

INITIAL MODAL TESTING OF A CANTILEVER BEAM

E. SKRODZKA and E. HOJAN

Institute of Acoustic A. Mickiewicz University
(60-769 Poznań, ul. Matejki 48/49)

This paper is devoted to modal testing and to the problem of contactless measurements of a system response signal. The aim of investigation was to establish the best, contactless method for obtaining experimental data used in the process of modal analysis of vibrating objects. Measurements were carried out for a cantilever beam. The beam was excited manually by a small/large impact hammer with a force transducer. The response signal was measured by a microphone probe and a condenser microphone. For comparison, the response signal was also measured by an accelerometer. The spectra of both the exciting and the response signals were delivered to a dual channel analyzer from which the Frequency Response Function (FRF) was obtained. On the basis of FRFs, modal parameters (modal frequencies and mode shapes) were established. A comparison was made between experimental results and theoretical calculations.

1. Introduction

Some measurements were carried out for a cantilever beam. Modal parameters (modal frequencies and mode shapes) were extracted from a set of frequency response functions (FRFs).

By the definition [1], the Frequency Response Function (FRF) is a ratio between a displacement response spectrum and a force spectrum (in this case the FRF is called *receptance*); or a ratio between a velocity response spectrum and a force spectrum (*mobility*). When the "output" quantity is an acceleration, the FRF is called *inertance* or *accelerance*.

Modal analysis is a technique applied in vibration analysis to describe the dynamic behaviour of structures. The analytical modal analysis [7] is an analysis of the structural mathematical model in order to find modal parameters of the structure. Mathematically it can be considered as the eigenvalue problem. Modal analysis has some limitations and imposes some assumptions on the object under investigation. The first assumption is that the structure is a linear system whose dynamics may be represented by a set of linear, second order differential equations. The second assumption is that the structure obeys Maxwell's reciprocity theorem. Maxwell's reciprocity theorem in terms of the frequency

response function measurements implies that the FRF measured between points i and j is identical with the FRF measured between points i and j . The third assumption is that the structure can be considered as a time-invariant system during the test. This assumption implies that the coefficients in linear, second order differential equations are constant and do not vary with time. The fourth assumption is that the dumping is small or proportional to the mass or stiffness. The last assumption states that excitation and response are measured at a point exactly.

Experimental modal analysis is a synthesis of the modal model on the basis of experimental data. Actually the mathematical structural model can be obtained. Experimental modal analysis is based on measurements of the set of Frequency Response Functions. Modal parameters like modal frequencies, modal dumping and mode shapes are extracted from these FRFs. Modal frequency is the system resonant frequency. Modal dumping is a dumping at a resonant frequency. A mode shape is a vibration pattern in a modal frequency.

When the set of modal parameters is found, curve fitting can be performed and mathematical modal model is obtained.

2. Aim

The main goals of investigation were:

a) Setting modal frequencies using two methods. According to the first of them (called the X -method), modal frequencies were those frequencies for which in the FRF modulus maxima appeared. In the second method (called the Y -method) modal frequencies were found as those for which a real part of an accelerance was zero and imaginary part reached an extremum [2];

b) An attempt to answer the question: is it possible to perform modal testing when the "response signal" is measured in a non-contact way by using a condenser microphone or a microphone probe?

Additional questions were those of the object linearity, mode shapes and choice of the best exciting tool (either a small impact hammer or a big impact hammer) for objects of medium mass.

Results were compared to theoretical calculations.

3. Measurements and theoretical calculations

The cantilever beam was an object under investigation. Physical parameters of the beam were: dimension $364 \times 45 \times 6.3$ mm, Young modulus of elasticity $E = 21 \cdot 10^{10}$ N/m², volume density $\rho = 7850$ kg/m³. There were chosen 6 measuring points, 2 cm, 8 cm, 13 cm, 20 cm, 28 cm and 34 cm distant from the rigid fixation point. In the first part of measurements the beam was excited in the point number 2 using either the BK 8202 big impact hammer (its mass was equal to about 209 g) or the BK 8203 small impact hammer (its mass was equal to about 4 g). In the second part of investigation the point number 4 was the excitation point. A response signal was measured either by an accelerometer or by a condenser microphone for free field measurements or by a microphone probe. The

microphone and the probe were 0.4 cm distant from the beam surface. Both the exciting and response signals were delivered to a dual channel analyzer. From the analyzer, the FRFs were obtained. The frequency range of measurements was of 0–4500 Hz.

