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OPTIMIZATION OF TWO MATCHING LAYERS FOR THE WIDE-BAND TRANSDUCER

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Two homogeneous elastic layers are situated between two homogeneous elastic materials. If the harmonic wave propagates in the direction perpendicular to the layers, then the reflection coefficient depends on the elastic constants of the layers, their thickness and frequency. If, instead of the monochromatic wave, the pulse is propagating, then the reflection coefficient depends on the frequency spectrum. The pulse in the form of two or four periods of the sine curve is considered. It is decomposed into a sum, of harmonic, monochromatic waves. In calculations the pulse was assumed to be a sum of 22 harmonic waves of different frequencies. The reflection coefficient for this sum was determined. The reflection coefficient possesses several minima. Only two of them are technially interesting. For one of them the thicknesses of the two layers are of the same order.

1. Monochromatic acoustic wave

Consider the case, when two homogeneous elastic layers are situated between two homogeneous elastic materials. The wave is produced in the first homogeneous material, propagates across the two layers and enters the second homogeneous material. One part of the energy of the incident wave is reflected. The reflection coefficient β is a function of thicknesses, densities and elastic constant of the layers. If $\beta=0$, the system is perfectly transparent, if $\beta=1$ the system is perfectly insulating. For two homogeneous materials given in advance, the elastic layers joining them may be chosen to minimize or maximize the reflection coefficient. Instead of two layers, a larger number of them may be used. The equations quoted in this chapter allow us to perform calculations for an arbitrary number of layers. However, for most practical acoustic applications, already one or two layers are sufficient.

From the mathematical point of view more interesting is the optimization of the transition zone between two materials, if the propagation speed and the density are continuous functions of the distance x, $c=c(x)$, $\rho=\rho(x)$. It is easy to write the governing equations, and to calculate the reflection coefficient β for $c(x)$, $\rho(x)$ given in advance. In very numerous situations the appropriate analytical formula may be obtained, cf, e.g [1]. It is impossible, however, to solve such problem exactly, since β can not be written as the functional of $c(x)$. This is due to the fact that the solutions of the ordinary differential equation can not be expressed by its coefficients.

Each of the four materials considered (two fixed half-spaces and two layers) is identified by the subscripts 0.1, 2, 3 (Fig. 1). Thicknesses of the layers are denoted by h_1, h_2, respectively. The harmonic waves of frequency ω propagate in the direction perpendicular to the layers. The displacement u in the k-th material consists of two harmonic waves, the first of amplitude A_k running to the right, and the second one of amplitude B_k running to the left.

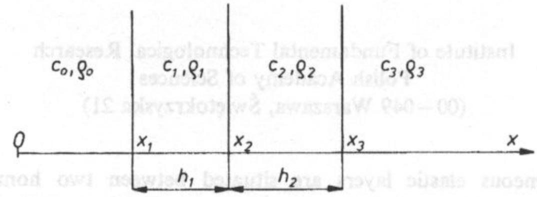

Fig. 1.

$$u = A_k \exp i\omega \left[t - \frac{x - x_k}{c_k} \right] + B_k \exp i\omega \left[t + \frac{x - x_k}{c_k} \right]. \tag{1.1}$$

At the boundaries between the layers both the displacement and the stress are continuous. It follows that the amplitudes A_k, B_k are connected by the matrix relations (cf. e.g. [1])

$$\begin{bmatrix} A_k \\ B_k \end{bmatrix} = M_k \begin{bmatrix} A_{k-1} \\ B_{k-1} \end{bmatrix}. \tag{1.2}$$

where

$$M_k = \frac{1}{2} \begin{bmatrix} (1+\kappa_k)\ \exp(-i\alpha_k) & (1-\kappa_k)\ \exp(i\alpha_k) \\ (1-\kappa_k)\ \exp(-i\alpha_k) & (1+\kappa_k)\ \exp(i\alpha_k) \end{bmatrix} \tag{1.3}$$

$$\kappa_k = \frac{\rho_{k-1}\ c_{k-1}}{\rho_k\ c_k}, \qquad \alpha_k = \omega \frac{h_{k-1}}{c_{k-1}}. \tag{1.4}$$

The transfer matrix M_k is non-singular, therefore its inverse M_k^{-1} always exists. Changing the formulae (1.2) for subsequent $k = 1, 2, 3$, the amplitudes A_3, B_3 may be expressed by the amplitude A_0, B_0 and vice versa.

