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CHARACTERIZATION OF SOUND ABSORBING MATERIALS USED IN NOISE CONTROL ENGINEERING

A. COPS AND W. LAURIKS*

Laboratorium voor Akoestiek en Warmtegeleiding
Department Natuurkunde, K.U. Leuven
(3001 Heverlee, Celestijnenlaan 200 D, Belgium)

1. Introduction

Sound absorbing materials are extensively used in the field of noise control engineering and architectural acoustics. Indeed they are, for an important part, responsible for the efficient sound insulation of partitions between rooms and of enclosures, for the acoustic comfort in rooms and industrial halls etc. Due to the continuing technological progress, research has been done and is still done in the motorcar, the aerospace and the manufacturing industries and others, to obtain absorbing materials who are simultaneously small in volume, have a low density, are rather cheap and have a good acoustic performance. From the other side it is often desirable to optimize the behaviour of acoustic materials for very specific purposes. To achieve highly qualified absorbing materials in a broad field of applications it is necessary to develop precise and, from time to time, sophisticated theoretical models which take into account the complex physical phenomena of acoustic materials.

The study of acoustic properties of absorbing materials has occupied scientist since the work of BERANEK [1, 2], MORSE et al. [3] and ZWIKKER and KOSTEN [4] in the early 1940's. A large number of different models, describing the sound propagation in porous materials, have been published since. An excellent review can be found in the publication of ATTENBOROUGH [5]. However most of these models require the introduction of parameters which can not be measured independently, making them inappropriate for the design of absorbing materials. Since 1956, the Biot theory allows in a very general and rigorous way the description of sound propagation in porous materials. This theory has been developed originally for application in the field of underwater geophysics, where the densities of the fluid (water) and the solids are comparable. Recently, the Biot theory has been used extensively to calculate the acoustic properties of porous materials used in noise control engineering, where the

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* Senior Research Assistant N.F.W.D

fluid in the materials is air in stead of water. It has been proven that in a number of cases, the theory is indispensable to explain the acoustic behaviour of different types of acoustic materials. In this document the theory of Biot, related to the sound propagation of acoustic waves in porous materials, will be presented and adopted for different types of single and multiple layers by using matrix models for the materials. From the theory it is possible to calculate the characteristics of the material, either the specific impedance or the sound absorption coefficient.

For practical purposes also precise standardized methods, to measure acoustical parameters, are developed and still have to be developed. In this document some standardized methods will be discussed. Comparisons of theoretical predictions and measured characteristics of porous materials will be discussed.

Measuring methods

1. Measurement of sound absorption in a reverberant room

The International Standard ISO-354(6) specifies a method of measuring the sound absorption coefficient of acoustical materials used as wall or ceiling treatments, or the equivalent sound absorption area, such as furniture, persons or space absorbers, such as baffles, in a reverberation room. It is not intended for measuring the absorption characteristics of weakly damped resonators.

The results obtained can be used for comparison purposes and for design calculations with respect to room acoustics and noise control. The measuring principle is the following: Measurement of reverberation times in a reverberation room with and without the test specimen. From these times, the calculation of the equivalent sound absorption area A of the test specimen is performed. In the case of plane test specimens, the sound absorption coefficient is obtained by dividing A by its surface area S. When the test specimen comprises several identical objects, the equivalent sound absorption area of an individual object, for example a baffle, is obtained by dividing A by the number of objects.

The equivalent sound absorption area A, in square metres, of the test specimen, shall be calculated using the formula

$$A = 55.3\,(V/c)(1/T_2 - 1/T_1) \tag{1}$$

where V is the volume, in cubic metres, of the empty reverberation room; c is the velocity of sound in air in metres per second; for temperatures between 15°C and 30°C, the velocity of sound in air, c, can be calculated form the formula: $c = 331 + 0.6t$, where t is the air temperature, in degrees Celsius. T_1 and T_2 are respectively the reverberation times, in seconds, of the empty room and after the test specimen has been introduced.

The sound absorption coefficient α_S of a plane absorber shall be calculated using the formula:

$$\alpha_S = A/S \tag{2}$$

where A is the equivalent sound absorption area, in square metres, calculated from equation (1); S is the area, in square metres, of the test specimen.
For discrete absorbers, the result should generally be expressed as equivalent sound absorption area per object, which is determined by dividing A by the number of objects tested. For a specified array of objects, the result should be given as equivalent sound absorption area of the whole configuration.

Recently an annex has been added to the ISO 354 Standard specifying the test specimen mountings for sound absorption tests. Specific prescriptions are given for specimens mounted directly against the room surface (type A mounting), for test specimens mounted with an air space behind it (type E mounting), for test specimens, such as curtains, drapperies, window shades or window blinds, hanging parallel to the room surface (type G mounting), for spray- or towel-applied materials, such as plaster (type I mounting) and for sound absorber pads and baffles where the sound absorption per unit of rectangular unit shall be used (type J mounting).

The feature of this measuring method is that it gives the results of the sound absorption coefficient or the equivalent sound absorption area of materials for at random sound incidence, as is generally the case for practical applications. A disadvantage of this method is that no information about more fundamental parameters like the specific acoustic impedance or wave propagation constants in the material can be obtained.

2. Sound absorbers — Rating of sound absorption

The International Standard Document ISO/CD 11654(7) specifies a method by which the frequency dependent values of the sound absorption coefficient can be converted into a single number. Before this is done the one-third octave band values of the sound absorption coefficient measured according to ISO 354 are converted into octave bands. The sound absorption coefficient α_{pi}, for each octave band i, is calculated as the arithmetic mean value of the three one-third octave sound absorption coefficient, α_{i1}, α_{i2}, α_{i3} within the octave band:

$$\alpha_{pi} = \frac{(\alpha_{i1} + \alpha_{i2} + \alpha_{i3})}{3} \tag{3}$$

α_p has to be rounded in steps of 0.05 and maximized to 1.00, that is if α_p calculated is > 1.00 then $\alpha_p = 1.00$.
α_p is used to calculate the weighted sound absorption coefficient α_w, from the reference curve in Fig. 1 and Table 1. Shift the reference curve in steps of 0.1 towards the measured value until the sum of the unfavourable deviations is less than or equal to 0.1. An unfavourable deviation occurs at a particular frequency when the measured value is less than the value of the reference curve. Only deviaitons in the unfavourable direction shall be counted. The weighted sound absorption α_w is defined as the value of the shifted reference curve at 500 Hz.

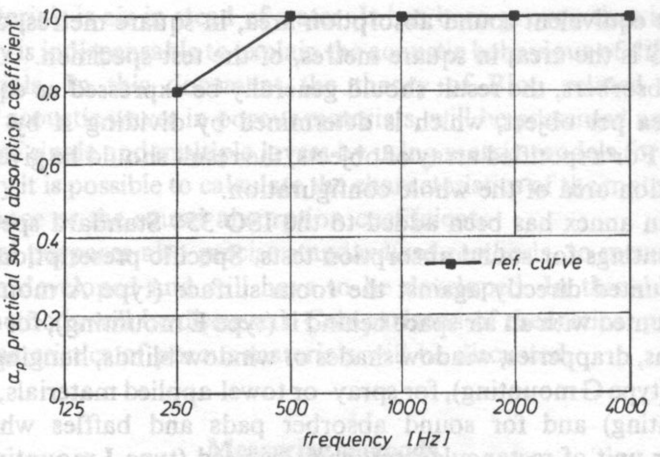


Fig. 1. Reference curve for evaluation of weighted sound absorption coefficient, α_w.

Table 1. Values of the reference curve in Fig. 1.

Frequency	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz
$\alpha_{pi,\,ref}$	0.8	1.0	1.0	1.0	1.0

If, for any octave band, $(\alpha_{pi} - \alpha_{pi,\,\text{ref, shifted}}) \geqslant 0.3$ then add a* sign after α_w — value, e.g. $\alpha_w = 0.5$*. This means that the sound absorption coefficient at one or more frequencies is considerably higher than the reference curve. An example of the calculation of α_w given in Fig. 2. Shift the reference curve in steps of 0.1 towards the measured value until the sum of the unfavourable deviations $\leqslant 0.1$. In the example the unfavourable deviations occur at 250 and 1000 Hz and the result is $\alpha_w = 0.6$.