Theoretically calculated frequencies [3] in the frequency range of interest for the above beam should be: 38 Hz, 239 Hz, 670 Hz, 1310 Hz, 2170 Hz, 3200 Hz and 4400 Hz. For these frequencies theoretical nodal points should be distant from the rigid fixation point of the cantilever as is listed in the Table 1.

Table 1. Distances of nodal points from the rigid fixation point of the cantilever

	x_0 [m]	x_1 [m]	x_2 [m]	x_3 [m]	x_4 [m]
$f_1 = 38$ Hz	0.0	—	—	—	—
$f_2 = 239$ Hz	0.0	0.282	—	—	—
$f_3 = 670$ Hz	0.0	0.182	0.316	—	—
$f_4 = 1310$ Hz	0.0	0.130	0.234	0.330	—
$f_5 = 2170$ Hz	0.0	0.101	0.182	0.263	0.337

4. Results

From the FRFs modal frequencies were extracted for the following pairs of measuring devices:

- the accelerometer — the big impact hammer,
- the accelerometer — the small impact hammer,
- the condenser microphone — the big impact hammer,
- the condenser microphone — the small impact hammer,
- the microphone probe — the big impact hammer,
- the microphone probe — the small impact hammer,

Examples of measured FRFs (point no. 2 — excitation, point no. 4 — response) for each pair of measuring devices are shown in Fig. 1–3.

For each pair of measuring devices frequencies “suspected” to be the modal frequencies should appear in 6 FRFs because of 6 measuring points at least 5 times. Distributions of theoretical nodal points and measuring points show that a measuring point can be placed in a nodal point at most once for each modal frequency. Another important parameter in the process of modal frequency extraction was a coherence function. “Suspected” frequencies for which the coherence function was less than 0.8 were neglected. For each pair of the measuring devices modal frequencies were found either as frequencies of the FRF modulus maximum (*X*-method) or as frequencies for which the FRF real part was zero and the FRF imaginary part had extreme value (*Y*-method). An assumption was made that the measured FRFs were accelerances. Because of the equipment accuracy it was impossible in *Y*-method to find the accurate frequency for which the FRF real component was equal to zero. In fact two frequencies were found for which the FRF real part had two different signs (positive-negative or negative-positive). From these two frequencies the one, corresponding to the greater value of the FRF imaginary part, was chosen as a “modal” frequency.

Using *X*-method and *Y*-method, there were found sets of 7 modal frequencies for the measurements that were carried out with the accelerometer in the response path (in the frequency range of interest there were also 7 natural frequencies!). When a response signal