$$\begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = M_3 M_2 M_1 \begin{bmatrix} A_0 \\ B_0 \end{bmatrix}, \qquad \begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = M_1^{-1} M_2^{-1} M_3^{-1} \begin{bmatrix} A_3 \\ B_3 \end{bmatrix}. \tag{1.5}$$

Two of four amplitudes may be taken at will. If we take $B_3 = 0$ and prescribe the value of A_0, then A_0, A_3, B_3 represent the amplitudes of the incident wave, the transmitted wave (both running to the right, Fig. 1) and the reflected wave (running to the left). If we take $A_0=0$, then B_3, B_0, A_0 represent the amplitudes of the incident and the reflected wave (both running to the left) and the reflected wave (running to the right).

Take $A_0=0$ and consider the term proportional to B_3 as the incident wave: and the terms proportional to A_3, B_0 as the reflected and transmitted waves, respectively. The other possible choice ($B_3=0$) leads to the same reflection coefficient, since a system of layers has no directional properties, [2].

In accord with the above relations, the following expressions for A_3, B_3 are obtained

$$8A_3 = B_0 \exp(-\alpha_1)* \tag{1.6}$$

$$*[(1-\kappa_1)(1+\kappa_2)(1+\kappa_3)\exp(+\alpha_2+\alpha_3)+(1-\kappa_1)(1-\kappa_2)(1-\kappa_3)\exp(+\alpha_2-\alpha_3)+ +(1+\kappa_1)(1-\kappa_1)(1+\kappa_3)\exp(-\alpha_2+\alpha_3)+(1+\kappa_1)(1+\kappa_2)(1-\kappa_3)\exp(-\alpha_2-\alpha_3)].$$

$$8 B_3 = B_0 \exp(-\alpha_1)* \tag{1.7}$$

$$*[(1-\kappa_1)(1+\kappa_2)(1-\kappa_3)\exp(+\alpha_2+\alpha_3)+(1-\kappa_1)(1-\kappa_2)(1+\kappa_3)\exp(+\alpha_2-\alpha_3)+ +(1+\kappa_1)(1-\kappa_2)(1-\kappa_3)\exp(-\alpha_2+\alpha_3)+(1+\kappa_1)(1+\kappa_2)(1+\kappa_3)\exp(-\alpha_2-\alpha_3)].$$

The right-hand sides of (1.6), (1.7) are complex numbers. Their squared moduli are given by the following expressions:

$$\begin{aligned}64\, A_3\bar{A}_3 = B_0\bar{B}_0 [D_1^2 + D_2^2 + D_3^2 + D_4^2 + 2(D_1D_3 + D_2D_4)\cos 2\alpha_2 \\ + 2(D_1D_2 + D_3D_4)\cos 2\alpha_3 + 2D_1D_4\cos(2\alpha_2 + 2\alpha_3) \\ + 2D_2D_3\cos(2\alpha_2 - 2\alpha_3)].\end{aligned} \tag{1.8}$$

$$\begin{aligned}64\, B_3\bar{B}_3 = B_0\bar{B}_0 [D_5^2 + D_6^2 + D_7^2 + D_8^2 + 2(D_5D_7 + D_6D_8)\cos 2\alpha_2 \\ + 2(D_5D_6 + D_7D_8)\cos 2\alpha_3 + 2D_5D_8\cos(2\alpha_2 + 2\alpha_3) \\ + 2D_6D_7\cos(2\alpha_2 - 2\alpha_3)].\end{aligned} \tag{1.9}$$

where the real parameters D_K depend on the speed radios κ_k only,

$$\begin{aligned}D_1 &= (1-\kappa_1)(1+\kappa_1)(1+\kappa_3), & D_2 &= (1-\kappa_1)(1-\kappa_2)(1-\kappa_3),\\ D_3 &= (1+\kappa_1)(1-\kappa_2)(1+\kappa_3), & D_4 &= (1+\kappa_1)(1+\kappa_2)(1-\kappa_3),\\ D_5 &= (1-\kappa_1)(1+\kappa_2)(1-\kappa_3), & D_6 &= (1-\kappa_1)(1-\kappa_2)(1+\kappa_3),\\ D_7 &= (1+\kappa_1)(1-\kappa_2)(1-\kappa_3), & D_8 &= (1+\kappa_1)(1+\kappa_2)(1+\kappa_3).\end{aligned} \tag{1.10}$$

