# THEORY OF SURFACE ACOUSTIC WAVE REVERSING MULTISTRIP COUPLER

## E. DANICKI

Institute of Fundamental Technological Research
Polish Academy of Sciences
(00-049 Warszawa, ul. Świętokrzyska 21)

A reversing, multistrip coupler device is presented that can find numerous applications in SAW devices. Fundamental theory and first experimental results are presented on an application of rmsc in SAW resonator.

## 1. Introduction

Surface acoustic wave (SAW) propagating on a surface of piezoelectric substrates is accompanied by a wave of electric potential on the surface. SAW can be excited by metal strips on the surface of a piezoelectric body when they are supplied with electric potential. Multistrip coupler is the directional coupler of two adjacent SAW channels where the coupling is provided by periodic metal strips covering both channels.

A new reversing multistrip coupler (rmsc) is proposed and analyzed where the strips are interlaced between the channels. This results in coupling of forward propagating potential wave in one channel to the backward propagating wave in the other channel in certain frequency bands. The device can find many applications in SAW technology, allowing construction of SAW pass-band and dispersive filters.

An interesting mathematical problem arises in modelling of the above system of interlaced strips. A "continuous" eigenvalue problem with mixed electrics and mechanical boundary conditions must be solved to characterize SAW propagation in periodic systems of thin metal strips. Another "discrete" eigenvalue problem is encountered in rmsc, resulting from the equality of certain strip currents and potentials in both acoustic channels.

Let us consider a surface acoustic wave propagating in a piezoelectric halfspace $y \geqslant 0$. We observe a wave of particle displacement $\mathbf{u} \exp(j\omega t - jk_v x)$ on the substrate surface, and a wave of electric potential $\varphi \exp(j\omega t - jkx)$ accompanying SAW due to the substrate piezoelectricity; $\omega$ and $k_v$ are angular frequency and wave-number. If the substrate surface is metallized, the SAW wave number takes another value, $k_0$, where $k_0 > k_v$, and instead of the surface electric potential which is zero, there is surface

electric charge density $\Delta D_\perp$ equal to the electric flux discontinuity on both sides of metallization. The relative velocity change $\Delta v/v = (k_v - k_0)/k_0$ is an important parameter characterizing piezoelectric substrates (another parameter [1] is the "effective surface permittivity" $\varepsilon_e$).

In the case of partial surface metallization in the form of a periodic metal strip deposited on the substrate surface, the corresponding wave-numbers of SAW propagating perpendicularly to the strips are $r_v$ and $r_0$ for free, and short-circuited strips, $k_v < r_v < r_0 < k_0$. Let the system of strips spans over two adjacent acoustic channels, and let the SAW beam propagates only in the upper channel (Fig. 1a). The electric potential induced on the strips by SAW is distributed over the lower channel as well. This is the travelling-wave potential which excites SAW in the lower channel. This is a synchronous excitation because SAWs in both channels have the same velocity. The system of strips is then a directional coupler of two acoustic channels, the microstrip coupler [2] (msc).
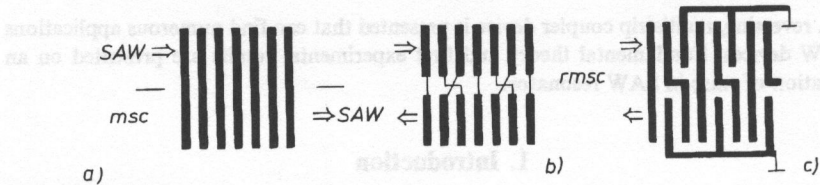


Fig. 1. a) Multistrip coupler (msc); b) Reversing msc in basic configuration and c) practical structure.

Let us consider periodic strips with three strips per wave-length of SAW. Thus, the following strip potentials are phase-shifted by $0°$, $-120°$, $-240°$, corresponding to $\exp(j\omega t - jrn\Lambda)$, $\Lambda = \dfrac{1}{3}\,\omega/r$ and $n = 0, 1, 2$. Let the system of strips every second and third strip interlaced between the channels (Fig. 2b). In the lower channel the following strip potentials have phases $0°$, $-240°$, $-120°$ equivalent to $0°$, $120°$, $240°$, respectively. This is a potential wave in synchronism with SAW propagating in the opposite directions as compared to SAW in the upper channel. The system shown in Fig. 1b is the reversing directional coupler [3 – 5] "rmsc".

It is somewhat difficult to make a planar system of strips with strips crossing one over the other. Among several possible solutions, in this paper we consider the system where every third strip is grounded (Fig. 1c). Such strip structures can easily be made with the help of microelectronic technology.