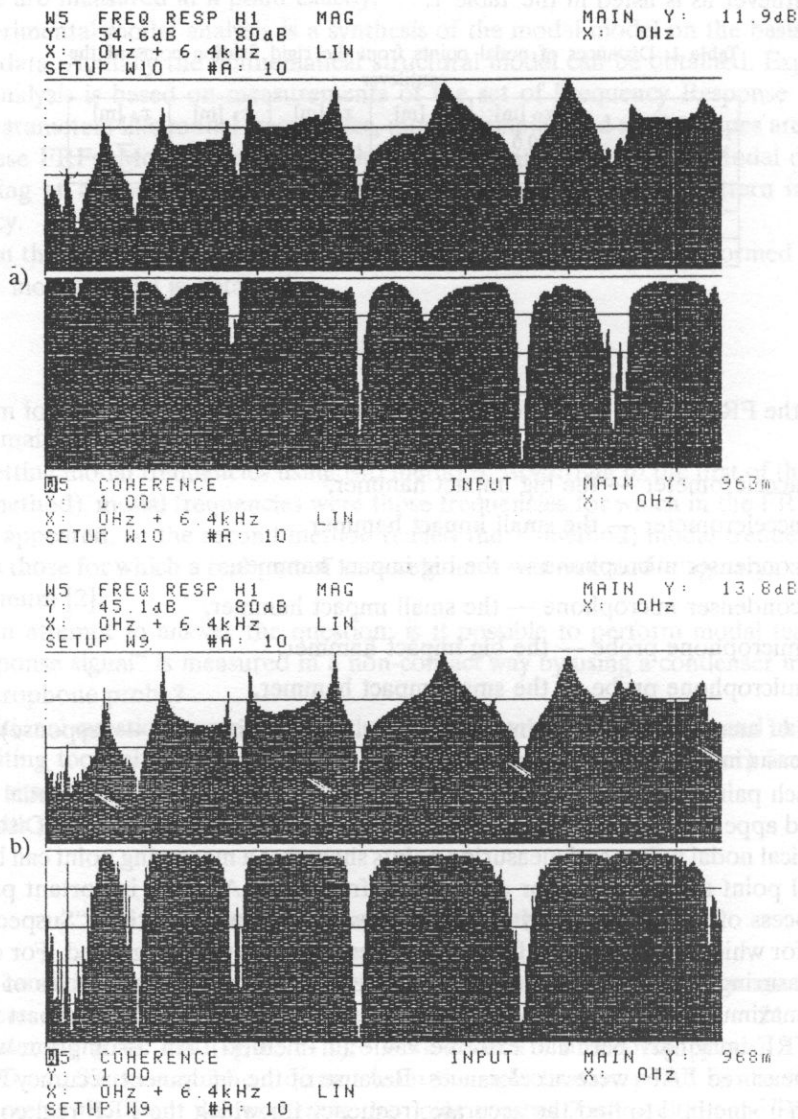


FIG. 1. Typical measured FRFs and coherence functions for
a) the accelerometer — the big hammer,
b) the accelerometer — the small hammer.

was measured by the microphone or the microphone probe, a sound pressure was a measured quantity. There is a limited range of frequency ($f < 1950$ Hz) in which the measured pressure is proportional to the acceleration of beams' vibrating point [6]. This leads to the

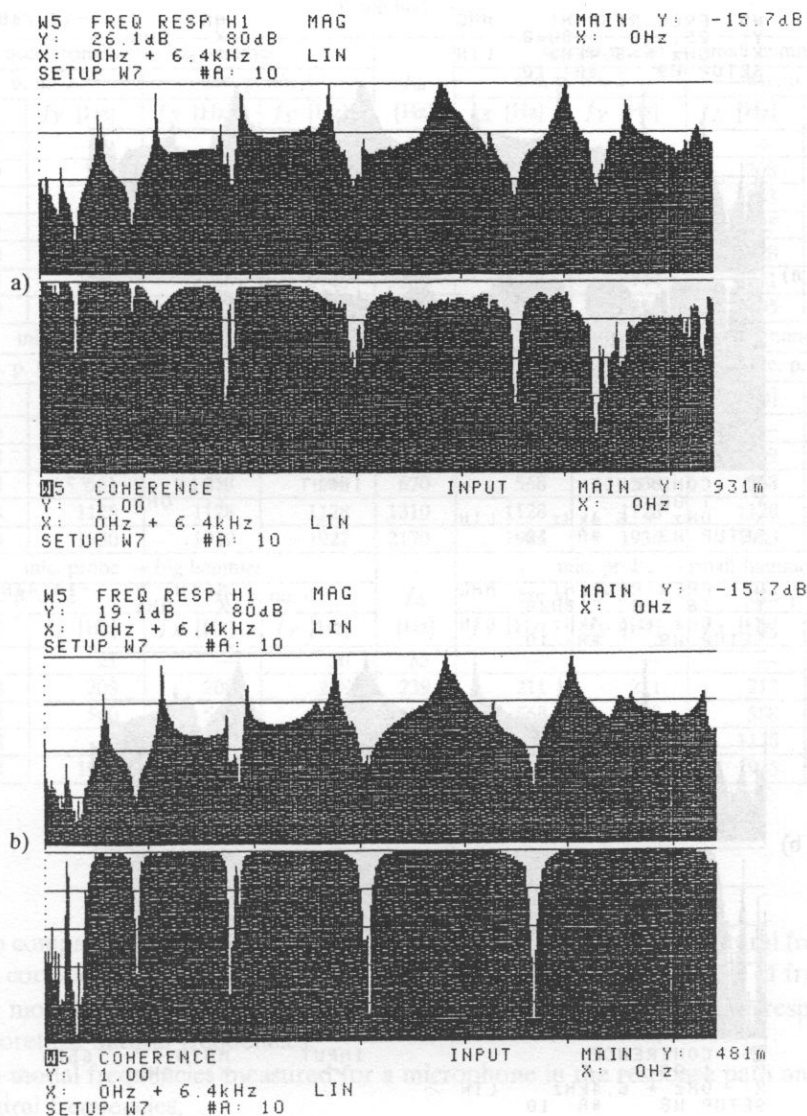


FIG. 2. Typical measured FRFs and coherence functions for
a) the condenser microphone — the big hammer,
b) the condenser microphone — the small hammer.

conclusion that only for frequencies less than 1950 Hz the measured FRF was the acceleration. This is the reason that only 5 modal frequencies measured by means of the condenser microphone probe were taken into account in further considerations.