Energy flux q_3 corresponding to the wave of amplitude A_3 and speed c_3 is proportional to the squared frequency

$$q_3 = \rho_3 c_3 \omega^2 A_3\bar{A}_3. \tag{1.11}$$

This flux is a vector quantity possessing the direction of wave propagation. Analogous relations hold for the remaining waves of amplitudes A_0, B_0, A_1,..., B_3. The reflection coefficient equals the ratio of the energy flux of the reflected wave and the energy flux of the incident wave. Therefore

$$\beta = \frac{A_3\bar{A}_3}{B_3\bar{B}_3}. \tag{1.12}$$

Obviously $0<\beta<1$. The first inequality follows from (1.12), since both the numerator and denominator are positive. The second inequality follows from the energy conservation law (reflected energy cannot be larger than the incident energy).

Since (1.12) is essential for the further calculations, we write explicitly the complete formula for β resulting from substitution of (1.7)–(1.9) into (1.12). there is

$$\begin{aligned}\beta = [&D_1^2+D_2^2+D_3^2+D_4^2+2\,(D_1D_3+D_2D_4)\cos 2\alpha_2\\ &+2(D_1D_2+D_3D_4)\cos 2\alpha_3+2\,D_1D_4\cos(2\alpha_2+2\alpha_3)\\ &+2\,D_2D_3\cos(2\alpha_2-2\alpha_3)]*\\ *\,[&D_5^2+D_6^2+D_7^2+D_8^2+2(D_5D_7+D_6D_8)\cos 2\alpha_2\\ &+2(D_5D_6+D_7D_8)\cos 2\alpha_3+2\,D_5D_8\cos(2\alpha_2+2\alpha_3)\\ &+2\,D_6D_7\cos(2\alpha_2-2\alpha_3)]^{-1}\end{aligned} \tag{1.13}$$

where D_K are defined by (1.10). Note that all parameters in the above equations are dimensionless.

The reflection coefficient β is a function of the frequency ω propagation speeds c_1 and c_2 (speeds c_0 and c_3 are fixed) and thicknesses h_1, h_2, $\beta=\beta(\omega, c_1, c_2, h_1, h_2)$.

In order to find for a fixed frequency ω the minimum value of β, the partial derivatives of the function (1.13) with respect to c_1, c_2, h_1 and h_2 must be calculated and put equal to zero. Then the speeds c_1, c_2, thicknesses h_1 and h_2 and value of the minimum reflection coefficient β may be calculated. The corresponding system of trigonometric equations is very complex and no satisfactory analytic treatment of the equations may be expected.

In the much easier special case of one layer only there exists the following solution. Take the propagating speed in the matching layer equal to the geometric mean of the two other speeds. Take the layer thickness equal to a quarter of the wave length $2\pi c_1/\omega$ in this layer,

$$c_1=\sqrt{c_0c_3}\;\; h_1=c_1\frac{\pi}{2\omega}, \qquad h_2=0, \quad c_2=\text{arbitrary} \tag{1.14}$$

From the relations (1.12), (1.13) it follows that for the above data

$$\beta_s=0 \tag{1.15}$$

Note that this result was obtained only for a monochromatic wave. In the applications the situation is more involved, since the real pulse is never monochromatic. In [5] and [6] attempts were made to match the impedances for wide-band pulse using two different layers. In the next chapter such optimization will be provided for wide-band spectrum corresponding to two different short acoustic pulses.

2. Wide-band pulse

Consider the case, when the ultrasound wave passes from a material of high impedance into a material of low impedance. In the typical biological applications the impedances are 30 and 1.5, respectively, [4]. The incident wave reflects partially on the biological inhomogeneities. The reflected wave carriers the information concerning the structure of the examined object. The reflected wave may be properly detected if no other wave arrives simultaneously at the experimentator. Therefore, at the instant when the reflected wave arrives to the measuring device, the incident wave must be already terminated. This fact forces the experimentator to produce in medium 0 very short pulses, e.g. four perioes of the sine curve only. Typical pulse used in ultrasonics is

$$u(t) = \begin{cases} 0 & \text{for } t < 0, \\ \sin \omega_0 t & \text{for } 0 > t < N 2\pi/\omega \\ 0 & \text{for } t > N 2\pi/\omega_0, \end{cases} \tag{2.1}$$

where ω_0 is a certain fixed frequency, and N natural number, $N=1, 2, 3, 4,\ldots$, In order to save space, consider here only even values of N, $N=2n$.

