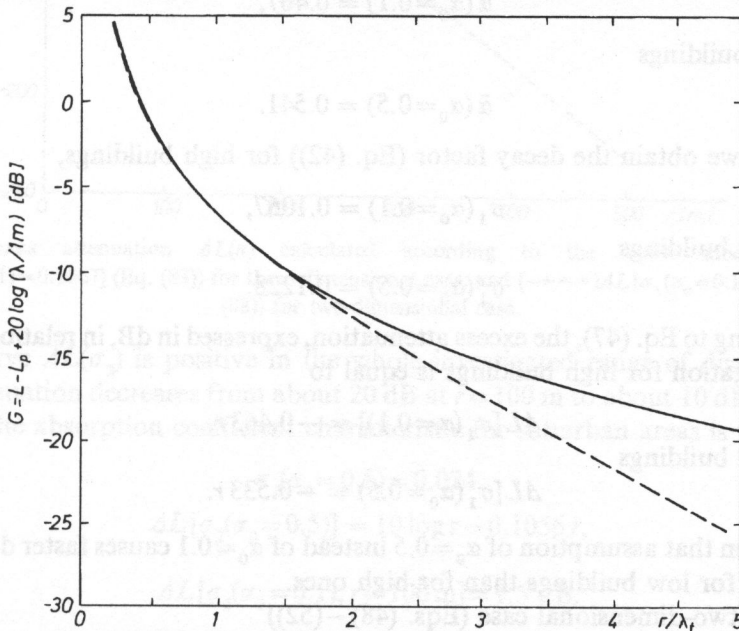

Fig. 6. Comparison of the Kuttruff model results (— — —) with the Bullen model results (———) [3].

model. The largest differences can be noticed in case of large distances. The Kuttruff model exhibits slower decrease of the sound intensity than the Bullen model.

Taking into account the range of free paths typical for urban areas [1], the distances (r) presented in Fig. 6 range from 250 m to 500 m.

3.3. Application of the Yeow model

In the Yeow model, according to Eq. (39),

$$Q_f = l_0 b_0 = 100 \text{ m}^2, \tag{74}$$

and according to Eqs. (21), (66)

$$n = n_r = 0.00104 \ 1/\text{m}^2, \tag{75}$$

what gives the packing function (Eq. (38)) value

$$f = 0.104. \tag{76}$$

According to Eq. (40)

$$g_1 = 4, \tag{77}$$

so that the path (Eq. (37)) is equal to

$$\lambda_{f1} = 16.23 \text{ m.} \tag{78}$$

According to Eqs. (41), (43), the average absorption coefficient for high buildings

$$\bar{\alpha}(\alpha_0 = 0.1) = 0.467, \tag{79}$$

or for low buildings

$$\bar{\alpha}(\alpha_0 = 0.5) = 0.541. \tag{80}$$

As a result we obtain the decay factor (Eq. (42)) for high buildings,

$$\sigma_1(\alpha_0 = 0.1) = 0.1067, \tag{81}$$

and for low buildings

$$\sigma_1(\alpha_0 = 0.5) = 0.1228. \tag{82}$$

According to Eq. (47), the excess attenuation, expressed in dB, in relation to the free field propagation for high buildings is equal to

$$\Delta L[\sigma_1(\alpha = 0.1)] = -0.463 \, r, \tag{83}$$

and for low buildings

$$\Delta L[\sigma_1(\alpha_0 = 0.5)] = -0.533 \, r. \tag{84}$$

It can be seen that assumption of $\alpha_0 = 0.5$ instead of $\alpha_0 = 0.1$ causes faster decay of the sound level for low buildings than for high ones.

For the two-dimensional case (Eqs. (48)−(52))

$$g_2 = 0.4, \tag{85}$$

$$\lambda_{f2} = 67.67, \tag{86}$$

$$\sigma_2(\alpha_0 = 0.1) = 0.007, \tag{87}$$

$$\Delta L[\sigma_2(\alpha_0 = 0.1) = 10 \log r - 0.03 \, r. \tag{88}$$

In Fig. 7 the calculated excess attenuation $\Delta L(\sigma_1)$ (Eq. 83) and $\Delta L(\sigma_2)$ (Eq. (88)) for $\alpha_0 = 0.1$ are plotted. The obtained values of attenuation are drastically different than those derived in the case of the two former models.