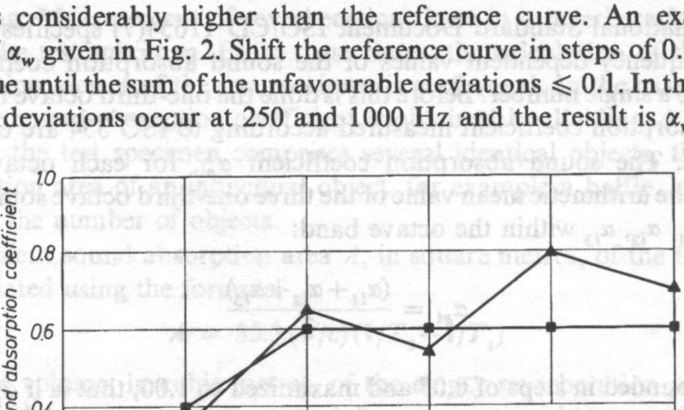


Fig. 2. Example of an α_w-calculation.

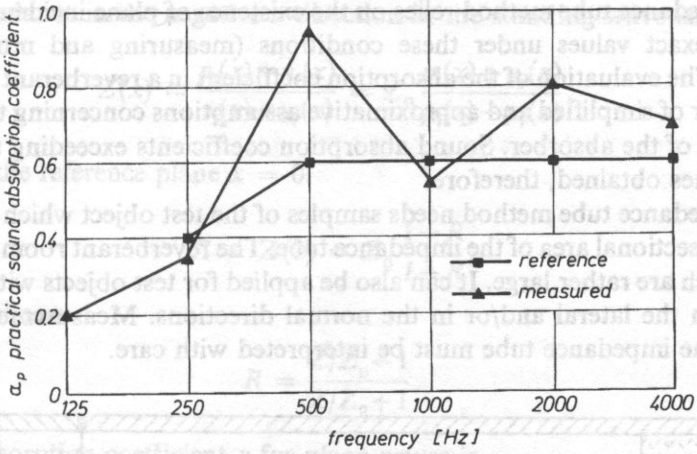


Fig. 3. Example of an $\alpha_w = 0.6^*$ calculation.

In Fig. 3 the corresponding example is shown when $(\alpha_{p,\ 500\,\mathrm{Hz}} - \alpha_{p,\ \mathrm{ref,\,shifted}}) = 0.35 \geqslant 0.30$ that is $\alpha_w = 0.6^*$.

The weighted sound absorption coefficient shall be expressed to one decimal place. This international standard is, in principle, applicable to all products for which the sound absorption coefficient has been determined according to ISO 354. It is, however, often not suitable for application on single items, such as chairs, baffles, etc.

3. Determination of the sound absorption coefficient and impedance or admittance by the impedance tube method

The International Standard Document ISO/CD. 10534 [8] describes the determination of the sound absorption coefficient, the reflection factor and the surface impedance or surface admittance of materials and objects. The values are determined for normal incidence by an evaluation of the standing wave pattern of a plane wave in a tube, which is generated by the superposition of an incident sinusoidal plane wave with the plane wave reflected from the test object. It is well suited for parameter studies and for the design absorbers, because only small samples of absorber material are needed.

There are some characteristic differences compared to the measurement of the sound absorption in a reverberant room (ISO 354). The impedance tube method can be applied for the determination of the reflection factor and the impedance or admittance, also. The sound is incident normally on the object surface. The reverberant room method will — under idealized conditions — determine the sound absorption coefficient for omnidirectional sound incidence.

The impedance tube method relies on the existence of plane incident sound waves and gives exact values under these conditions (measuring and mounting errors excluded). The evaluation of the absorption coefficient in a reverberant room is based on a number of simplified and approximative assumptions concerning the sound field and the size of the absorber. Sound absorption coefficients exceeding the values one are sometimes obtained, therefore.

The impedance tube method needs samples of the test object which are of the size of the cross-sectional area of the impedance tube. The reverberant room method needs objects which are rather large. It can also be applied for test objects with pronounced structures in the lateral and/or in the normal directions. Measurements with such objects in the impedance tube must be interpreted with care.

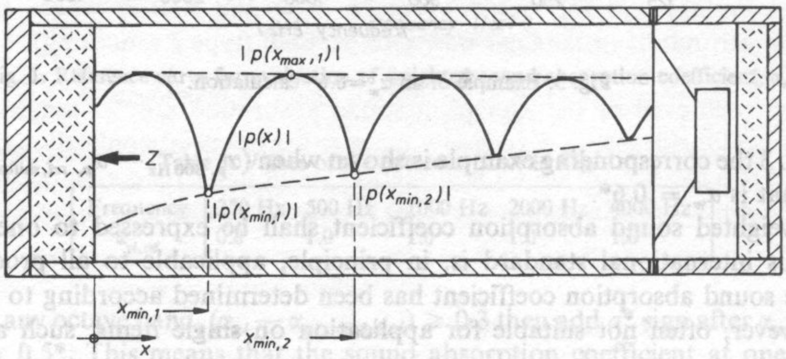


Fig. 4. Representation of the standing wave pattern in the tube.

The test object is mounted at one end of the straight, rigid smooth and tight impedance tube, see Fig. 4.

The incident sound wave p_i is assumed to be plane, harmonic in time with frequency f and angular frequency $\omega = 2\pi f$ (the time factor $e^{j\omega t}$ will be omitted in what follows), and directed along the axis of the impedance tube in the negative x-direction:

$$p_i(x) = P_0 e^{jk_0 x}\,;\; k_0 = \frac{\omega}{c_0} = \frac{2\pi f}{c_0}. \tag{4}$$

The amplitude P_0 is arbitrary.
The wave which is reflected from the test object having a reflection factor R is then:

$$p_r(x) = RP_0 e^{-jk_0 x}. \tag{5}$$

The particle velocities of the waves (counted positive in the negative x-direction) are respectively:

$$v_i = \frac{1}{Z_0} p_i(x)\,;\; v_r(x) = -\frac{1}{Z_0} p_r(x). \tag{6}$$

The field impedance in the negative x-direction in the standing wave is:

$$Z(x) = \frac{p_i(x)+p_r(x)}{v_i(x)+v_r(x)} = Z_0 \frac{p_i(x)+p_r(x)}{p_i(x)-p_r(x)}. \tag{7}$$

Especially at the reference plane $x = 0$:

$$Z = Z(0) = Z_0 \frac{1+R}{1-R}, \tag{8}$$

from which follows:

$$R = \frac{Z/Z_0 - 1}{Z/Z_0 + 1}. \tag{9}$$

The sound absorption coefficient α for plane waves is

$$\alpha = 1 - |R|^2. \tag{10}$$

A pressure maximum in the standing wave is at a place where p_i and p_r are in phase

$$|p_{\max}| = |P_0| \cdot (1+|R|). \tag{11}$$

A pressure minimum is at a place of opposite phase

$$|p_{\min}| = |P_0| \cdot (1-|R|). \tag{12}$$

Using the standing wave ratio $s = |p_{\max}| \,/\, |p_{\min}|$:

$$s = \frac{1+|R|}{1-|R|} \quad \text{and} \quad |R| = \frac{s-1}{s+1} \tag{13}$$

The sound absorption then follows from the relation (10), with $|p_{\max}|$ and $|p_{\min}|$ at a given frequency.
If the sound pressure in the impedance tube is measured in a logarithmic scale (in dB), and the difference in level between the pressure maximum and the pressure minimum is $\Delta L =$ dB, then:

$$s = 10^{\Delta L/20} \tag{14}$$

The sound absorption coefficient then follows from:

$$\alpha = \frac{4 \cdot 10^{\Delta L/20}}{(10^{\Delta L/20}+1)^2} \tag{15}$$

The phase angle Φ of the complex reflection factor $R = |R| e^{j\Phi}$ follows from the phase condition for a pressure minimum in the standing wave:

$$\Phi + (2n-1) \cdot \pi = 2k_0 x_{\min, n} \tag{16}$$

for the n-th minimum ($n = 1, 2, ...$) in front of the reference plane (towards the sound source).
From this:

$$\phi = \pi\left(\frac{4x_{\min,n}}{\lambda_0} - 2n + 1\right), \tag{17}$$

and for the first minimum ($n = 1$):

$$\Phi = \pi\left(\frac{4x_{\min,1}}{\lambda_0} - 1\right). \tag{18}$$

The complex reflection factor is then

$$R = R' + jR''$$
$$R' = |R|\cdot\cos\Phi;\ R'' = |R|\cdot\sin\Phi. \tag{19}$$

According to Eq. (8) one obtaines the normalized impedance $z = Z/Z_0$:

$$z = z' + jz''$$
$$z' = \frac{1-R'^2-R''^2}{(1-R')^2+R''^2};\ z'' = \frac{2R''}{(1-R')^2+R''^2}. \tag{20}$$

Finally the normalized admittance is obtained as the reciprocal of the normalized impedance:

$$g = Z_0/Z = Z_0 G, \tag{21}$$

with $G = 1/Z = v/p$ the admittance, is the ratio of the sound particle velocity to the sound pressure at that point.