Time shift transforms the function (2.1) into the odd function of time

$$u(t) = \begin{cases} 0 & \text{for } t < -n\, 2\pi/\omega_0, \\ \sin \omega_0 t & \text{for } -n\, 2\pi/\omega_0 < t < n\, 2\pi/\omega_0, \\ 0 & \text{for } t > n\, 2\pi/\omega_0. \end{cases} \tag{2.2}$$

Since the medium is nondispersive, the pulse propagates with speed c_0 in the medium 0 without change of the profile and duration (but in media 1, 2 and 3 it has another profile). The time-dependent displacement in medium 0 is therefore

$$u(x, t) = \begin{cases} 0 & \text{for } t < x/c_0 - 2\pi n/\omega_0, \\ \sin \omega_0 (t - x/c_0) & \text{for } x/c_0 - 2\pi n/\omega_0 < t < x/c + 2\pi n/\omega_0 \\ 0 & \text{for } t > x/c + 2\pi n/\omega_0. \end{cases} \tag{2.3}$$

This motion is not the monochromatic harmonic wave.

Apply the Fourier sine transform to the odd function $f(t)$

$$f(t) = \int_0^\infty I(\omega) \sin \omega t \, d\omega, \qquad I(\omega) = \frac{2}{\pi} \int_0^\infty f(t) \sin \omega t \, dt. \tag{2.4}$$

If N is odd, then $u(t)$ after time shift is an even function of time, and the cosine Fourier transform must be applied. If N is not an integer, then the exponential Fourier transform must be applied. However, this case is not interesting, since such displacement is not a continuous function of time. The formula (2.4) allows us to represent the founction $f(t)$ in form of a sum of harmonic waves.

For the function (2.2) the spectral intensity $I(\omega)$ may be calculated from the formula

$$I(\omega) = \frac{2}{\pi} \int_0^{2\pi n/\omega_0} \sin \omega_0 t \sin \omega t \, dt. \tag{2.5}$$

The integration is elementary. Figure 2 a gives the intensity $I(\omega)$ for $N=4$ the pulse consisting of four periods of the sine function.