The curve $\Delta L(\sigma_1)$ shows a very steep decrease with distance: about 50 dB at $r = 100$ m, and about 230 dB at $r = 500$ m.
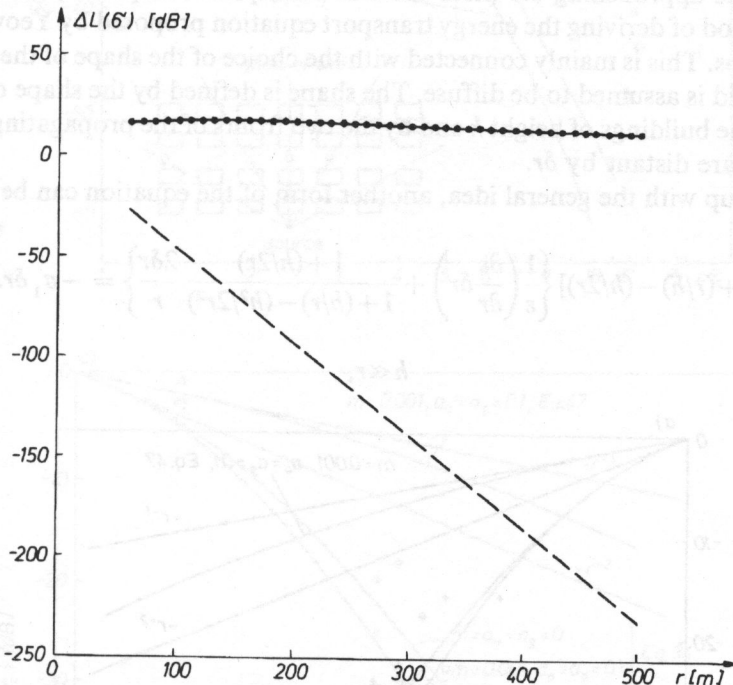

Fig. 7. Excess attenuation $\Delta L(\sigma)$ calculated according to the Yeow model: ($---$) $\Delta L[\sigma_1(\alpha_0 = 0.1) = 0.1067]$ (Eq. (83)) for three-dimensional case, and ($-\bullet-\bullet-$)$\Delta L[\sigma_2(\alpha_0 = 0.1) = 0.007]$ (Eq. (88)) for two-dimensional case.

The curve $\Delta L(\sigma_2)$ is positive in the whole investigated range of distances. The excess attenuation decreases from about 20 dB at $r = 100$ m to about 10 dB at $r = 500$. When the absorption coefficient characteristic for suburban areas is taken, then

$$\sigma_2(\alpha_0 = 0.5) = 0.024, \tag{89}$$

$$\Delta L[\sigma_2(\alpha_0 = 0.5)] = 10 \log r - 0.1056 \, r, \tag{90}$$

what gives

$$\Delta L[\sigma_2(\alpha_0 = 0.1), r = 100 \, \text{m}] = 9.5 \, \text{dB}, \tag{91}$$

$$\Delta L[\sigma_2(\alpha_0 = 0.1), r = 500 \, \text{m}] = -25.5 \, \text{dB}. \tag{92}$$

The above results show that the two-dimensional case is more sensitive to the value of the obstacle absorption coefficient than the three-dimensional case.

In case of the Yeow model, a substantial difference appears between the three-dimensional and two-dimensional cases.