4. *Two-microphone impedance measurement tube method*

The two-microphone method (9) of measuring the acoustic absorption coefficient involves the decomposition of broad-band stationary random signal into its incident (P_i) and the reflected (P_r) components. The singal is generated by a sound source, and the incident and reflected components are determined from the relationship between the acoustic pressure measured by microphones at two locations on the wall of the tube (see Fig. 5).

From the incident and reflected components of the sound pressure at the two microphone positions, three frequency response functions are calculated; H_1 the frequency response function between the two microphone channels, and H_i the frequency response function associated with the incident component, and H_r, the frequency response function associated with the reflected component. Using these values, the complex reflection coefficient R is calculated from the following equation:

$$R = \left(\frac{H_1 - H_i}{H_r - H_1}\right) \cdot e^{j2k(1+s)} \tag{22}$$

where k is the wave number, l is the distance between the first microphone location and the front of the sample (in mm), and s is the spacing between the microphones (in mm). Using the value for the reflection coefficient, the normalized impedance z and the sound absorption coefficient α can be calculated from the equations (8) respectively (10).

For each measurement made on a sample, the following date can be calculated and displayed in the frequency range of interest from the above mentioned equations: the acoustic absorption coefficient (magnitude only); the acoustic reflection coefficient, the normalized impedance, the frequency response function or the calibration result, each displayed as magnitude, phase, real part or imaginary part.

The two-microphone theory assumes plane-wave propagation, no mean flow and no losses due to absorption at the tube wall. This absorption is kept to a minimum in the normaly used two-microphone impedance measuring tube.

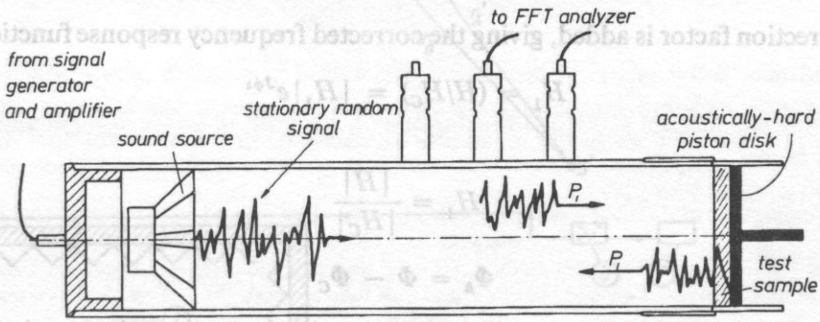


Fig. 5. Set-up of the impedance measurement tube

As described above, the frequency response function is calculated from the cross spectrum of the two microphone signals, so any phase or amplitude mismatch between these microphone channels will corrupt its calculated value. During the calibration procedure, the frequency response function is calculated with the two microphones interchanged, and then again in there initial positions. The geometric mean of these two results is a complex value which can be added to any subsequent frequency response function that is calculated using the same setup, effectively eliminating errors due to any mismatches in the microphone channels.

During the calibration procedure, the calibration frequency response functions for the microphones in the standard positions (H_{C1}) and the interchanged positions (H_{C2}) are calculated as:

$$H_{C1} = |H_{C1}|\, e^{j\Phi_1}, \tag{23}$$

and:

$$H_{C2} = |H_{C2}|\, e^{j\Phi_2}, \tag{24}$$

where Φ_1 is the phase of the calibration frequency response function H_{C1}, Φ_2 is the phase of the calibration frequency response function H_{C2} and j is $\sqrt{-1}$.

From these values, the calibration factor (H_C) is calculated as:

$$H_C = |H_C|\ e^{j\Phi_C} \tag{25}$$

where

$$|H_C| = \sqrt{|H_{C1}|/|H_{C2}|}, \tag{26}$$

$$\phi_C = (1/2)\ (\phi_1 + \phi_2). \tag{27}$$

This calibration factor can now be added to any frequency response function that is calculated using the tube set-up, giving a value that is uneffected by the amplitude or phase mismatches between the microphone channels.

For example, the following frequency response function is measured, with the microphones in the standard positions:

$$H = |H|\, e^{j\phi} \tag{28}$$

The correction factor is added, giving the corrected frequency response function (H_1):

$$H_1 = (H/H_C) = |H_1|\, e^{J\phi_h} \tag{29}$$

where

$$H_1 = \frac{|H|}{|H_C|} \tag{30}$$

$$\Phi_h = \Phi - \Phi_C \tag{31}$$

This frequency response function H_1 is the value that is used to calculate the acoustic properties of the test sample.

5. *Two-microphone free field method*

The two-microphone free field method [10, 11, 12] used to calculate the normalized surface impedance of a test sample involves the measurement of the Fourier transform of the pressure at the point half between the two microphones (see Fig. 6):

$$P = \frac{P_A + P_B}{2} \tag{32}$$

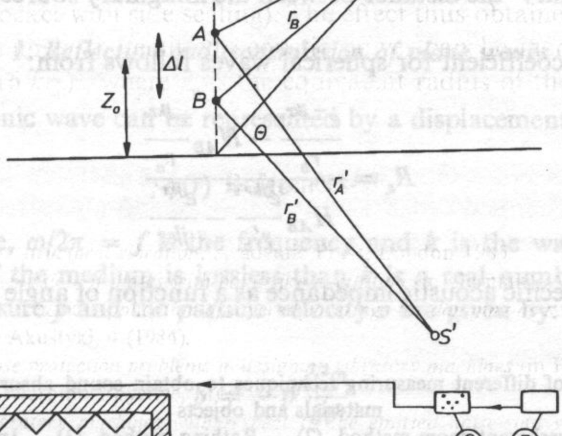
where P_A and P_B are the Fourier transforms of the sound pressure at the microphones A and B. The particle velocity at that point can be expressed by the Fourier transform of the pressure gradient:

$$V_z = \frac{P_A - P_B}{jw\rho\Delta z}. \tag{33}$$

For plane acoustic waves the normalized impedance at the measuring point is given by:

$$z^* = -\frac{1}{\rho c}\frac{P}{V_z} = \frac{jw\Delta z}{2c}\frac{1+H_{AB}}{1+H_{AB}}, \tag{34}$$

with $H_{AB} = P_A/P_B$.



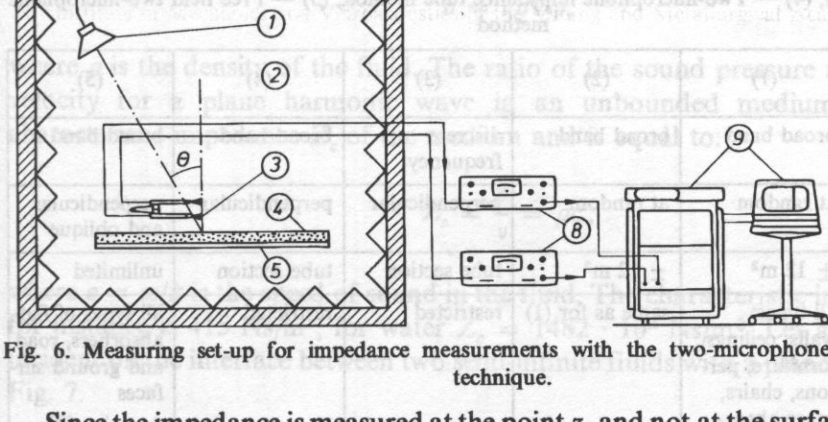


Fig. 6. Measuring set-up for impedance measurements with the two-microphone transfer function technique.