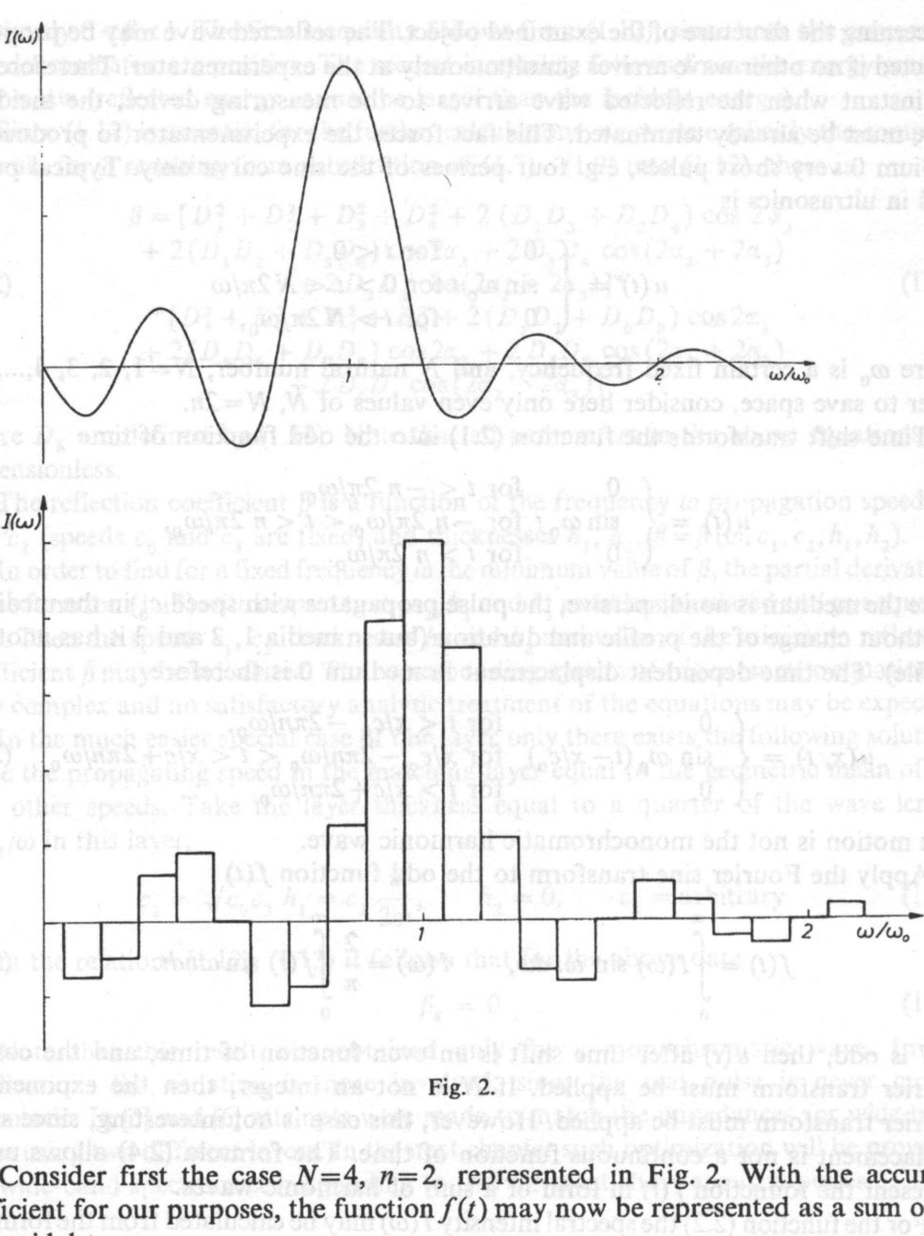

Fig. 2.

Consider first the case $N=4$, $n=2$, represented in Fig. 2. With the accuracy sufficient for our purposes, the function $f(t)$ may now be represented as a sum of 18 sinusoidal terms

$$\begin{aligned} f(t) = & -.60\sin(.1\omega_0 t) - .38\sin(.2\omega_0 t) + .41\sin(.3\omega_0 t) + \\ & + .71\sin(.4\omega_0 t) - .94\sin(.6\omega_0 t) - .73\sin(.7\omega_0 t) + 1.03\sin(.8\omega_0 t) + \\ & + 3.18\sin(.9\omega_0 t) + 4.00\sin(\omega_0 t) + 2.89\sin(1.1\omega_0 t) + .86\sin(1.2\omega_0 t) + \\ & - .55\sin(1.3\omega_0 t) - .64\sin(1.4\omega_0 t) + .40\sin(1.6\omega_0 t) + .20\sin(1.7\omega_0 t) + \\ & - .17\sin(1.8\omega_0 t) - .24\sin(1.9\omega_0 t) + .16\sin(2.1\omega_0 t), \end{aligned} \tag{2.6}$$

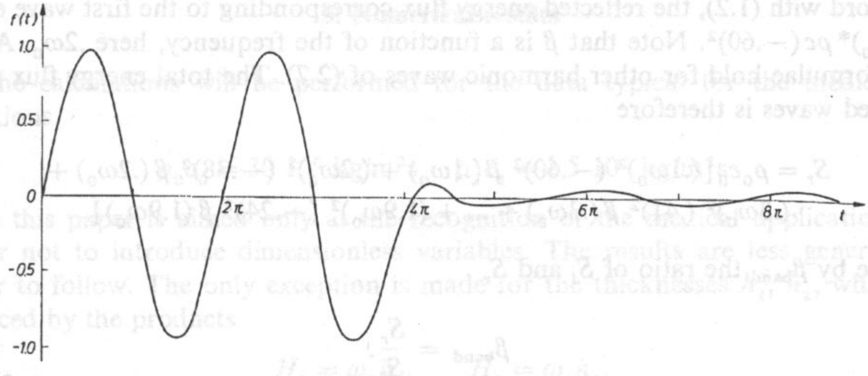

Fig. 3.

The above function corresponds to the intensity $I(\omega)$ shown in Fig. 2b. It is the approximation of the smooth intensity shown in in Fig. 2a. Obviously $f(-t)=f(t)$, as demands by (2.2). Figure 3 shows the function $f(t)$ calculated from (2.4). Each of the harmonic motions of (2.6) results in the medium 0 in a harmonic monochromatic wave. The time-dependent displacement in medium 0 equals therefore the sum of 18 monochromatic waves

$$\begin{aligned} u(x, t) = -.60 \sin\{.2\omega_0 (t-x/c_0)\} - .38 \sin\{.2\omega_0 (t-x/c_0)\} \\ +.41 \sin\{.3\omega_0 (t-x/c_0)\} + .71 \sin\{.4\omega_0 (t-x/c_0)\} + ... \\ -.17 \sin\{1.8\omega_0 (t-x/c_0)\} - .24 \sin\{1.9\omega_0 (t-x/c_0)\} + .16 \sin\{2.1\omega_0 (t-x/c)\}, \end{aligned} \tag{2.7}$$