In the next Section, some general theoretical results necessary for description of msc and rmsc are presented. We apply a perfect strip model, that is, we neglect strip elasticity [6] and mass, and also assume perfect strip conductivity. This allows us to apply directly the developed method [7, 8] for analyzing waves in periodic strips. The following Sections present the theory of rmsc. In Conclusions, we discuss possible applications of rmsc in SAW devices like SAW filters, resonators, and dispersive delay lines for signal processing.

## 2. SAW in periodic system of strips

### 2.1. Simplified description of piezoelectric halfspace

Let us apply a traction $T_{yj} \exp(j\omega t - jkx)$ to the surface $y=0$ of piezoelectric halfspace. If $k > k_s$ where $k_s$ is cut-off wave number of bulk waves, the response of the substrate will be described by a Hermitian Green's matrix

$$u_i = z_i z_j^* \frac{T_{yj}}{k-k_v} + z_i \frac{\kappa}{k-k_v} \frac{\Delta D_\perp}{\sqrt{\varepsilon_e}}, \qquad \varphi = z_j^* \frac{\kappa}{k-k_v} \frac{T_{yj}}{\sqrt{\varepsilon_e}} + \frac{k-k_0}{k-k_v} \frac{\Delta D_\perp}{k\varepsilon_e}, \qquad (1)$$

where $\kappa^2 = (k_0 - k_v)/k_0 = \Delta v/v$, and $z_i = z_i(k)$, $z_i(-k) = z_i^*(k)$.

Wave numbers $k_v$ and $k_0$ are the eigenvalues of boundary problems for free and metallized piezoelectric halfspace $[T_{yj}, \Delta D_\perp]^T = G(\omega; k) [u_i, \varphi]^T = 0$ and $[T_{yj}, \varphi]^T = G(\omega, k) [u_i, \Delta D_\perp]^T = 0$, correspondingly. In narrow bounds $r \approx k_v, k_0$, both $G$ and $G'$ resulting from the equations of motions of a piezoelectric body can be approximated by the linear functions of $k$; this was exploited to obtain Eqs. (1) otherwise we should apply a more general approximation [9].

Constraining $k$ to the area close to $(k_v, k_0)$, which is usually very narrow as compared to $k_v - k_s$, $z_i$ can be applied as a constant

$$z_i = -\frac{\kappa}{\sqrt{\varepsilon_e}} \frac{U_i^{(v)}}{\Phi^{(v)}} = k_v \kappa \sqrt{\varepsilon_e} \frac{U_i^{(0)}}{D_\perp^{(0)}}, \quad \varepsilon_e = -\frac{U_i^{(v)}}{k_v \Phi^{(v)}} \frac{D_\perp^{(0)}}{U_i^{(0)}}, \quad \kappa^2 \approx -\frac{v}{4} \Phi^{(v)*} D_\perp^{(0)}, \qquad (2)$$

where capital letters denote normalized wave-field amplitudes of SAW propagating on free (index $v$), or metallized (index $0$), substrate surface $y=0$.

As mentioned earlier, we apply approximation of perfect weightless conducting strips, that is $T_{yj}=0$. Equations (1) can be transformed to

$$\left.\begin{aligned} A &= \frac{u_i}{z_i} \approx \frac{\sqrt{\varepsilon_e}}{\kappa}\left(-\varphi + \frac{1}{k\varepsilon_e}\Delta D_\perp\right) \\ \varepsilon_e (k-k_v) k\varphi - (k-k_0)\Delta D_\perp &= 0 \end{aligned}\right\} \quad \text{for Re}\{k\} > 0,$$

$$\text{apply: } k \to -k \text{ and } z_i \leftrightarrow z_i^* \qquad \text{for Re}\{k\} < 0. \tag{3}$$

It should be stressed that the bulk waves, which can be generated in the body by the surface traction if $k < k_s$, are not included in the above description, thus neglected.

To complete characterization of SAW by its wave number $k$ and wave-field amplitudes $\varphi$ and $A$, let us introduce the SAW amplitude $a$, by definition involved in the relation for SAW Poynting vector magnitude $\Pi = \frac{1}{2}|a|^2$. It can be obtained from Eqs. (1) that

$$a = A\sqrt{\omega/2} \tag{4}$$

with accuracy slightly dependent on the electric boundary conditions, as far as the SAW wave number is close to $(k_v, k_0)$.