Since the impedance is measured at the point z_0 and not at the surface, a correction has to be made. It can be shown [10, 11] that the normalized specific impedance at the sample surface is given by:

$$z(\theta) = \frac{z^* + j/\cos\theta \cdot \tan(\omega z_0 \cos\theta/c)}{1 + j\cdot\cos\theta\cdot\tan(\omega z_0\cos\theta/c)}\,. \tag{35}$$

Several alternatives on this two-microphone free field method have been worked out e.g. by CHUNG [13, 14] and CHU [15] for plane waves.

The most important disadvantage of the preceeding methods is the plane wave assumption. Therefore the spherical wave estimate technique has been developed [16, 17]. If the distance between the source and the measuring surface is finite, the sound pressure above the surface is approximated by:

$$p = \frac{e^{jkr}}{r} + R_s \frac{e^{jkr'}}{r'}, \tag{36}$$

with R_s the spherical reflection coefficient, r the distance between the source and the measuring point and r' the distance between the imaginary source and the measuring point.

The reflection coefficient for spherical waves follows from:

$$R_s = \frac{\dfrac{e^{-jkr_b}}{r_b} - H_{AB}\dfrac{e^{jkr_a}}{r_a}}{H_{AB}\dfrac{e^{jkr'_a}}{r'_a} - \dfrac{e^{jkr'_b}}{jkr}} \tag{37}$$

The normalized specific acoustic impedance as a function of angle of incidence can be obtained from:

Table 2. Comparison of different measuring techniques to obtain sound absorption characteristics of materials and objects

Techniques: (1) — Reverberant room method, (2) — Rathing method, (3) — Impedance Tube method (single microphone), (4) — Two-microphone impedance tube method, (5) — Free field two-microphone method

Measuring methods	(1)	(2)	(3)	(4)	(5)
Source signal	broad band	broad band	discrete frequency	broad band	broad band
Sound incidence	at random	at random	perpendicular	perpendicular	perpendicular and oblique
Sample surface	± 12 m²	± 12 m²	tube section	tube section	unlimited
Test materials	all types: walls, ceilings, furniture, persons, chairs, space absorbers etc...	same as for (1)	restricted	restricted	all types of flat absorbers, road and ground surfaces
Measuring time	fast	fast	time consuming	fast	fast
Test results	α (can be >1) A (eq.abs.area) A/test object	α (always $\leqslant 1$)	α (always $\leqslant 1$) R $z = Z/Z_0$	α (always $\leqslant 1$) R $z = Z/Z_0$	α (always $\leqslant 1$) R $z = Z/Z_0$

$$z_s(\theta) = \frac{1}{\cos\theta} \cdot \frac{1+R_s}{1-R_s} \cdot \left(1 - \frac{1}{jkr}\right) \tag{38}$$

An advantage of the spherical wave estimate technique is that measurements can be done in surroundings with background noise.

In Table 2 the features of the 5 different measuring methods to obtain absorption characteristics of sound absorbing materials are compared with each other.

Theory

1. Reflection and transmission of plane waves

A plane harmonic wave can be represented by a displacement potential:

$$\varphi(r) = Ae^{-jkr+j\omega t}, \tag{39}$$

A is the amplitude, $\omega/2\pi = f$ is the frequency and k is the wave number of the harmonic wave. If the medium is lossless than k is a real number, otherwise it is complex. The pressure p and the particle velocity v are given by:

$$p = -p\frac{\delta^2\varphi}{\delta t^2}, \tag{40}$$

$$v = i\omega\nabla\varphi, \tag{41}$$

where ρ is the density of the fluid. The ratio of the sound pressure and the particle velocity for a plane harmonic wave in an unbounded medium is called the characteristic impedance Z_c of the medium and is equal to:

$$Z_c = \frac{p}{v} = \rho c, \tag{42}$$

where $c = \omega/k$ is the speed of sound in the fluid. The characteristic impedance of air for instance is 415 Ns/m^3; for water $Z_c = 1482 \cdot 10^3$ Ns/m^3. Let a plane wave be incident on the interface between two semi-infinite fluids with a plane boundary as in Fig. 7.

The incident wave can be written as:

$$\varphi_{\text{inc}} = Ae^{-jk(x_1\sin\theta+x_3\cos\theta)+j\omega t}, \tag{43}$$

with θ the angle of incidence. The reflected plane wave can be written as:

$$\varphi_{\text{refl}} = RAe^{-jk(x_1\sin\theta+x_3\cos\theta)+j\omega t}, \tag{44}$$

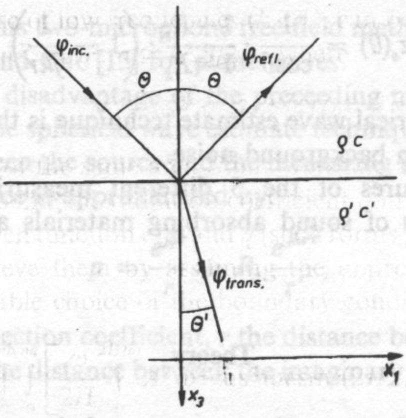


Fig. 7. A plane sound wave incident on a boundary between two semi-infinite fluids.

with R the amplitude of the reflection coefficient at the interface. In the general case, R is a complex number and consists of a magnitude and a phase: $R = |R|\, e^{j\phi}$, with $0 \leqslant |R| \leqslant 1$. The transmitted wave can be written as:

$$\varphi_{\text{trans}} = TAe^{-jk(x_1 \sin\theta + [illegible])}, \tag{45}$$

with T the transmission coefficient.

The ratio of the acoustic pressure and the normal component of the palocity at the interface is called the normal surface impedance Z. In general Z is a complex, frequency dependent number $Z = R + jX$.

It can be shown that:

$$Z = \frac{p}{v_3} = \frac{\rho c}{\cos\theta} \cdot \frac{1+R}{1-R}, \tag{46}$$

and

$$R = \frac{Z\cos\theta - \rho c}{Z\cos\theta + \rho c}. \tag{47}$$

In a lot of applications, the absorption coefficient α is used as a parameter:

$$\alpha = 1 - |R|^2. \tag{48}$$

Since it is a ree it is a real number, α gives less information about the acoustic behaviour of the interface than the surface impedance. Combining Eq. (47) and (48) results in:

$$\alpha = \frac{4R\cos\theta}{(R\cos\theta + 1)^2 + (X\cos\theta)^2}. \tag{49}$$

2. Single wave propagation for materials with low flow resistivity

Empirical relations between flow resistivity and characteristic impedance and propagation constant