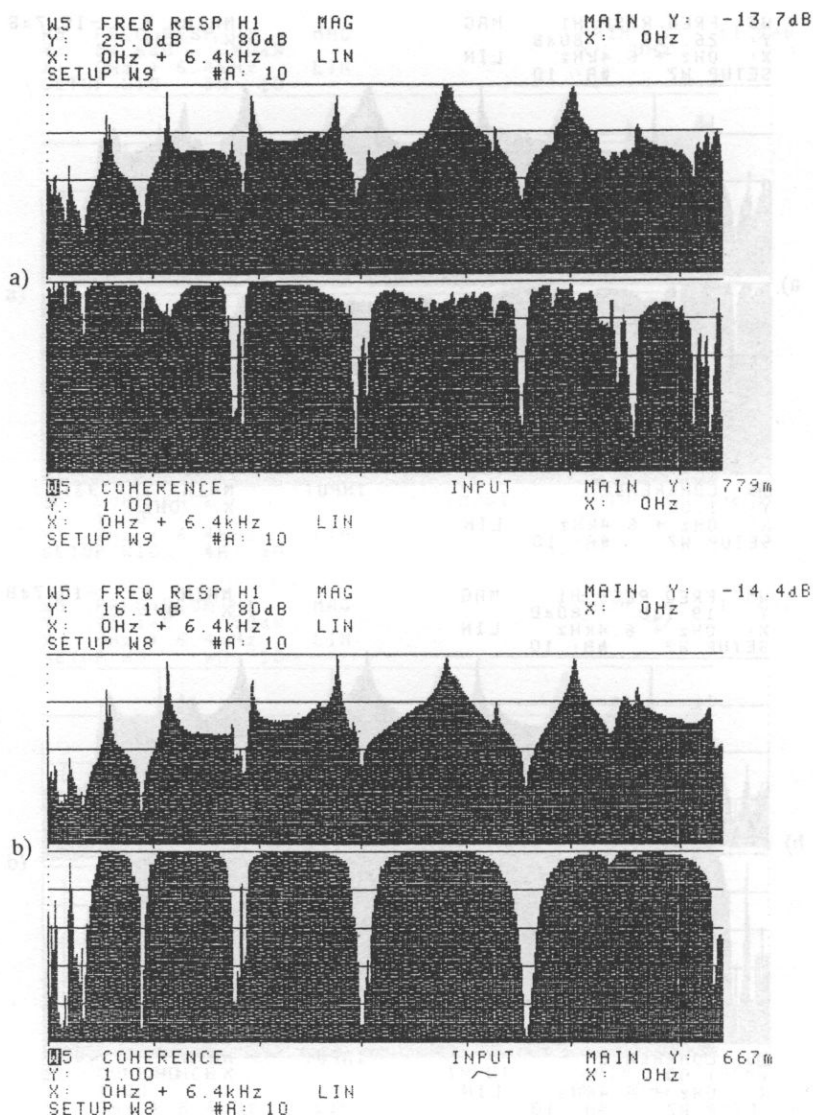


FIG. 3. Typical measured FRFs and coherence functions for
a) the microphone probe — the big hammer,
b) the microphone probe — the small hammer.

Some of these frequencies had not the same value for 6FRFs. They differed from each other by the value of measurement accuracy or more. In such cases the weighted mean value was calculated. The appropriate value of the coherence function was chosen as the weight. The results obtained are listed in the Table 2.

Table 2. Modal frequencies: theoretical — f_{th} , obtained using the X -method — f_X , obtained using the Y -method — f_Y