## 2.2. Eigenvalue boundary problem

The considered boundary problem concerns wave-propagation in periodic systems, thus applying Floquet's theorem; the solution is sought in the form (term $\exp j\omega t$ dropped)

$$E_{\parallel}(x) = \sum_{n=-\infty}^{\infty} E_n e^{-j(r+nK)x}, \quad \Delta D_{\perp}(x) = \sum_{n=-\infty}^{\infty} D_n e^{-j(r+nK)x}, \tag{5}$$

where $r$ is assumed in the first Brillouin zone $(0 < r < K)$, and $E_{\parallel}(x) = -\partial_x \varphi(x)$.

Complex amplitudes $D_n$ and $E_n$ are dependent on each other on the strength of Eqs. (3), where we apply $k = r + nK$

$$D_n = -j\varepsilon_e S_{n+r/K} \frac{r+nK-k_v}{r+nK-k_0} E_n,$$

$$E_n = j(r+nK)\varphi_n, \quad S_v = \begin{cases} 1 & \text{for } v \geqslant 0 \\ -1 & \text{for } v < 0 \end{cases}. \tag{6}$$

It is assumed below, that $K > k_0$. In fact, rmsc works at $K \approx 3 k_0 \gg k_s$, allowing for several simplifications in the following considerations, primarily $(r+nK-k_v)/(r+nK-k_0) \approx 1$ for all $n$ except $n=0$ or $n=-1$, and $r$ in the assumed domain.

Electric field is shielded under perfectly conducting strips and the electric charge can be different from zero only on strips (Fig. 2), that is
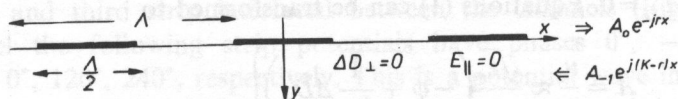


Fig. 2. Periodic system of strips on piezoelectric halfspace.

$$E_{\parallel}(x) = 0, \quad \text{on strips,} \quad l\Lambda - w < x < l\Lambda + w, \tag{7}$$

$$\Delta D_{\perp}(x) = 0, \quad \text{between strips,} \quad (l-1)\Lambda + w < x < l\Lambda - w,$$

are mixed electric boundary conditions at $y = 0$ plane.

The $l$-th strip potential and current

$$V_l = V(r)e^{-jrl\Lambda}, \quad I_l = I(r)e^{-jrl\Lambda}, \tag{8}$$

depend on $r$. Below, we consider the two most important cases

• short-circuited (grounded) strips, where strip potentials are zero

$$V(r) = \sum_{n=-\infty}^{\infty} \varphi_n = -j \sum_{n=-\infty}^{\infty} \frac{1}{r+nK} E_n = 0; \tag{9}$$

• free open strips, where the electric current flowing to a strip is zero

$$I(r)=j\omega \int_{-w}^{w} \Delta D_\perp(x)\,dx = j2\omega \sin r\frac{\Lambda}{2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{r+nK}D_n=0. \tag{10}$$

## 2.3. Method of solution

Following the method [7], we apply the representation for the given $n$-th harmonic components $E_n$ and $D_n$ in the form of finite series including certain new unknowns

$$E_n=\sum_{m=M}^{N+1}\alpha_m S_{n-m}P_{n-m}(\Delta),\qquad D_n=-j\varepsilon_e\sum_{m=M}^{N+1}\alpha_m P_{n-m}(\Delta), \tag{11}$$

where $S_v$ is as defined above Eqs. (6) and $P_v(\Delta)=P_v$ — Legendre function (Appendix), $\Delta=\cos Kw=0$ in the case considered in this paper, the limits $M\leqslant 0$ and $N\geqslant 0$ are certain integers.

The representation (11), satisfying the electric boundary conditions (7) (see Appendix), will satisfy also Eqs. (6) if

$$\sum_m \alpha_m\left(1-S_{n-m}S_{n+r/K}\frac{r+nK-k_v}{r+nK-k_0}\right)P_{n-m}=0. \tag{12}$$

In the considered case $K\gg k_0$, the number of unknowns is not large. Indeed, let us note that the solution (11) satisfies Eq. (12) for every $n<M$ and $n>N$ automatically. Assuming $M=-1$ and $N=0$, we get Eqs. (6) satisfied by the solution in the above two domains of $n$, independently of $\alpha_m$, $m\in[-1,1]$ provided that $K>k_0$ and $0<r<K$.

Solving Eqs. (12) and applying Eqs. (3) $(A(x)=\Sigma A_n e^{-j(r+nK)x})$ we obtain

$$\alpha_1=\alpha_0\frac{\kappa^2}{2}\frac{r}{r-k},\qquad A_0=-j\alpha_0\frac{\kappa\sqrt{\varepsilon_e}}{r-k},$$

$$\alpha_{-1}=\alpha_0\frac{\kappa^2}{2}\frac{K-r}{K-r-k},\qquad A_{-1}=-j\alpha_0\frac{\kappa\sqrt{\varepsilon_e}}{K-r-k}, \tag{13}$$

where $k=(k_v+k_0)/2=\omega/k$, $\alpha_0$ is arbitrary constant.