A plastic foam can be considered as an elastic solid frame containing bubbles. In high porosity low density foams, the gas bubbles have approximately the shape of dodecahedra. The lines of intersection and the membranes between different cells occupy only a few percent of the total volume fraction for most materials used in noise control and architectural applications. If all the membranes are ruptured, the foam is called reticulated. For most practical purposes, the sound propagation in reticulated foams can be simplified since the viscous coupling between frame and air is weak. In a large frequency range, the sound propagation in such materials is mainly determined by a wave propagating in the air in the material. In that case, the characteristic impedance and the propagation constant (or equivalently, the speed of sound $c = \omega/k$ and the complex density $\rho = Zk/\omega$) are defined by the flow resistivity ρ of the material, which is defined as the ratio of the air pressure difference Δp accross a test specimen and the steady volumetric air flow rate Q crossing the test specimen, normalized for the cross-sectional area A and the thickness d of the test specimen:

$$\sigma = \frac{\Delta p}{Q} \frac{A}{d} \quad (\mathrm{Ns/m^4}). \tag{50}$$

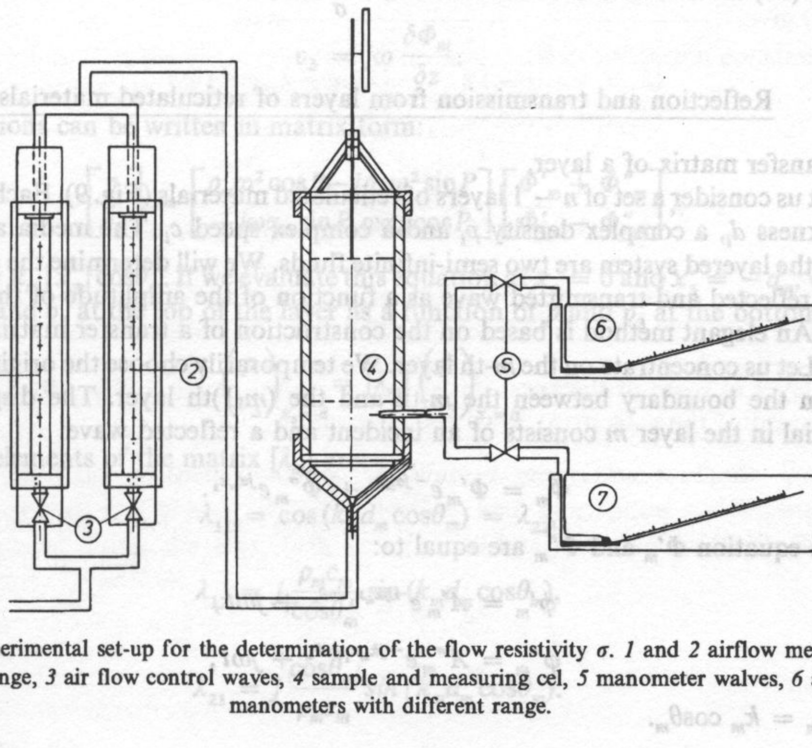


Fig. 8. Experimental set-up for the determination of the flow resistivity σ. *1* and *2* airflow meters with different range, *3* air flow control waves, *4* sample and measuring cel, *5* manometer walves, *6* and *7* oil manometers with different range.

The flow resistivity of reticulated foams varies from a few thousands Ns/m^4 to approximately 20000 Ns/m^4. The flow resistivity can be measured with the apparatus shown in Fig. 8 (18).

The relation between the flow resistivity and the acoustic properties of open cell porous materials has been determined by several authors for fibrous materials and plastic foams [19, 20, 21]. These relations are based on a number of measurements of Z_c and k as a function of σ. The results are [21]:

$$\mathrm{Re}(Z_c) = \rho_0 c_0 \left[1 + 0.1087 \left(\frac{\rho_0 f}{\sigma} \right)^{-0.6731} \right], \tag{51}$$

$$\mathrm{Im}\,(Z_c) = -\rho_0 c_0\, 0.2082 \left(\frac{\rho_0 f}{\sigma} \right)^{-0.6193}, \tag{52}$$

$$\mathrm{Re}(k) = \frac{\omega}{c_0} \left[1 + 0.0608 \left(\frac{\rho_0 f}{\sigma} \right)^{-0.7173} \right], \tag{53}$$

$$\mathrm{Im}\,(k) = -\frac{\omega}{c_0} \left[1 + 0.1323 \left(\frac{\rho_0 f}{\sigma} \right)^{-0.6601} \right]. \tag{54}$$

In these equations ρ_0 is the density of air and c_0 is the sound speed in air. Equations (51) to (54) are considered valid if $0.01 \leqslant \frac{(\rho_0 f)}{\sigma} \leqslant 1$.

Reflection and transmission from layers of reticulated materials

Transfer matrix of a layer.

Let us consider a set of $n - 1$ layers of reticulated materials (Fig. 9). Each layer has a thickness d_i, a complex density ρ_i and a complex speed c_i. The media above and below the layered system are two semi-infinite fluids. We will determine the amplitude of the reflected and transmitted wave as a function of the amplitude of the incident wave. An elegant method is based on the construction of a transfer matrix for each layer. Let us concentrate on the *m*-th layer. We temporarily choose the origin of the x_3 axis on the boundary between the *m*-th and the (*m*-1)th layer. The displacement potential in the layer *m* consists of an incident and a reflected wave:

$$\Phi_m = \Phi'_m e^{-j\alpha_m x_3} + \Phi''_m e^{j\alpha_m x_3}. \tag{55}$$

In this equation Φ'_m and Φ''_m are equal to:

$$\phi'_m = A'_m e^{-jk_m x_1 \sin\theta_m} + j\omega t,$$

$$\phi''_m = A''_m e^{-jk_m x_1 \sin\theta_m} + j\omega t,$$

and $\alpha_m = k_m \cos\theta_m$.

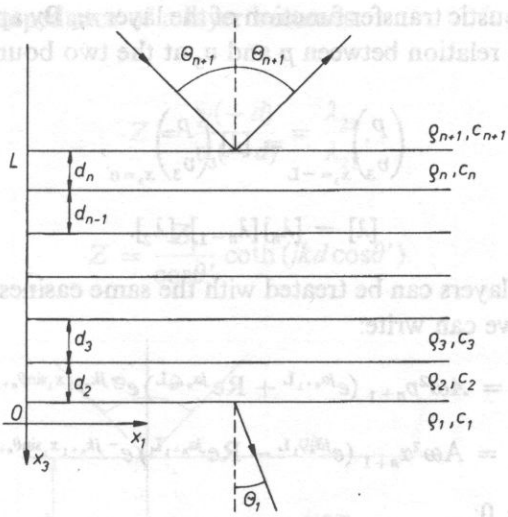


Fig. 9. A system of *n*-1 layers of acoustic material.

The normal component of the velocity and the sound pressure are:

$$p = -\rho \frac{\delta^2 \Phi_m}{\delta t^2}, \tag{56}$$

$$v_3 = j\omega \frac{\delta \Phi_m}{\delta z}. \tag{57}$$

The equations can be written in matrix form:

$$\begin{bmatrix} p \\ v_3 \end{bmatrix} = \begin{bmatrix} \rho_m \omega^2 \cos P & -j\rho_m \omega^2 \sin P \\ -j\omega\alpha_m \sin P & \omega\alpha_m \cos P \end{bmatrix} \begin{bmatrix} \Phi'_m + \Phi''_m \\ \Phi'_m - \Phi''_m \end{bmatrix}, \tag{58}$$

where $P = k_m |x_3| \cos\theta_m$. If we evaluate this equation at $x_3 = 0$ and $x_3 = -d_m$, we can express p and v_3 at the top of the layer as a function of p and v_3 at the bottom face:

$$\begin{pmatrix} p \\ v_3 \end{pmatrix}_{x_3=-d} = [\lambda_m] \begin{pmatrix} p \\ v_3 \end{pmatrix}_{x_3=0'}. \tag{59}$$

The four elements of the matrix $[\lambda_m]$ are:

$$\lambda_{11} = \cos(k_m d_m \cos\theta_m) = \lambda_{22},$$

$$\lambda_{12} = j \frac{\rho_m c_m}{\cos\theta_m} \sin(k_m d_m \cos\theta_m),$$

$$\lambda_{21} = j \frac{\cos\theta_m}{\rho_m c_m} sin\,(k_m d_m \cos\theta_m).$$