It could be said that the two former models have stronger theoretical grounds than the Yeow model, but the last one is confirmed [43] by field measurements (Fig. 8). In spite of the fact that the buildings arrangement is regular, and that in the most cases buildings are relatively long, the range of fairly good agreement can be found between the theoretical and experimental curves at large distances and under high packing function value approaching 0.5 (near the half of a space occupied by obstacles).
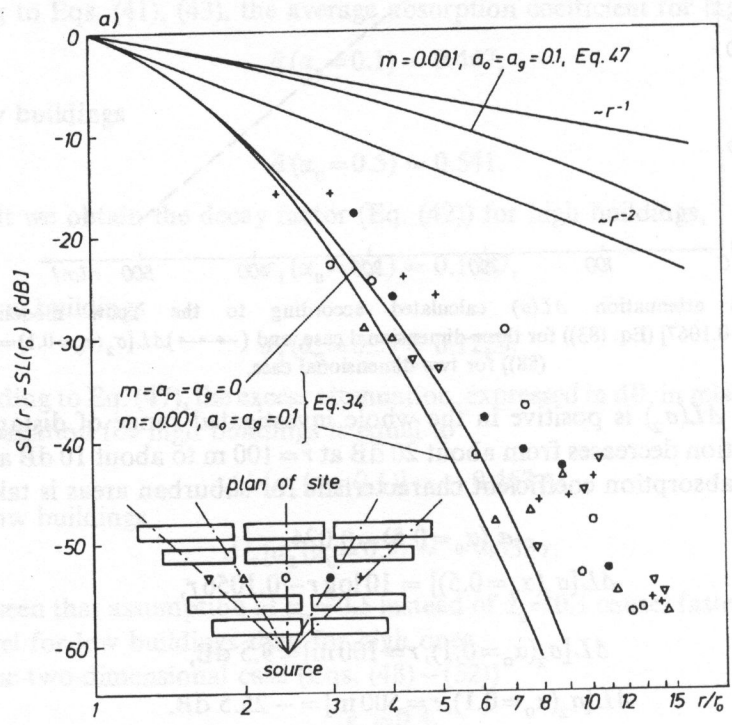
The method of deriving the energy transport equation proposed by Yeow contains arbitrary steps. This is mainly connected with the choice of the shape of the cell inside which the field is assumed to be diffuse. The shape is defined by the shape of the strip containing the buildings of height h and by the two fronts of the propagating spherical wave which are distant by δr.

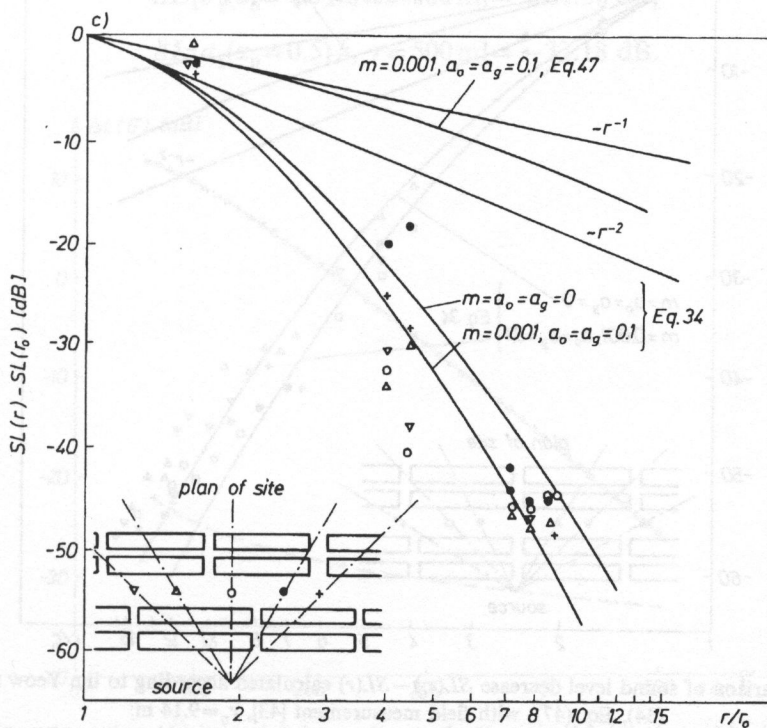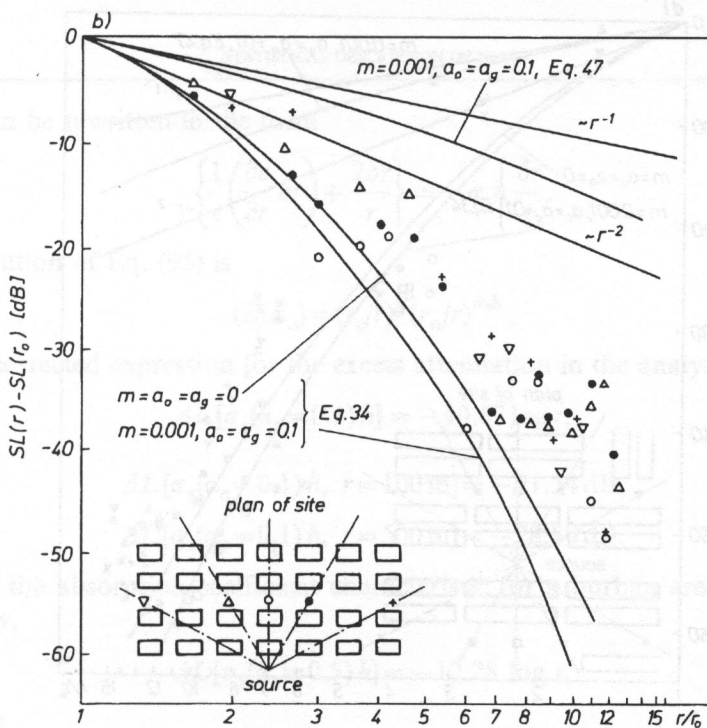Keeping up with the general idea, another form of the equation can be obtained:

$$[1+(r/h)-(h/2r)]\left\{\frac{1}{\varepsilon}\left(\frac{\partial\varepsilon}{\partial r}\delta r\right)+\frac{1+(h/2r)}{1+(h/r)-(h^2/2r^2)}\frac{2\delta r}{r}\right\}=-\sigma_1\delta r. \qquad (93)$$

For

$$h \ll r, \qquad (94)$$

Fig. 8. Comparison of sound level decrease $\Delta L(r) = W(r) - W(r_0)$ calculated according to the Veer model (Eq.

[217]

d)

$m = 0.001, a_o = a_g = 0.1, Eq. 47$

$-r^{-1}$

$m = a_o = a_g = 0$

$m = 0.001, a_o = a_g = 0.1$ $\Big\}$ Eq. 34

$-r^{-2}$

plan of site

source

$SL(r) - SL(r_o)$ [dB]

r/r_o

e)

$m = 0.001, a_o = a_g = 0.1, Eq. 47$

$-r^{-1}$

$-r^{-2}$

$m = a_o = a_g = 0$

$m = 0.001, a_o = a_g = 0.1$ $\Big\}$ Eq. 34

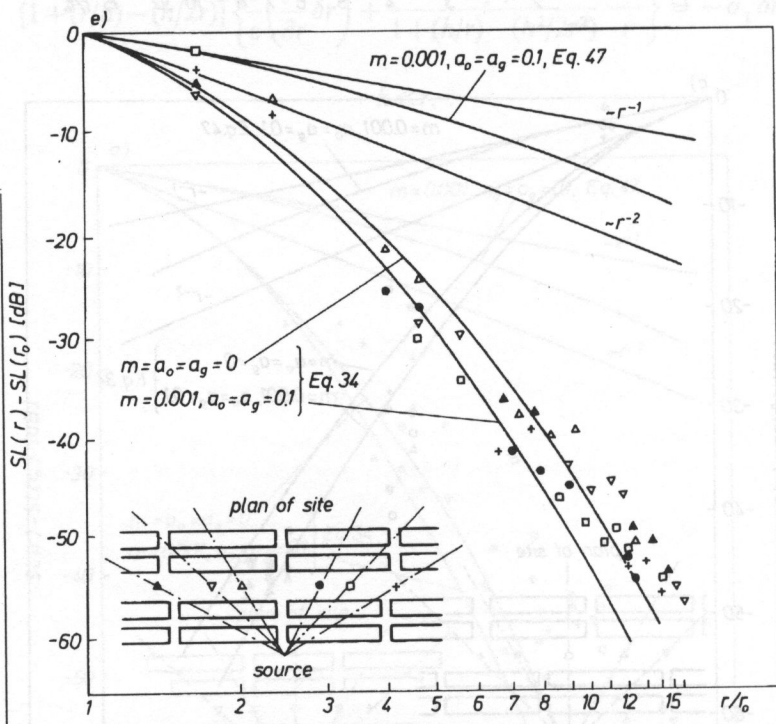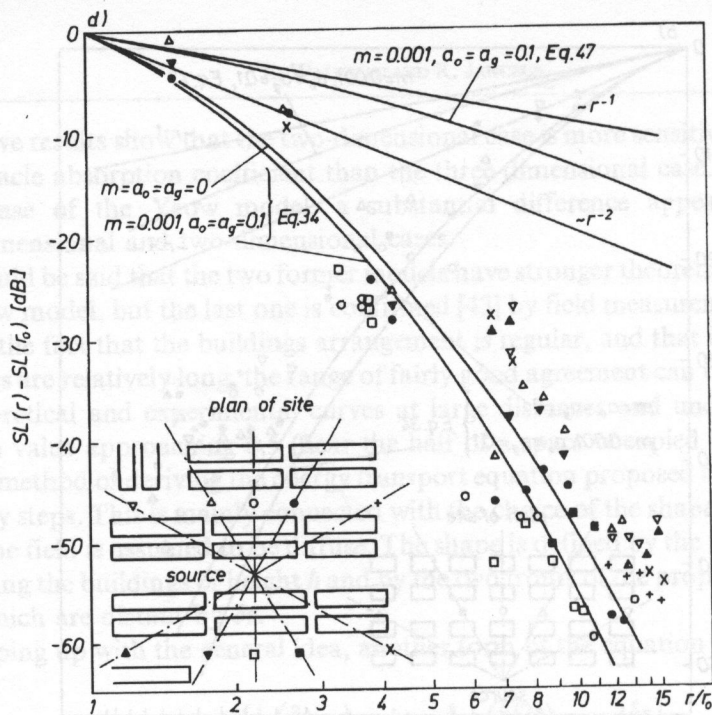plan of site

source

$SL(r) - SL(r_o)$ [dB]

r/r_o

Fig. 8. Comparison of sound level decrease $SL(r_0) - SL(r)$ calculated according to the Yeow model (Eq. (34), Eq. (47)) with field measurement [43], $r_0 = 9.14$ m:
(a) — $f = 0.35$, $Q = 1012$ m², $h = 4.88$ m; (b) — $f = 0.23$, $Q = 203$ m², $h = 8.23$ m; (c) — $f = 0.37$, $Q = 1055$ m², $h = 8.23$ m; (d) — $f = 0.39$, $Q = 1231$ m², $h = 8.23$ m; (e) — $f = 0.45$, $Q = 1177$ m², $h = 8.23$ m.

Eq. (93) can be rewritten in the form

$$\left\{\frac{1}{\varepsilon}\left(\frac{\partial \varepsilon}{\partial r} \delta r\right)+\frac{2\delta r}{r}\right\}=-\sigma_1 h \frac{\delta r}{r}. \tag{95}$$

The solution of Eq. (95) is

$$(E/E_0)=(r_0/r)^2 (r_0/r)^{\sigma_1 h}. \tag{96}$$

Thus, the corrected expression for the excess attenuation in the analyzed example is

$$\Delta L[\sigma_1(\alpha_0=0.1)h] = -10.67 \log r, \tag{97}$$

what gives

$$\Delta L[\sigma_1(\alpha_0=0.1)h, \ r=100\,\text{m}] = -21.34\,\text{dB}, \tag{98}$$

$$\Delta L[\sigma_1(\alpha_0=0.1)h, \ r=500\,\text{m}] = -28.80\,\text{dB}. \tag{99}$$

The use of the absorption coefficient characteristic for suburban area gives a little faster decay,

$$\Delta L[\sigma_1(\alpha_0=0.5)h] = -12.28 \log r, \tag{100}$$

what yields

$$\Delta L[\sigma_1(\alpha_0=0.5)h, \ r=100\,\text{m}] = -24.56\,\text{dB}, \tag{101}$$