## 2.3. Propagation of SAW in a periodic system of strips

The last condition to be satisfied is either Eq. (9) or (10) which can be rewritten in the form (evaluation of the series over $n$, see Appendix)

$$V(r)=\alpha_0\frac{\pi}{jK\sin \pi r/K}V_r,\qquad I(r)=\alpha_0\,\omega\varepsilon_e\Lambda I_r,$$

$$V_r=\sum_m\frac{\alpha_m}{\alpha_0}(-1)^m P_{-m-r/K},\qquad I_r=\sum_m\frac{\alpha_m}{\alpha_0}P_{-m-r/K}. \tag{14}$$

The corresponding dispersion relation are

$$V_r = 0, \quad r = r_0, \text{ for short-circuited strips,}$$
$$I_r = 0, \quad r = r_v, \text{ for open strips}$$

which explicitly are (apply '$-$' for $r_0$ and '$+$' for $r_v$)

$$(r-k)(K-r-k) \mp \kappa^2 K p(K/2-k) = 0, \quad p = \frac{\sin \pi r/K}{\pi (P_{-r/K})^2} \approx \frac{r}{K}\left(1 - \frac{r}{K}\right) \quad (15)$$

Well outside the Bragg stop-band ($K/2 \neq k$; in case of rmsc $K \approx 3k$), we have

$$r_0 \approx k\left(1 + \frac{1}{2}\frac{\Delta v}{v}\left(1 - \frac{k}{K}\right)\right), \quad r_v \approx k\left(1 - \frac{1}{2}\frac{\Delta v}{v}\left(1 - \frac{k}{K}\right)\right) \quad (16)$$

Finally, the symmetry in Eqs. (14) allows us to write

$$I(r) = j2\omega\varepsilon_e V(r)\frac{(r-r_v)(K-r-r_v)}{(r-r_0)(K-r-r_0)}\sin \pi r/K, \quad (17)$$

and

$$A_0 = V(r)\frac{\kappa\sqrt{\varepsilon_e}}{\pi P_{-r/K}}\frac{K-r-k}{(r-r_0)(K-r-r_0)}K\sin \pi r/K, \quad (18)$$

$$A_{-1} = V(r)\frac{\kappa\sqrt{\varepsilon_e}}{\pi P_{-r/K}}\frac{r-k}{(r-r_0)(K-r-r_0)}K\sin \pi r/K,$$

where, following Eqs. (3)

$$A_0(r)\, e^{-jrx} \quad \text{and} \quad A_{-1}(r)\, e^{-j(r-K)x} \quad (19)$$

are forward and backward propagating SAWs, depicted in Fig. 2.

## 3. Theory of rmsc

### 3.1. Modelling of electric field in the system

It is seen in Fig. 1c that the considered system is periodic with period $3\Lambda$. Hence, the Floquet theorem requires the following representation for strip potentials and currents

$$\sum_{n=0}^{2} V(s+nK')e^{-j(s+nK')x}, \quad \sum_{n=0}^{2} I(s+nK')e^{-j(s+nK')x}, \quad x = l\Lambda \quad (20)$$

where, in order to adopt the results of the previous Section, we must apply

$$0 < r = s + nK' < K, \quad K' = \frac{K}{3}. \quad (21)$$

In what follows, we consider rmsc working in a narrow frequency band where coupling between forward wave in the upper acoustic channel and backward wave in the lower channel is the strongest. This takes place at

$$s = K' + \delta, \qquad |\delta| \ll K', \qquad r_v \approx K' \approx r_0 \approx k = \omega/v \qquad (22)$$

so that in Eqs. (21) we must apply either $n = 0, 1, 2$ for $\delta < 0$, or $n = -1, 0, 1$ when $\delta > 0$, and similarly in Eqs. (20).

On the strength of the previous section and the above assumption

$$I(s + nK') = y_n V^n, \qquad V^n = V(s + nK'),$$

$$y_0 = Y \frac{d_v + \delta}{d_0 + \delta}, \qquad y_1 = Y \frac{d_v - \delta}{d_0 - \delta}, \qquad (23)$$

$$Y = j\omega\varepsilon_e\sqrt{3}, \qquad d_v = K' - r_v, \qquad d_0 = K' - r_0,$$

and $y_2 \approx 0$ (or $y_{-1} \approx 0$ in case $\delta > 0$), see Eq. (17).