Instead of the non-harmonic pulse (2.1). we face now the superposition of 18 harmonic waves of different frequencies $.1\omega_0$, $.2\omega_0$, $.3\omega_0$,... $2.1\omega_0$ being the fractions of the center frequency ω_0. The waves of frequency much larger than the center frequency ω_0 have very small amplitudes. For each such wave we may apply the formulae derived in the first chapter.

Since each of the separate waves of (2.7) propagates and reflects independently from the others, the energy flux is the sum of individual energy fluxes. Take first into account the first wave of (2.7), namely the wave $-.60 \sin\{.1\omega_0 (t-x/c_0)\}$ of frequency $.1\omega_0$. It carries the energy flux $\rho_0 c_0 (.1\omega_0)^2 - (.60)^2$. The energy flux is a function of time and space. The above formula should be understood as giving the average value for one period. In electrical engineering it corresponds to resistive load and the real power. The part corresponding to reactive load has zero mean value and is not taken into account. In acoustics it corresponds to the sound intensity. The second wave carries the energy flux $\rho_0 c_0 (.2\omega_0)^2 (-.38)^2$. Analogous energies are carried by the other waves. It follows that the total energy flux of all incident waves is given by the formula

$$\begin{aligned} S_i = \rho_0 c_0 [(.1\omega_0)^2 (-.60)^2 + (.2\omega_0)^2 (-.38)^2 \\ +(.3\omega_0)^2 (.41)^2 + ... + (1.9\omega_0)^2 (-.24)^2]. \end{aligned} \tag{2.8}$$

In accord with (1.2), the reflected energy flux corresponding to the first wave equals $\beta(.1\omega_0)^* \rho c(-.60)^2$. Note that β is a function of the frequency, here $.2\omega_0$. Analogous formulae hold for other harmonic waves of (2.7). The total energy flux of the reflected waves is therefore

$$S_r = \rho_0 c_0 [(.1\omega_0)^2 (-.60)^2 \beta(.1\omega_0) + (.2\omega_0)^2 (-.38)^2 \beta(.2\omega_0) + (.3\omega_0)^2 (.41)^2 \beta(.3\omega_0) + \ldots + (1.9\omega_0)^2 (-.24)^2 \beta(1.9\omega_0)]. \tag{2.9}$$

Denote by β_{band} the ratio of S_i and S_r

$$\beta_{\text{band}} = \frac{S_r}{S_i}. \tag{2.10}$$

This is the effective reflection coefficient for the pulse (2.1)

In the next chapter we shall perform the calculations for the short pulse $N=2$, Fig. 4. The function $I(\omega)$ is wider and has a lower maximum than that shown in Fig. 2a. The relation analogous to (2.5) corresponding to this pulse consists of 22 following terms:

$$\begin{aligned} f(t) = &-.07\sin(.1\omega_0 t) - .63\sin(.2\omega_0 t) - .66\sin(.3\omega_0 t) + \\ &-.44\sin(.4\omega_0 t) + .58\sin(.6\omega_0 t) + 1.19\sin(.7\omega_0 t) + 1.68\sin(.8\omega_0 t) + \\ &+ 1.97\sin(.9\omega_0 t) + 2.00\sin(\omega_0 t) + 1.78\sin(1.1\omega_0 t) + 1.38\sin(1.2\omega_0 t) + \\ &+ .88\sin(1.3\omega_0 t) + .39\sin(1.4\omega_0 t) - .24\sin(1.6\omega_0 t) - .32\sin(1.7\omega_0 t) + \\ &-.27\sin(1.8\omega_0 t) - .14\sin(1.9\omega_0 t) + .11\sin(2.1\omega_0 t) + .16\sin(2.2\omega_0 t) + \\ &-.14\sin(2.3\omega_0 t) + .08\sin(2.4\omega_0 t). \end{aligned} \tag{2.11}$$