$$\Delta L[\sigma_1(\alpha_0=0.5)h, \ r=500\,\text{m}] = -33.18\,\text{dB}. \tag{102}$$
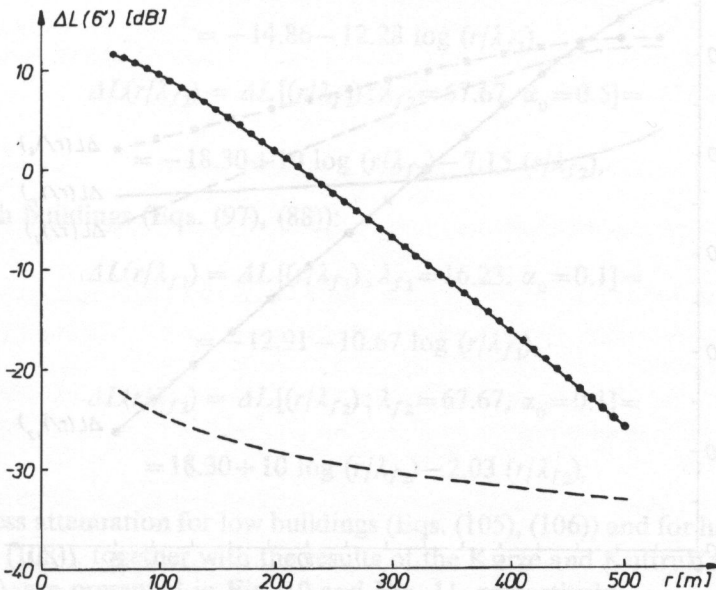

Fig. 9. Excess attenuation $\Delta L(\sigma)$ calculated according to the Yeow model: $(---)$ $\Delta L[\sigma_1 (\alpha_0=0.5)h]$ (Eq. (100)) for three-dimensional case, and $(-\bullet-\bullet-)$ $\Delta L[\sigma_2(\alpha_0=0.5)=0.024]$ (Eq. (90)) for two-dimensional case.

The excess attenuation, with $\alpha_0 = 0.5$, for the corrected three-dimensional case (Eq. (100)) and for the two-dimensional case (Eq. (90)) is presented in Fig. 9. Now, the results obtained are not so different from the Kurze and the Kuttruff models as before.

4. Comparison of the three models

It seems to be useful to make a general comparison of the three models, the dimensionless parameter (r/λ) being used.

The excess attenuation in the Kurze model (Eq. (63)) is derived from the diffusion equation in the three-dimensional space (Eq. (18)). The absorbed energy is described by the coefficient $\alpha'(\alpha_s)$ (Eqs. (13), (62)). Without other restrictions, the excess attenuation

$$\Delta L(r/\lambda_z) = 10 \log \{\exp(-r/\lambda_z) + 3(r/\lambda_z) \exp[-0.56(r/\lambda_z)]\} \tag{103}$$

can be presented as a function of dimensionless parameter (r/λ_z), what makes it free of ambiguity in calculation of the free path length.

The excess attenuation in the Kuttruff model (Eq. (70)) is derived from the Maxwell−Boltzman equation for the phonon distribution function (Eq. (17)). Two-dimensional propagation is assumed with the specific attenuation factor α (Eq. (31)). The excess attenuation,
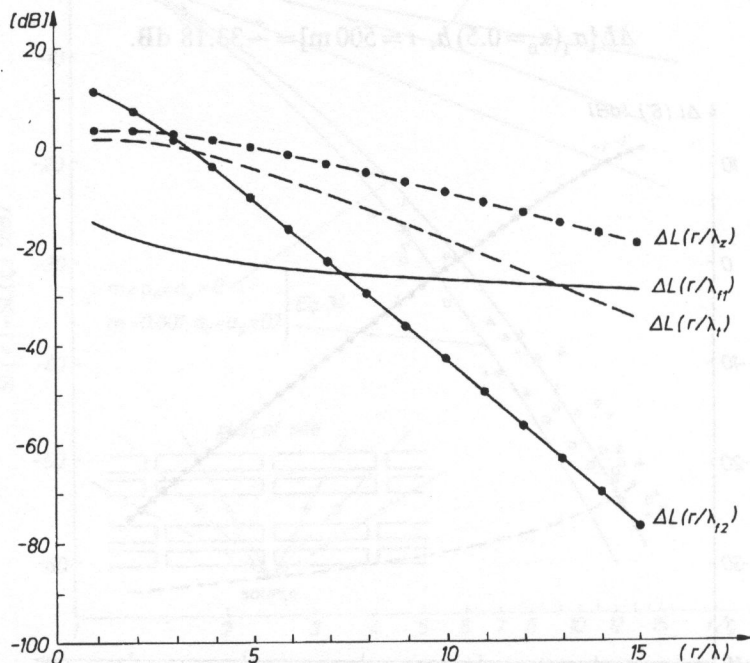