This matrix is the acoustic transfer function of the layer *m*. By application of equation (59), we can find the relation between *p* and υ_3 at the two boundaries of the layered system:

$$\begin{pmatrix} p \\ \upsilon_3 \end{pmatrix}_{x_3=-L} = [\lambda] \begin{pmatrix} p \\ \upsilon_3 \end{pmatrix}_{x_3=0}, \tag{60}$$

$$[\lambda] = [\lambda_n][\lambda_{n-1}]..[\lambda_2].$$

The system with *n*-1 layers can be treated with the same easiness as for one layer. At the face $x_3 = -L$, we can write:

$$p_{n+1} = A\omega^2 p_{n+1} (e^{j\alpha_{n+1}L} + \mathrm{R}e^{j\alpha_{n+1}L}) e^{-jk_{n+1}x_1 \sin\theta_{n+1} + j\omega t}, \tag{61}$$

$$\upsilon_{3n+1} = \mathrm{A}\omega^2 \alpha_{n+1} (e^{j\alpha_{n+1}L} - \mathrm{R}e^{j\alpha_{n+1}L}) e^{-jk_{n+1}x_1 \sin\theta_{n+1} + j\omega t}, \tag{62}$$

and at the face $x_3 = 0$:

$$p_1 = A\omega^2 \rho_1 \mathrm{T}e^{-jk_1 x_1 \sin\theta_1 + j\omega t}, \tag{63}$$

$$\upsilon_{31} = \mathrm{A}\omega\alpha_1 \mathrm{T}e^{-jk_1 x_1 \sin\theta_1 + j\omega t}, \tag{64}$$

In these equations *R* is the reflection coefficient and *T* the transmission coefficient of the system. *A* is the amplitude of the incident wave. The Equations (59) to (64) give:

$$R = \frac{(\lambda_{12}\alpha_1 + \lambda_{11}\omega\rho_1)\alpha_{n+1} - (\lambda_{22}\alpha_1 + \lambda_{21}\omega\rho_1)\omega\rho_{n+1}}{(\lambda_{12}\alpha_1 + \lambda_{11}\omega\rho_1)\alpha_{n+1} + (\lambda_{22}\alpha_1 + \lambda_{21}\omega\rho_1)\omega\rho_{n+1}} e^{2j\alpha_{n+1}L}, \tag{65}$$

$$T = \frac{2\alpha_{n+1}\omega\rho_{n+1}}{(\lambda_{12}\alpha_1 + \lambda_{11}\omega\rho_1)\alpha_{n+1} + (\lambda_{22}\alpha_1 + \lambda_{21}\omega\rho_1)\omega\rho_{n+1}} e^{2j\alpha_{n+1}L}. \tag{66}$$

Application to one layer of foam, stuck on a hard backing

A layer of foam of thickness *d* is stuck on a hard backing (Fig. 10). The normal component of the velocity at $x_3 = 0$ is equal to:

$$\upsilon_3(0) = 0. \tag{67}$$

From Eq. (59) we obtain υ_3 and *p* at $x_3 = -d$:

$$\upsilon_3(-d) = \lambda_{21} p(0), \tag{68}$$

$$p(-d) = \lambda_{22} p(0). \tag{69}$$

The normal surface impedance of a layer becomes:

$$Z = \frac{p(-d)}{v_3(-d)} = \frac{\lambda_{22}}{\lambda_{21}}, \tag{70}$$

or:

$$Z = \frac{Z_c}{\cos\theta'} \coth(jkd\cos\theta'). \tag{71}$$

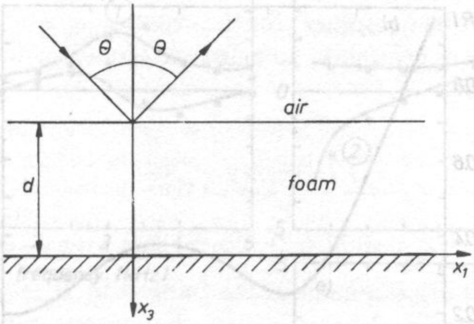


Fig. 10. One layer of foam, stuck on a hard backing.

In this equation, Z_c is the characteristic impedance, k the propagation constant. θ' is the angle of refraction and is given by Snell's law:

$$\cos\theta' = \sqrt{1 - (k_0 \sin\theta / k)^2}, \tag{72}$$

with k_0 the wave number in air and θ the angle of sound incidence.

The Figure 11 shows the normal surface impedance at normal sound incidence of a 5 cm thick foam layer, stuck on a hard floor. The low resistivity of the material is 5000 Ns/m^4. The characteristic impedance Z_c and the propagation constant k have been calculated using the Eqs. (51) to (54) and the normal surface impedance with Eq. (71). The dots are experimental results, obtained with the free field method described in chapter measuring methods, point 5. For comparison, the values of the reflection and absorption coefficient have been calculated using the Eqs. (47) and (48). The $\lambda/2$ anti resonance at 3000 Hz can clearly be noticed.

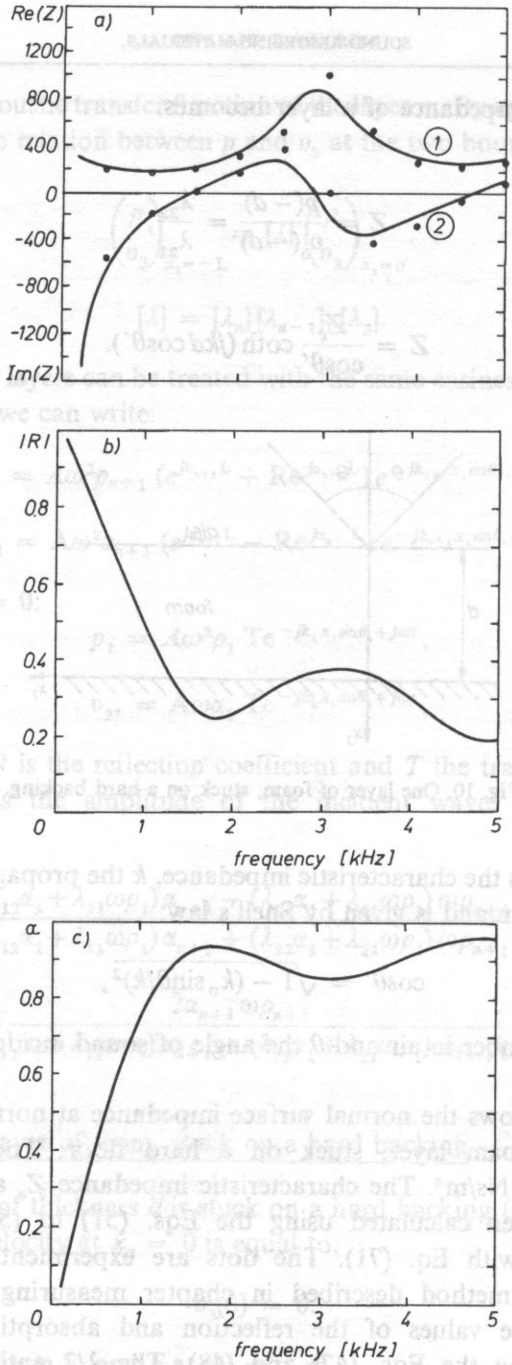


Fig. 11. a) Real (1) and imaginary (2) part of the normal surface impedance at normal sound incidence of a 5 cm thick reticulated foam, stuck on a hard backing, as a function of frequency. The foam flow resistivity is 5000 Ns/m⁴: ——— Eqs. (71) and (72) and (47) and (48). • experimental results from two-microphone technique. b) Magnitude of the reflection coefficient of the same foam layer. c) Absorption coefficient of the same foam layer.