### 3.2. Discrete eigen-problem

It is seen from Eqs. (20) that we can consider only three strips numbered $l = 0, 1, 2$, Fig. 3a, and having potentials $V_l$, $U_l$ and currents flowing to them $I_l$ and $J_l$, in the upper and lower channels, respectively. Kirchhoff's laws yield
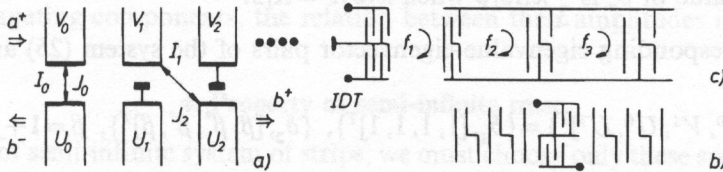


Fig. 3. a) One section of (rmsc); b) SAW resonator and c) dispersive delay line with rmsc.

$$V_0 = V^0 + V^1 + V^2 = U_0 = U^0 + U^1 + U^2,$$

$$V_1 = zV^0 + \alpha zV^1 + \alpha^2 zV^2 = U_2 = z^2 U^0 + \alpha^2 z^2 U^1 + \alpha^4 z^2 U^2,$$

$$V_2 = z^2 V^0 + \alpha zV^1 + \alpha^2 zV^2 = 0,$$

$$U_1 = zU^0 + \alpha zU^1 + \alpha^2 zU^2 = 0, \quad I_0 + J_0 = y_0 V^0 + y_1 V^1 + y_0 U^0 + y_1 U^1 = 0,$$

$$I_1 + J_2 = zy_0 V^0 + \alpha zy_1 V^1 + z^2 y_0 U^0 + \alpha^2 z^2 y_1 U^1 = 0,$$

$$\alpha = \exp(-jK'\Lambda) = \exp(-j2\pi/3), \quad 1 - \alpha = -j\sqrt{3}\alpha, \quad 1 + \alpha = -\alpha^2, \quad \alpha^3 = 1,$$

$$z = \exp(-js\Lambda) = \alpha z', \quad z' = \exp(-j\delta\Lambda) \approx 1,$$

resulting in the following homogeneous set of linear equations:

$$\begin{bmatrix} (d_0+\delta) & -\alpha(d_0-\delta) & \alpha(d_0+\delta) & -(d_0-\delta) \\ (d_0+\delta) & -(d_0-\delta) & z'(d_0+\delta) & -z'(d_0-\delta) \\ (d_v+\delta) & (d_v-\delta) & (d_v+\delta) & (d_v-\delta) \\ (d_v+\delta) & \alpha(d_v-\delta) & \alpha z'(d_v+\delta) & z'(d_0-\delta) \end{bmatrix} \begin{bmatrix} V^0/(d_0+\delta) \\ V^1/(d_0-\delta) \\ U^0/(d_0+\delta) \\ U^1/(d_0-\delta) \end{bmatrix} = 0. \tag{25}$$

The determinant of the above system of equations should be equal to zero

$$\left\{ \sqrt{3}\left[(d_0+\delta)(d_v-\delta)-(d_v+\delta)(d_0-\delta)\right]\cos\delta\,\frac{\Lambda}{2} - \right.$$

$$\left. -\left[(d_0+\delta)(d_v-\delta)+(d_v+\delta)(d_0-\delta)\right]\sin\delta\,\frac{\Lambda}{2}\right\}^2 + 12(d_0^2-\delta^2)(d_v^2-\delta^2)=0 \tag{26}$$

the most important solutions of which ($\delta$ is small as assumed previously) are

$$\delta_1^2 = d_0 d_v, \quad \text{and} \quad \delta_2^2 \approx d_0 d_v\left(1-\frac{1}{\sqrt{3}}\Lambda(d_v-d_0)\right). \tag{27}$$

There are stopbands in both SAW modes, having wave numbers $s_1 = K' + \delta_1$ and $s_2 = K' + \delta_2$. This happens if $\delta_n$ is complex, that is $d_0 d_v = (K'-r_0)(K'-r_v) < 0$. From Eqs. (16) we obtain the relative stopband width equal to $\frac{2}{3}\Delta v/v$, thus the maximum imaginary value of $\delta_n$ is $\frac{1}{3}k\Delta v/v$ when $k \approx K' = K/3$.