Fig. 10. Excess attenuation: $\Delta L(r/\lambda_z)$ (Eq. (103)) — in the Kurze model, $\Delta L(r/\lambda_z)$ (Eq. (104)) — in the Kuttruff model, and the Yeow model with $\alpha_0 = 0.5$: for three-dimensional propagation $\Delta L(r/\lambda_{f_1})$ (Eq. (105)), and two-dimensional propagation $\Delta L(r/\lambda_{f_2})$ (Eq. (106)).

$$\Delta L(r/\lambda_t) = 10 \log \{\exp(-r/\lambda_t) + 2.44 \, (r/\lambda_t)^{3/2} \exp[-0.87(r/\lambda_t)]\}, \qquad (104)$$

as the function of dimensionless parameter (r/λ_t) is slightly influenced by the choice of the ground surface shape (Fig. 4). Here, the value for the rectangular one is adopted (Eq. (72)).

The excess attenuation curves (Eqs. (103), (104)), presented in Fig. 10, are founded on the diffusion process patterns. The fact that $\Delta L(r/\lambda_z)$ corresponds to the three-dimensional propagation, and $\Delta L(r/\lambda_t)$ — to the two-dimensional propagation is reflected by different method of calculation of the free paths. This fact, and different descriptions of the attenuation process influence the functional form of the expressions (Eqs. (103), (104)). Being the functions of the dimensionless parameter (r/λ), they still keep on being different. The difference increases with increasing value of (r/λ).

The excess attenuation in the Yeow model for three-dimensional propagation (Eqs. (97), (100)) and two-dimensional propagation (Eqs. (88), (90)) contains the average attenuation coefficients (Eq. (43)) which have different values for high and low buildings (Eq. (41)). The excess attenuation cannot be presented in a universal form as a function of dimensionless parameter (r/λ), but it can be presented for the urban system under consideration. This mean a direct introduction of the free paths calculated for the system λ_{f1} (Eq. (78)), λ_{f2} (Eq. (86)) into Eqs. (97), (100) and Eqs. (88), (90), respectively.

The excess attenuation for low buildings (Eqs. (100), (90)) is:

$$\Delta L(r/\lambda_{f1}) = \Delta L[(r/\lambda_{f1}); \lambda_{f1} = 16.23, \, \alpha_0 = 0.5] =$$

$$= -14.86 - 12.28 \log (r/\lambda_{f1}), \qquad (105)$$

$$\Delta L(r/\lambda_{f2}) = \Delta L[(r/\lambda_{f2}); \lambda_{f2} = 67.67, \, \alpha_0 = 0.5] =$$

$$= -18.30 + 10 \log (r/\lambda_{f2}) - 7.15 \, (r/\lambda_{f2}), \qquad (106)$$

and for high buildings (Eqs. (97), (88)):

$$\Delta L(r/\lambda_{f1}) = \Delta L[(r/\lambda_{f1}); \lambda_{f1} = 16.23, \, \alpha_0 = 0.1] =$$

$$= -12.91 - 10.67 \log (r/\lambda_{f1}), \qquad (107)$$

$$\Delta L(r/\lambda_{f2}) = \Delta L[(r/\lambda_{f2}); \lambda_{f2} = 67.67, \, \alpha_0 = 0.1] =$$

$$= 18.30 + 10 \log (r/\lambda_{f2}) - 2.03 \, (r/\lambda_{f2}), \qquad (108)$$

The excess attenuation for low buildings (Eqs. (105), (106)) and for high buildings (Eqs. (107), (108)), together with the results of the Kurze and Kuttruff models (Eqs. (103), (104)), are presented in Fig. 10 and Fig. 11, respectively.