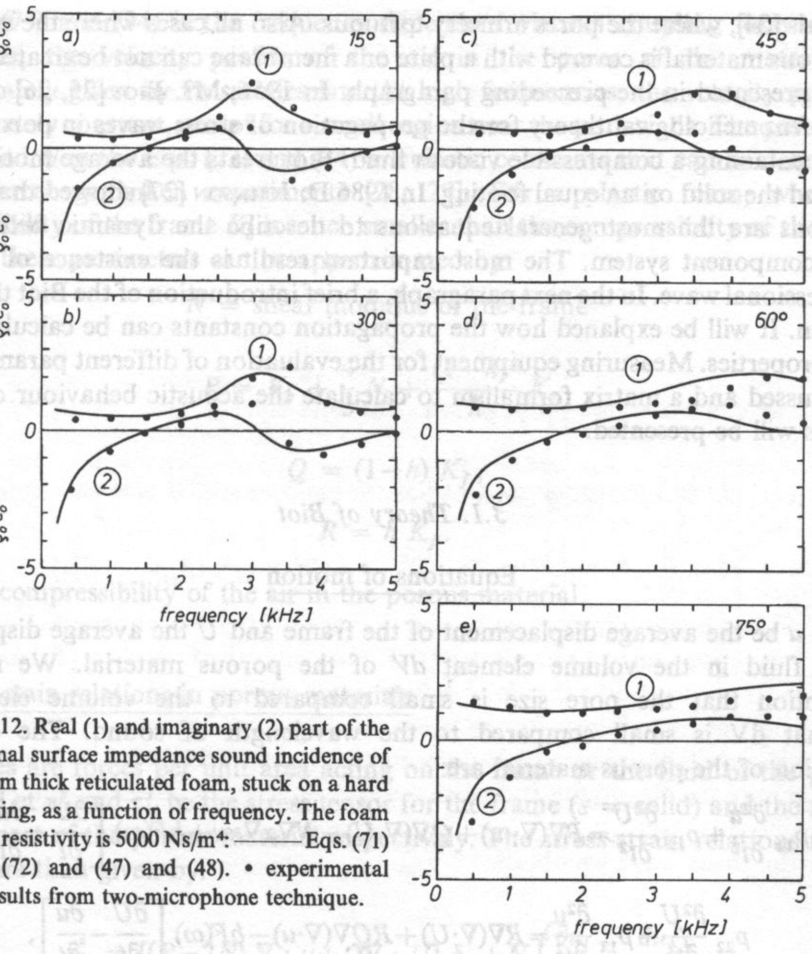


Fig. 12. Real (1) and imaginary (2) part of the normal surface impedance sound incidence of a 5 cm thick reticulated foam, stuck on a hard backing, as a function of frequency. The foam flow resistivity is 5000 Ns/m^4: —— Eqs. (71) and (72) and (47) and (48). • experimental results from two-microphone technique.

Figure 12 shows the normal surface impedance of the same foam layer for the angles of incidence $\theta = 15°, 30°, 45°, 60°$ and $75°$.

The above described method allows for the calculation of the normal surface impedance of any combination of layers of reticulated foam including air gaps. The method is very well suited for design purposes.

3. Acoustical properties of materials with medium and high flow resistivity

The model presented in the previous paragraph allows for the calculation of the acoustic behaviour of materials with low flow resistivity. In a lot of cases, these conditions are not satisfied, and an incident sound wave will generate a movement of the air in the pores as well as a movement of the frame. This may occur for instance in materials with medium and high flow resistivity [22, 23] or partially reticulated materials [24], where the pores are very tortuous. Also all cases where the surfaces of the porous material is covered with a plate or a membrane can not be treated with the theory presented in the preceeding paragraph. In 1956, Mr. Biot [25, 26] developed a semi-fenomenological theory for the propagation of stress waves in porous elastic solids containing a compressible viscous fluid. Biot treats the average motion of the fluid and the solid on an equal footing. In 1986 D. Johnson [27] showed that the Biot equations are the most general equations to describe the dynamic behaviour of a two-component system. The most important result is the existence of a second compressional wave. In the next paragraph, a brief introduction of the Biot theory will be given. It will be explaned how the propagation constants can be calculated from foam properties. Measuring equipment for the evaluation of different parameters will be discussed and a matrix formalism to calculate the acoustic behaviour of layered systems will be presented.

3.1. Theory of Biot

Equations of motion

Let u be the average displacement of the frame and U the average displacement of the fluid in the volume element dV of the porous material. We make the assumption that the pore size is small compared to the volume element dV and that dV is small compared to the wavelength of sound. The equations of motion of the porous material are:

$$\rho_{11}\frac{\partial^2 u}{\partial t^2}+\rho_{12}\frac{\partial^2 U}{\partial t^2}=P\nabla(\nabla\cdot u)+Q\nabla(\nabla\cdot U)-N\nabla x\nabla xu+bF(\omega)\left[\frac{\partial U}{\partial t}-\frac{\partial u}{\partial t}\right], \quad (73)$$

$$\rho_{22}\frac{\partial^2 U}{\partial t^2}+\rho_{12}\frac{\partial^2 u}{\partial t^2}=R\nabla(\nabla\cdot U)+RQ\nabla(\nabla\cdot u)-bF(\omega)\left[\frac{\partial U}{\partial t}-\frac{\partial u}{\partial t}\right], \quad (74)$$

In these equations ρ_{11}, ρ_{22} and ρ_{12} are determined as follows:

$$\rho_{11} = \rho_1 + \rho_a, \quad (75)$$

$$\rho_{22} = h\rho_f + \rho_a, \quad (76)$$

$$\rho_{12} = -\rho_a, \quad (77)$$

ρ_1 and ρ_f are the densities of the frame and air respectively and ρ_a is the mass coupling term, which can be related to the structure factor k_s and the porosity h by the equation:

$$\rho_a = h\rho_f(k_s - 1). \quad (78)$$

The mass coupling term allows for the force acting on one component of the porous material whenever the other component is accelerated. The coeficient $bF(\omega)$ in the Eqs. (73) and (74) is the frequency dependent viscous coupling term. At low frequencies, the velocity profile of the air in the pores of the material are approximately given by Poisseulles law. At high frequencies, the velocity profile is approximately constant, except for a small region near the pore walls. The parameters P, Q, R and N in the Eqs. (73) and (74) are elastic constants that can be determined with different gedanken experiments [28, 29]. For a plastic foam, where the compressibility of the frame K_b is much smaller than the compressiblity of the frame material, these parameters can be approximated by:

$$N = \text{shear modulus of the frame} \tag{79}$$

$$P = K_b + \frac{4}{3} N + \frac{(1-h)^2}{h} K_f, \tag{80}$$

$$Q = (1-h) K_f, \tag{81}$$

$$R = h K_f. \tag{82}$$

K_f is the compressibility of the air in the porous material.

Stress-stain relations in porous materials

Stresses are forces per unit area acting on the frame or the fluid of the porous material. Let τ^s_{ij} and τ^f_{ij} be the stress tensor for the frame (s = solid) and the fluid (f = fluid) part of the porous material respectively. The stress-strain relations for the material are than given by:

$$\tau^s_{ij} = [(P-2N)\ \nabla \cdot u + Q\nabla \cdot U]\ \delta_{ij} + N\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{83}$$

$$\tau^f_{ij} = -hp\delta_{ij} = [R\nabla \cdot U + Q\nabla \cdot u]\ \delta_{ij}. \tag{84}$$

In these equations we can clearly notice the influence of a third coupling mechanism between the movement of the air and frame as a result of the term Q.

Propagation constants in an elastic porous material

The average displacement vectors of the frame and gas can be written as a function of a scalar and a vector potential [30]:

$$u = \nabla\varphi + \nabla x H, \tag{85}$$

$$U = \nabla\psi + \nabla x G \tag{86}$$

Insertion of Eqs. (85) and (86) into (73) and (74) yields two sets of equations: one for longitudinal displacements and one for transverse displacements [31, 32]. For the longitudinal displacements, assumption of harmonic time dependence results in:

$$\varphi = \varphi_1 + \varphi_2, \tag{87}$$

$$\psi = \mu_1 \varphi_1 + \mu_2 \varphi_2, \tag{88}$$

with:

$$(\nabla^2 + k_1^2)\,\varphi_1 = 0, \tag{89}$$

$$(\nabla^2 + k_2^2)\,\varphi_2 = 0. \tag{90}$$

In these equations, the propagation constants are:

$$k_{1,2} = \frac{\omega^2}{2(PR-Q^2)}\left[(\tilde{\rho}_{11}R + \tilde{\rho}_{22}P - 2\tilde{\rho}_{12}Q) \pm \sqrt{\Delta}\right], \tag{91}$$

$$\Delta = (\tilde{\rho}_{11}R + \tilde{\rho}_{22}P - 2\tilde{\rho}_{12}Q)^2 - 4(PR-Q^2)(\tilde{\rho}_{11}\tilde{\rho}_{22} - \tilde{\rho}_{12}^2). \tag{92}$$

Hence, two longitudinal waves φ_1 and φ_2, with a different propagation constant k_1 and k_2, can propagate in a porous material. They are called the slow (P_1) and the fast (P_2) wave. It can be shown that for the slow wave, frame and air move approximately in phase opposition; whereas for the fast wave they move in phase. As a consequence, the slow wave will have much higher attenuation than the fast wave. It is important to note that both the P_1 and the P_2 wave have an amplitude in the solid and in the liquid phase.