The corresponding eigenvalue-eigenvector pairs of the system (25) are

$$\{\delta,[V^0,V^1,U^0,U^1]^T\}=\{\delta_1,[1,1,1,1]^T\}, \quad \{\delta_2,[\beta,\beta^*,\beta^*,\beta]^T\}, \quad \beta=1+j\,\frac{2}{\sqrt{3}}. \tag{28}$$

### 3.3. SAW wave-field in the system

The potential waves discussed above $V^n \exp(-j(s+nK')x)$ in the upper channel and $U^n \exp(-j(s+nK'x)$ in the lower one are accompanied by the corresponding particle-displacement waves at the substrate surface. They are

$$A_0^c(r+nK')e^{-j(s+nK')x}, \quad A_{-1}^c(r+nK')e^{-j(s+nK'-K)x}, \tag{29}$$

where amplitudes $A_n^c$, $n=0,-1$ can be evaluated from Eqs. (18), $c$ denotes the channel.

Most important wave components are those the wave numbers of which fall in the vicinity of SAW numbers, $r_0$ or $-r_0$, because they are closely related to SAW amplitudes, Eq. (3). Following the applied assumptions $K' \approx k \approx r_0$, one concludes that the important wave-component are $A_0$ related to $V^0$, $U^0$ and $A_{-1}$ related to $V^1$, $U^1$. For convenience, their corresponding amplitudes are expressed below by means of SAW amplitudes $a$ and $b$ in the upper and the lower channels,

$$a_0 = \frac{\kappa\sqrt{3\omega\varepsilon_e/2}}{\pi P_{-1/3}} \frac{V^0}{K'-r_0+\delta}, \qquad a_{-1} = \frac{\kappa\sqrt{3\omega\varepsilon_e/2}}{\pi P_{-1/3}} \frac{V^1}{K'-r_0-\delta},$$

$$b_0 = \frac{\kappa\sqrt{3\omega\varepsilon_e/2}}{\pi P_{-1/3}} \frac{U^0}{K'-r_0+\delta}, \qquad b_{-1} = \frac{\kappa\sqrt{3\omega\varepsilon_e/2}}{\pi P_{-1/3}} \frac{U^1}{K'-r_0-\delta}, \tag{30}$$

where $\delta_1$ or $\delta_2$ should be substituted for $\delta$; thus, amplitudes $a$ and $b$ should be provided another index $n = 1, 2$.

Note that $V^0, V^1, U^0, U^1$ are eigenvectors dependent on $\delta_n$ as given by Eqs. (28). Let the amplitude of SAW mode corresponding to eigenvalue $\delta_1$ be $a$, and that corresponding to $\delta_2 - b$. Hence, the SAW wave-fields in the system in upper and lower channels are, respectively,

$$a(1+\gamma_1 e^{j2k'x})e^{-j(K'+\delta_1)x} + b(\beta + \beta^*\gamma_2 e^{j2k'x})e^{-j(K'+\delta_2)x},$$

$$a(1+\gamma_1 e^{j2k'x})e^{-j(K'+\delta_1)x} + b(\beta + \beta^*\gamma_2 e^{j2k'x})e^{-j(K'+\delta_2)x}, \tag{31}$$

$$\gamma_n = \frac{K'-r_0+\delta_n}{K'-r_0-\delta_n}, \quad n=1, 2, \quad \delta_2 \approx \delta_1 = -\sqrt{(K'-r_0)(K'-r_v)}.$$

Let us finally note that we can apply either $+\delta_n$ or $-\delta_n$ in the above relations (fortunately, $\beta$ does not depend on a sign of $\delta_n$), or both of them, in case of the finite structure. The corresponding waves are modes propagating to the right (if $\mathrm{Im}\{\delta_n\} < 0$) and left (if $I_m\{\delta_n\} > 0$), each modes composed of the forward, and back-ward-propagating components, the relation between their amplitudes involving $\delta_n$.

## 4. Property of semi-infinite rmsc

In case of semi-infinite system of strips, we must choose only these solutions for $\delta_n$ which fulfill radiation condition at infinity ($x \to \infty$). For example, in stopband the corresponding $\delta_n$ must have a negative imaginary value. Assuming that the proper values are chosen for $\delta_n$. Equations (31) completely describes SAW wave-field in the system.

Let us assume there is an incident SAW in the upper channel only (Fig. 3a)

$$a^+ e^{-jkx} \tag{32}$$

We need the boundary conditions at the border between rmsc and the free area ($x = 0$). The only simple possibility to get this condition is to compare SAW wave-fields on both sides of the border which have similar wave numbers. This concerns equality of both particle displacements, and stress on both sides of $x = 0$ (note that stress is expressed by spatial derivatives of the particle-displacement field), and this is, in fact, the reason that we must put equality for wave components with similar wave numbers, including sign, separately. Hence, we obtain at $x = 0$

$$a+\beta b=a^{+}\Big\} \atop a+\beta^{*}b=0 \Big\}, \qquad \begin{cases} a^{-}=a\gamma_{1}+b\gamma_{2}\beta^{*} \\ b^{-}=a\gamma_{1}+b\gamma_{2}\beta \\ b^{+}=a\gamma_{1}+b\gamma_{2}\beta^{*} \end{cases} \tag{33}$$

where $a^{-}$ and $b^{-}$ are the SAW amplitudes sought for, Fig. 3a.