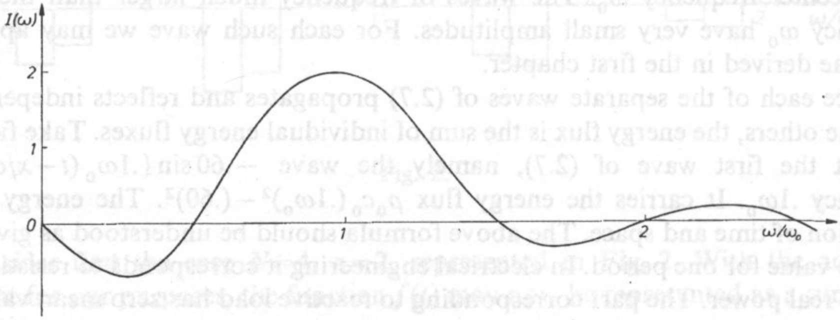

Fig. 4.

Numerical summation similar to that presented in Fig. 3 for (2.5) proves that the above formula provides sufficiently good approximation for the calculation of optimal matching layers.

3. Numerical results

The calculations will be performed for the data typical for the medical applications

$$\rho_0 c_0 = 30\ 10^6\ \text{kg/m}^2\text{s}, \quad \rho_3 c_3 = 1.5\ 10^6\ \text{kg/m}^2\text{s} \tag{3.1}$$

Since this paper is aimed only at the recognition of the medical applications, we prefer not to introduce dimensionless variables. The results are less general, but easier to follow. The only exception is made for the thicknesses h_1, h_2, which are replaced by the products

$$H_1 = \omega_0 h_1, \qquad H_2 = \omega_0 h_2. \tag{3.2}$$

Let us start with the pulse consisting of four periods of the sine curve. The matching layer (1.14) is

$$(c_1)_S = 6.7082\ 10^6\ \text{m/s}, \qquad (H_1)_S = 10.532\ \text{m/s}. \tag{3.3}$$

If the wave is monochromatic of frequency ω_0, then the reflection coefficient β_s for this wave equals zero. However. for the pulse represented in Fig. 3, the reflection coefficient for the layer (3.3) calculated from (2.7)–(2.9) is not zero,

$$\beta_S = .10931. \tag{3.4}$$

This is not a minimum even for one layer. A slightly better result may be obtained for other thicknesses and propagation speed,

$$(c_1)_m = 6.7091\ 10^6\ \text{m/s}, \qquad (H_1)_m = 10.34\ \text{m/s}, \qquad \beta_m = .10695. \tag{3.5}$$

This is the wide-band minimum for one layer.

In order to find two layers leading to lower β, the numerical analysis of (2.9) was performed. There exist many minima, but only some of them are interesting. For

$$\begin{aligned} c_1 &= 12.7\ 10^6\ \text{m/s}, \qquad H_1 = .172\ \text{m/s}, \\ c_2 &= 3.30\ 10^6\ \text{m/s}, \qquad H_2 = .043\ \text{m/s}, \end{aligned} \tag{3.6}$$

there exists a minimum

$$\beta = .03023 \tag{3.7}$$

It is more than three times lower than β_s, as given by (3.4), or than β_m, as given by (3.5)

Essential for the possibility of manufacturing the two layers (3.5) is the knowledge of the neighbourhood of the minimum. If the values of β near the point (3.5) are high, then it would be rather difficult to achieve value of β close to (3.6). Figure 5 shows the map of points c_1, c_2 for which $.035 < \beta < .040$ provided H_1, H_2 are fixed. The ovals are centered at (3.5). The length of the horizontal line represents propagation speed of 10^6 m/s.

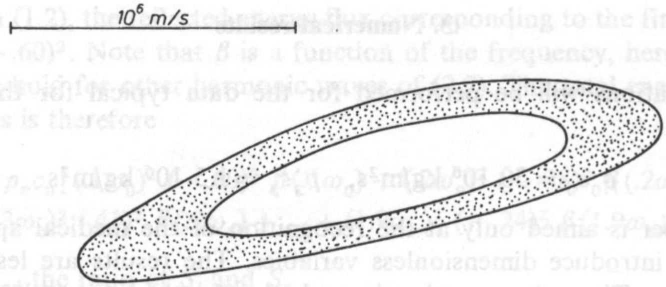

Fig. 5.