The Yeow model is founded on the concept of energy distribution by formation of subregions where the field is assumed to be diffuse. The concept substantially differs
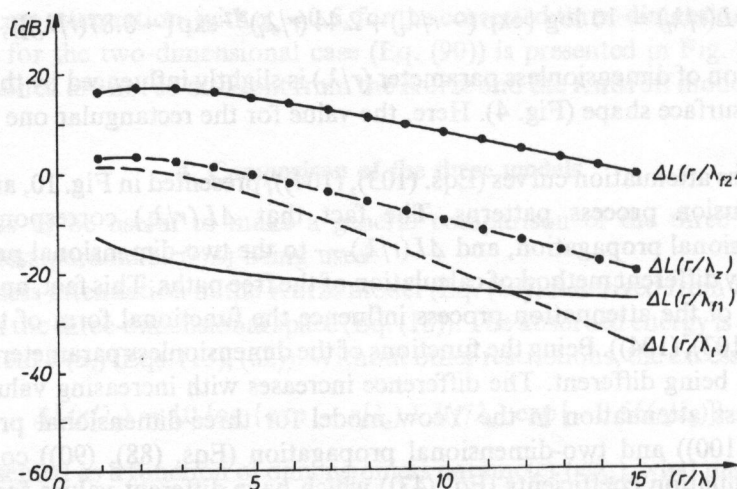
Fig. 11. Excess attenuation: $\Delta L(r/\lambda_z)$ (Eq. (103)) — in the Kurze model, $\Delta L(r/\lambda_t)$ (Eq. (104)) — in the Kuttruff model, and the Yeow model with $\alpha_0 = 0.1$: for three-dimensional propagation $\Delta L(r/\lambda_{f_1})$ (Eq. (107)), and two-dimensional propagation $\Delta L(r/\lambda_{f_2})$ (Eq. 108)).

from the diffusion process applied in the Kurze and the Kuttruff models. In spite of it, some regions (Fig. 10, 11) exist where the different models give approximately the same value of the excess attenuation.

When the diffuse models are assumed to give the functional dependence of the excess attenuation on the distance (r): $\Delta L(r;\lambda_z,\alpha')$, $\Delta L(r;\lambda_t,\alpha)$, $\Delta L(r;\lambda_{f_1},\alpha_0)$, $\Delta L(r;\lambda_{f_2},\alpha_0)$, where the model parameters (λ_z,α'), (λ_t,α), (λ_{f_1},α_0), (λ_{f_2},α_0) can be determined in a partly arbitrary way, then certain classes of urban systems can be found in which the models to the same results.

5. Discussion

The aim of the paper was to present the three different methods of construction of the diffuse models for sound propagation.

The diffuse models disregard all the effect connected with the wave nature of sound propagation and shadow effect appearing in geometrical acoustics. They neglect the shapes of buildings and their exact locations. Application of diffuse model requires formation of a homogeneous field, what is possible after a sufficiently large number of collisions with randomly placed buildings which can be treated as almost isotropic scatters.

This is the reason why the diffuse model cannot give a precise description of the acoustical field in a built-up area.

The numerical examples given show how large quantitative differences result from the application of the different models.

The authors of the models are aware of the approximate character of their models. They consider their models as tools for a general investigation of spatial and temporal

behavior of acoustical field between buildings resulting from the statistical nature of the traffic noise [16].

The advantage of the models consists in a simple form of the governing equation. They demonstrate the general principles of noise penetration into the area under investigation.

The free path (λ), as the basic parameter of the models, is differently estimated in various models. The absorption process is also differently described in the models. When the dimensionless parameter (r/λ) is introduced, the different models still result in different expression for the excess attenaution, but some common regions may be observed (Fig. 10, Fig. 11).

The diffuse field model of noise propagation seems to be useful for certain classes of urban systems in the case when an empirical model of noise propagation is developed. Then, using the functional form of the excess attenuation, the two parameters: the free path and the factor describing attenuation must be established experimentally for the class of urban systems considered.

The present paper represents a preliminary step in comparing the model [9], [40], describing noise propagation in an urban area system at a microscopic level, with other models (such as the diffuse ones). A more detailed analysis of the problem will be published in the future.

Aknowledgments

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