We obtain the following solution, describing the property of semi-infinite rmsc working in the stopband, or close to the stopband ($\delta_n$ small assumed)

$$a^{-}=\beta^{*}\frac{\gamma_{1}-\gamma_{2}}{\beta^{*}-\beta}\approx 0, \quad \omega_{o}=vK'\left(1-\frac{1}{3}\,\Delta v/v\right), \quad \omega_{v}=vK'\left(1+\frac{1}{3}\,\Delta v/v\right),$$

$$b^{-}=\frac{\beta^{*}\gamma_{1}-\beta\gamma_{2}}{\beta^{*}-\beta}\,a^{+}\approx\gamma a^{+}, \quad b^{+}\approx 0, \quad \gamma\approx\frac{1-\sqrt{(\omega-\omega_{v})/(\omega-\omega_{0})}}{1+\sqrt{(\omega-\omega_{v})/(\omega-\omega_{0})}}, \tag{34}$$

where $\gamma=\gamma_{1}\approx\gamma_{2}$, $|\gamma|=1$ in the stopband, and $\gamma\to 0$ for frequencies outside the stopband.

Let us summarize the features of the discussed rmsc:

• its scattering property is perfect in narrow stopband,

• there is no back reflection in the same channel ($a^{-}\approx 0$),

• there is no transmission in forward direction in the other channel ($b^{+}\approx 0$), as far as we consider rmsc in the frame of the above developed simple theory.

## 5. Conclusions

The structure of rmsc can be applied in several SAW devices. First of all it can serve like a "mirror" and the simultaneous track-changer of SAW in SAW resonators Fig. 3b, making their performance better for at least three reasons:

• reduction of bulk-wave spurious signals (bulk waves excited by IDT in one channel is not detected by IDT in the other channel)

• the "mirror" good reflection performance is limited to narrow frequency band what reduces the spurious passband of the resonator,

• the reflection of SAW by rmsc is of "regeneration" nature so that SAW diffraction effects are seriously limited.

First experimental result is shown in Fig. 4 concerning a one-port SAW resonator. Matrix coefficient $S_{11}$ is presented as a function of frequency. The device has been made on LiNbO$_3$ which is a rather strong piezoelectric. This makes the $Q$-factor low, about 1000. The perfect frequency response is obtained without special measures, what is frequently necessary in conventional resonators.

Figure 3c shows a dispersive delay line, similar to the so-called RAC SAW device, but with technologically difficult surface grooved reflective array replaced by rmsc. In this rmsc the strip period changes along the structure so that different frequencies are "track-changed" in different places. This makes the SAW path between IDTs dependent on the frequency — that is, dispersive delay line.
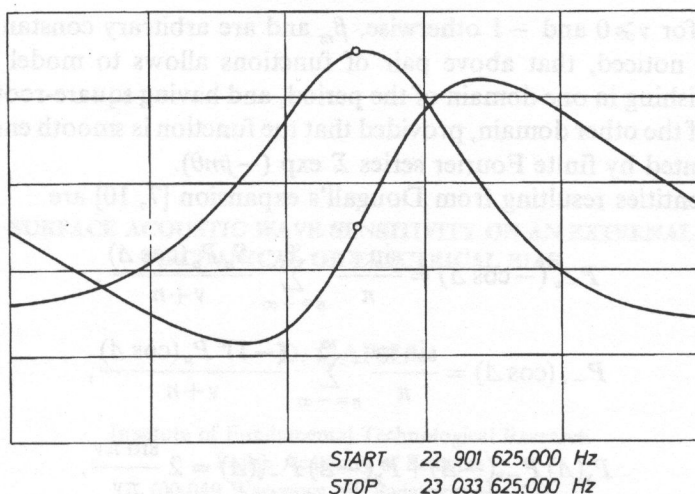
START    22 901 625.000 Hz
STOP     23 033 625.000 Hz

Fig. 4. Frequency response of one-port SAW resonator exploiting rmsc's.