accelerometer — big hammer				f_{th} [Hz]	accelerometer — small hammer			
exc. p. no 2		exc. p. no. 4			exc. p. no. 2		exc. p. no. 4	
f_X [Hz]	f_Y [Hz]	f_X [Hz]	f_Y [Hz]		f_X [Hz]	f_Y [Hz]	f_X [Hz]	f_Y [Hz]
28	—	35	16	38	—	—	—	—
209	208	208	208	239	208	208	208	208
561	560	563	568	670	561	565	563	563
1125	1125	1128	1128	1310	1128	1128	1125	1128
1928	1928	1928	1928	2170	1928	1928	1928	1928
2745	2740	2746	2746	3200	2743	2744	2741	2741
3809	3845	3802	3820	4400	3792	3796	3795	3801
microphone — big hammer				f_{th} [Hz]	microphone — small hammer			
exc. p. no 2		exc. p. no. 4			exc. p. no. 2		exc. p. no. 4	
f_X [Hz]	f_Y [Hz]	f_X [Hz]	f_Y [Hz]		f_X [Hz]	f_Y [Hz]	f_X [Hz]	f_Y [Hz]
24	23	30	28	38	—	—	—	—
208	208	209	208	239	212	212	209	209
568	561	568	568	670	568	568	568	568
1128	1128	1128	1128	1310	1128	1128	1128	1128
1928	1930	1929	1922	2170	1936	1936	1933	1935
mic. probe — big hammer				f_{th} [Hz]	mic. probe — small hammer			
exc. p. no 2		exc. p. no. 4			exc. p. no. 2		exc. p. no. 4	
f_X [Hz]	f_Y [Hz]	f_X [Hz]	f_Y [Hz]		f_X [Hz]	f_Y [Hz]	f_X [Hz]	f_Y [Hz]
—	21	—	30	38	—	—	—	—
208	208	208	208	239	211	211	212	208
568	560	568	560	670	568	568	568	568
1128	—	1128	—	1310	1128	1118	1128	1136
1928	1933	1928	1936	2170	1935	1935	1935	1935

5. Discussion

1. To compare the extracted modal frequencies and the theoretical natural frequencies, average correlation coefficients were calculated between the following sets of frequencies:

a) modal frequencies measured when there was accelerometer in the response path and theoretical natural frequencies.

b) modal frequencies measured for a microphone in the response path and theoretical natural frequencies,

c) modal frequencies measured for a microphone probe in the response path and theoretical natural frequencies,

d) modal frequencies measured for an accelerometer in the response path and modal frequencies measured for a microphone in the response path,

e) modal frequencies measured for an accelerometer in the response path and modal frequencies measured for a microphone probe in the response path.

Their values are shown on Fig. 4.

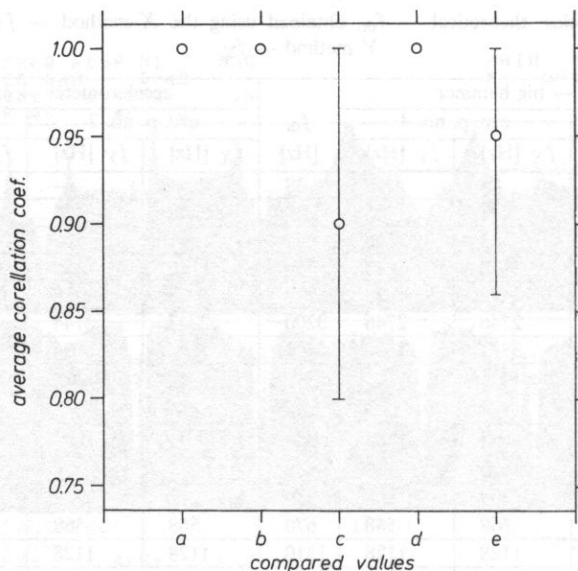


FIG. 4. Average correlation coefficients for results mentioned in 5.1.

It is clear that results obtained by means of the microphone probe in the response path are in a worse agreement with theoretical predictions and with the results obtained when there was the accelerometer in the response path. A good agreement between the results mentioned in b) and d) can be caused by the fact that the microphone was the free-field microphone, designed to compensate for the disturbance caused by its own presence in the sound field [5].

2. In order to compare the *X*-method and the *Y*-method of extracting modal frequencies, the following average correlation coefficients were calculated:

- X*-method results — theoretical predictions,
- Y*-method results — theoretical predictions,
- X*-method results — *Y*-method results,
- X*-method results with the accelerometer in the response channel — theoretical predictions,
- Y*-method results with the accelerometer in the response channel — theoretical predictions,
- X*-method results with the microphone in the response channel — theoretical predictions,

g) Y-method results with the microphone in the response channel — theoretical predictions,

h) X-method results with the microphone probe in the response channel — theoretical predictions,

i) Y-method results with the microphone probe in the response channel — theoretical predictions,

Their values are plotted on Fig. 5.