It is interesting to note there exists another minimum, situated far from the minimum (3.6)

$$c_1 = 7.79\ 10^6\ \text{m/s}, \qquad H_1 = .120\ \text{m/s},$$
$$c_2 = 2.04\ 10^6\ \text{m/s}, \qquad H_2 = .159\ \text{m/s}. \tag{3.8}$$
$$\beta = .07195. \tag{3.9}$$

This minimum is not so low as that given by (3.7). Note that the thicknesses of the layers are of the same order. In some situations this fact may simplify the manufacturing process. The reflection coefficient β, $.0775<\beta<.0875$ occupies the area between the two ovals, Fig. 6. Inside the inner oval the reflection coefficient

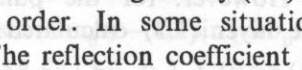

Fig. 6.

satisfies the inequality $\beta<0.7775$. Both ovals are centered at (3.8), the horizontal line represents 10^6 m/s. Note that the minimum is wide, therefore no extra accuracy is needed for manufacturing the layers.

Two examples of technically uninteresting minima are

$$c_1 = 11.24\ 10^6\ \text{m/s}, \qquad H_1 = 50.86\ \text{m/s},$$
$$c_2 = 3.25\ 10^6\ \text{m/s}, \qquad H_2 = 15.19\ \text{m/s}.$$
$$\beta = .12676.$$

This minimum is local minimum of the reflection coefficient. Note that its value is larger than β_s, as given by (3.4). A very narrow minimum exists at

$$c_1 = 6.70\ 10^6\ \text{m/s}, \qquad H_1 = 10.20\ \text{m/s},$$
$$c_2 = 17.50\ 10^6\ ,\!/\text{s}, \qquad H_2 = .114\ \text{m/s}.$$
$$\beta = .10726.$$

Pass now to the very short pulse consisting of two periods only, $N=2$, $n=1$. In accord with the spectral decomposition Fig. 4, we base on the formula (2.11). The matching layer (1.14) coincides with that given by (3.3), since it is determined by the central frequency ω_0, and is independent of the distribution of frequencies in the band. The wide-band pulse (2.11) reflecting on the layer (1.14) has the reflection coefficient

$$\beta_s = .21918. \tag{3.10}$$

The value of β is slightly smaller for the following single layer, namely

$$(c_1)_m = 6.710\ 10^6 \text{ m/s}, \quad (H_1)_m = 9.914 \text{ m/s}, \quad \beta_m = .20546. \tag{3.11}$$

This is the wide-band minimum for one layer. Note that both (3.3) and (3.11) give layers slightly thicker than $(H_1)_S$, and propagation speeds slightly lower than $(c_1)_S$

The formula (2.10) leads to the conclusion that at

$$\begin{aligned} c_1 &= 13.25\ 10^6 \text{ m/s}, \quad H_1 = .198 \text{ m/s}, \\ c_2 &= 3.37\ 10^6 \text{ m/s}, \quad H_2 = .0506 \text{ m/s}, \end{aligned} \tag{3.12}$$

there exists a minimum

$$\beta = .05961. \tag{3.13}$$

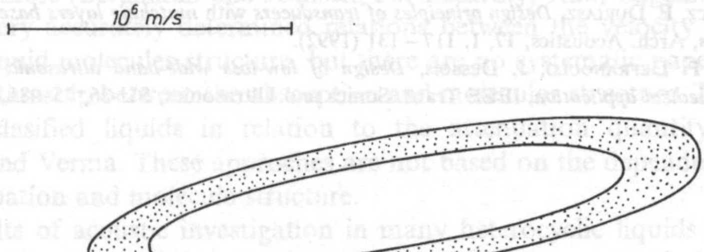

Fig. 7.

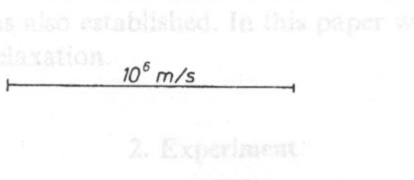

Fig. 8.

Figure 7 shows the map of values c_1, c_2 corresponding to: $065 < \beta < 0.7$ It is seen that the valley is rather wide. At the point

$$\begin{aligned} c_1 &= 8.36\ 10^6 \text{ m/s}, \quad & H_1 &= .125 \text{ m/s}, \\ c_2 &= 2.31\ 10^6 \text{ m/s}, \quad & H_2 &= .217 \text{ m/s}. \end{aligned} \tag{3.14}$$

there exist another minimum

$$\beta = .1770. \tag{3.15}$$

The valey corresponding to this minimum is rather narrow. The values $.180 < \beta < .185$ are situated between the two ovals, Fig. 8. Note that the gain due to the second layer is larger for the short pulse.

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