## Acknowledgment

## Appendix

The identity can be found [10] for a periodic function with period $2\pi$ (it is assumed that $0 < \theta < \pi$ and Re $\{\mu\}$ 1/2)

$$\Gamma\left(\frac{1}{2} - \mu\right) \sum_{n=0}^{\infty} P_n^{\mu}(\cos \theta) \cos\left(n + \frac{1}{2}\right)v = \begin{cases} \dfrac{(\pi/2)^{1/2} \sin^{\mu} \theta}{(\cos v - \cos \theta)^{\mu + 1/2}}, & 0 \leqslant v < \theta \\ 0, & \theta < v < \pi \end{cases}$$

where $P$ is the Legendre function $(P_v^0 = P_v)$

$$P_{-v-1}^{\mu}(x) = P_v^{\mu}(x), \quad P_n(-x) = (-1)^n P_n(x), \ n \geqslant 0, \ P_v(0) \approx 1 - v, \ 0 < v < 1.$$

The first equation can be rewritten for $-\pi < \theta < \pi$, $0 < \Delta < \pi$ as follows:

$$\sum_{n=-\infty}^{\infty} \alpha_m P_{n-m}(\cos \Delta) e^{-jn\theta} = \begin{cases} \sqrt{2} \dfrac{\alpha_m \exp(-jm\theta)}{\sqrt{\cos \theta - \cos \Delta}} e^{j\theta/2}, & |\theta| < \Delta \\ 0, & \Delta < |\theta| < \pi \end{cases}$$

$$\sum_{n=-\infty}^{\infty} \beta_m S_{n-m}(\cos \Delta) e^{-jn\theta} = \begin{cases} 0, & |\theta| < \Delta \\ -jS_{\theta} \sqrt{2} \dfrac{\beta_m \exp(-jm\theta)}{\sqrt{\cos \Delta - \cos \theta}} e^{j\theta/2}, & \Delta < |\theta| < \pi \end{cases}$$

where $S_v = 1$ for $v \geqslant 0$ and $-1$ otherwise, $\beta_m$ and are arbitrary constants.

It can be noticed, that above pair of functions allows to model any periodic function, vanishing in one domain of the period, and having square-root singularities at the edges of the other domain, provided that the function is smooth enough in order to be represented by finite Fourier series $\Sigma \exp(-jm\theta)$.

Useful identities resulting from Dougall's expansion [7, 10] are

$$P_{-v}(-\cos \Delta) = \frac{\sin \pi v}{\pi} \sum_{n=-\infty}^{\infty} \frac{S_n P_n(\cos \Delta)}{v+n},$$

$$P_{-v}(\cos \Delta) = \frac{\sin \pi v}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n P_n(\cos \Delta)}{v+n},$$

$$P_v(\Delta) P_{-v}(-\Delta) + P_v(-\Delta) P_{-v}(\Delta) = 2 \frac{\sin \pi v}{\pi v}.$$

## References

[1] K.A. INGEBRIGTSEN, *Surface waves in piezoelectrics*, J. Appl. Phys., **40**, pp. 2681–2686, 1969.

[2] F.G. MARSHALL, E.G. PAIGE, *Novel acoustic-surface-wave directional coupler with diverse applications*, Electr. Lett., **7**, 460–464 (1971).

[3] W. POLLOCK, Y. SCHOEFIELD, R.F. MILSOM, R.J. MURRAY, I. FLINN, *Low-loss SAV filter using single-phase IDTs and no external tuning*, Ultrason. Symp. Proc., 87–92 (1983).

[4] A.L. CHAMORRO, *Low noise SAW filter design using reflector multistrip coupler*, IEEE Ultras. Symp. Abstracts, PO-6, Dec. 1991.

[5] H. ENGAN, private communication, Jan. 1992.

[6] E. DANICKI, *Generation, and Bragg reflection of SAW in nearly periodic system of elastic metal strips on piezoelectric halfspace*, subm. for publ. in JASA, 1992.

[7] K. BLOTEKJAER, K.A. INGEBRIGTSEN, H. SKEIE, *A method for analyzing waves in structures consisting of metal strips on dispersive media*, IEEE Trans., **ED-20**, pp. 1133–1138 1973; *Acoustic surface waves in piezoelectric materials with periodic metal strip on the surface*, ibid., pp. 1139–1146.

[8] E. DANICKI, *Theory of periodic interdigital transducer of surface acoustic waves*, J. Techn. Phys., **19**, pp. 89–102 (1978).

[9] E. DANICKI, *Propagation of transverse surface acoustic waves in rotated Y-cut quartz substrates under heavy periodic metal electrodes*, IEEE Trans., **SU-30**, 303–312 (1983).

[10] A. ERDELYI, W. MAGNUS, F. OBERHETTINGER, and F.G. TRICOMI, *Higher transcendental functions*, Mc Graw Hill, New York 1953, v. 1.