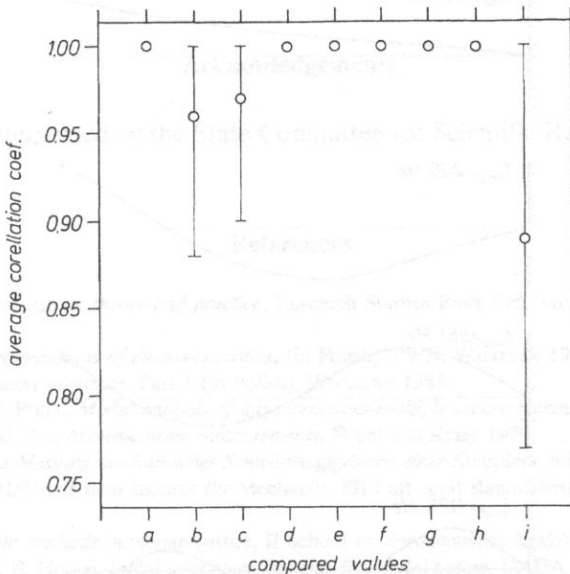


FIG. 5. Average correlation coefficients for results mentioned in 5.2.

It can be seen from Fig. 5 that the results obtained by the X-method correlate with theoretical values on the same level. The same can be seen for Y-method results except those for the microphone probe. This is the reason for slightly worse correlation between the X-method results and the Y-method results. Analysis of Fig. 5 leads to the conclusion that X-method and Y-method are of the same value. Poor results for the microphone probe are caused by a wrong definition of the FRF.

It seems to be worth pointing out that for frequencies extracted by the Y-method, FRF imaginary components reach extremes. It means that in same frequencies it is a maximum and in others — a minimum. It is caused by the fazor rotation of 180° for every next modal frequency.

3. Setting in order the average correlation coefficients considering excitation point no. 2 and no. 4 (without results for the microphone probe) leads to conclusions that the beam is the linear object. Object's linearity is one of the main assumptions of modal testing.

Taking into account Fig. 1–3 it can be seen that better coherence functions were obtained for the big impact hammer. It seems to be a rule using this tool for modal testing of all objects, except the cases when its high crest factor can cause damaging of the structure.

4. Examples of experimental mode shapes obtained from FRFs measurements for excitation by the big hammer and for the accelerometer in the response path are shown on Fig. 6. They are similar to those predicted by the theory.

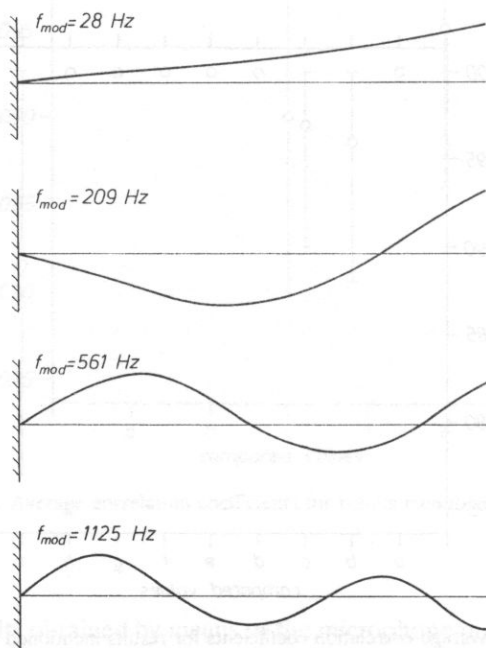


FIG. 6. Examples of measured mode shapes for the cantilever beam.

It should be pointed out that amplitudes of motion of measuring points are not envelopes of imaginary parts of FRFs.

These amplitudes were obtained by multiplication the FRF modulus by the cosine of a relative phase angle between points under investigation. Such procedure was caused, on one hand, by the fact that the extracted modal frequencies were close to real modal frequencies but not equal to them and, as a consequence, in the measured modal frequencies FRF real parts were not equal to zero. On another hand, modal damping were in the range of 0.02–0.1. Both these facts gave their contribution to mode shapes.

6. General conclusions

- There is no difference between the *X*-method and the *Y*-method of extracting modal frequencies from FRFs. However, the *Y*-method seems to be more suitable, especially when experimental data are digitally processed.

- In a general case it is almost impossible to perform non-contact modal testing using in the response path a condenser microphone or a microphone probe. When a microphone is a condenser microphone for free field measurements, modal testing can be done but only in the narrow frequency range. The bigger is objects linear dimension the narrower is a frequency range.
- A big impact hammer with a force transducer should be used for every object under investigation providing it does not damage the structure.
- The cantilever beam under investigation was the linear object and its mode shapes were similar to those predicted by theory.

Acknowledgements

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