For the shear wave we obtain:

$$G = \mu_3 H, \tag{93}$$

with:

$$\nabla^2 H + k_3^2 H = 0,$$

The propagation constant is equal to:

$$k_3^2 = \frac{\omega^2}{N}\,\frac{\tilde{\rho}_{11}\tilde{\rho}_{22} - \tilde{\rho}_{12}^2}{\tilde{\rho}_{22}}. \tag{94}$$

Determination of material parameters

The porosity

In the Biot equation, the porosity h is the amount of air in the porous material that can participate to the sound propagation. For open cell reticulated foams, the porosity can be easily determined if the density ρ of the frame material is known.

$$h = 1 - m/V\rho. \tag{95}$$

In this equation, V is the volume and m is the mass of the sample. For materials containing closed cells the situation is more complicated. Since the gas in the closed cells can not be accessed from the outside, it should be considered as a part of the frame. The major influence of the closed cells is that they modify the compressibility of the frame. The acoustic porosity can be measured with a device shown in Fig. 13.

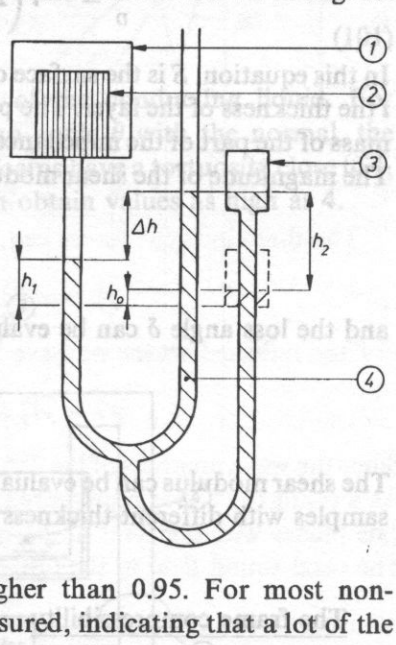


Fig. 13. Measuring set-up for the determination of the porosity h. *1* — sample volume, *2* — sample, *3* — controlling gas volume, *4* — manometer.

The porosity of reticulated foams is typically higher than 0.95. For most non-reticulated foams a porosity higher than 0.90 is measured, indicating that a lot of the cell membranes have been ruptured or pierced.

The shear modulus

Since losses due to the viscoelasticity of the frame have to be taken into account, the dynamic shear modulus should be used in the equation of Biot. The dynamic shear modulus can be several times higher than its static value. The shear modulus can be measured with a divice shown in Fig. 14. Two samples of equal thickness are glued

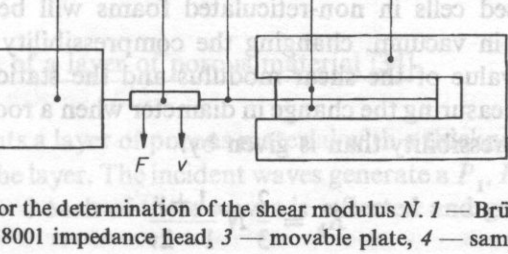


Fig. 14. Measuring set-up for the determination of the shear modulus N. *1* — Brüel and Kjaer 4810 shaker, *2* — Brüel and Kjaer 8001 impedance head, *3* — movable plate, *4* — sample, *5* — rigid plate.

between a solid wall and a thin movable plate. A shaker generates shear waves in the two layers. An impedance head measures the force exercised on the plate, together with the velocity of the plate. These two outputs can be fed to a two-channel FFT analyzer and the frequency response function is given by

$$\frac{F}{v} = -i\left(2S\sqrt{\rho N}\,\mathrm{cotg}\left(\sqrt{\frac{\rho}{N}}\,\omega l\right) - m\omega\right) \tag{96}$$

In this equation, S is the surface of the sample, ρ the density, N the shear modulus and l the thickness of the layer. The parameter m is the mass of the movable plate plus the mass of the part of the impedance head that participates in the movement of the plate. The magnitude of the shear modulus can be evaluated from the resonance frequency:

$$|N| = \frac{ml^2}{2S}\,\omega_{\mathrm{res}}^2 \tag{97}$$

and the loss angle δ can be evaluated from the width 2σ of the resonance peak

$$\delta = \frac{2\sigma}{\omega_{\mathrm{res}}} \tag{98}$$

The shear modulus can be evaluated as a function of frequency by measuring different samples with different thickness l.

The frame compressibility

The frame compressibility K_b can be evaluated from the shear modulus and the Young modulus E, which can be measured on a rod of porous material vibrating in longitudinally [33]

$$K_b = \frac{EN}{3(3N-E)} \tag{99}$$

The dynamic measurement of the Young modulus of the frame should be performed in vacuum, since the relative motion of the air and the frame influence the results considerably, due to the viscosity of the air. This can be safely done with reticulated foams, but the closed cells in non-reticulated foams will be destroyed when the material is brought in vacuum, changing the compressibility of the frame can be obtained from the value of the shear modulus and the static measurement of the Poisson ratio ν, by measuring the change in diameter when a rod of porous material is stretched. The compressibility than is given by:

$$K_b = \frac{2}{3}N\,\frac{1+\nu}{1-2\nu} \tag{100}$$

The tortuosity

The tortuosity k_s can be measured with a device if the frame material is an electrical insulator (Fig. 15). The air in the pores is replaced by an electrical conducting liquid, and the electrical resistivity r of the slab of the material is measured. The tortuosity is than given by:

$$k_s = h(r/r_f) \tag{101}$$

In this equation r_f is the electrical resistivity of the conducting liquid. For a material with parallel cylindrical pores making an angle θ with the normal, the tortuosity is equal to $k_s = 1/\cos^2\theta$. Most reticulated foams have a tortuosity close to 1, whereas the tortuosity of non-reticulated foams can obtain values as high as 4.

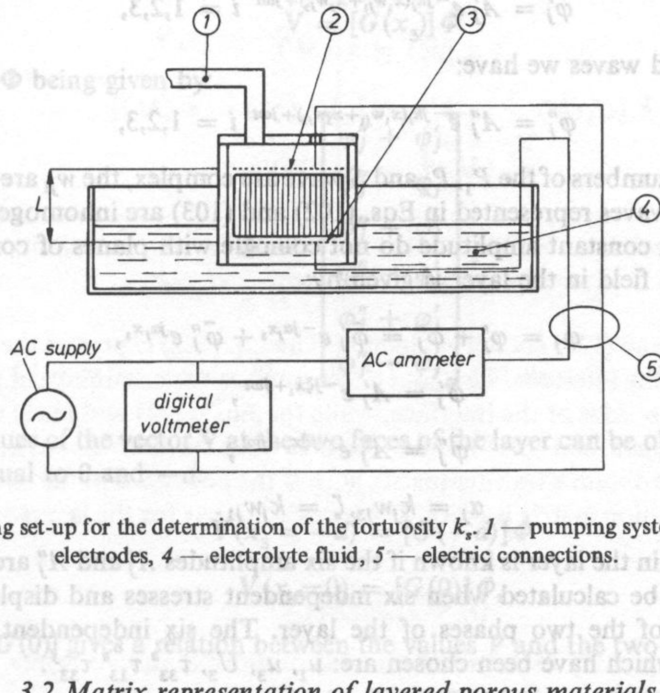


Fig. 15. Measuring set-up for the determination of the tortuosity k_s. *1* — pumping system, *2* and *3* — grid electrodes, *4* — electrolyte fluid, *5* — electric connections.

3.2 Matrix representation of layered porous materials

Matrix formalism of a layer of porous material [34]

Figure 16 represents a layer of porous material with a thickness d. A plane wave is incident from above the layer. The incident waves generate a P_1, P_2 and a S wave in the porous layer. At $x_3 = 0$ each of these waves is reflected and generates a P_1, P_2 and S wave.

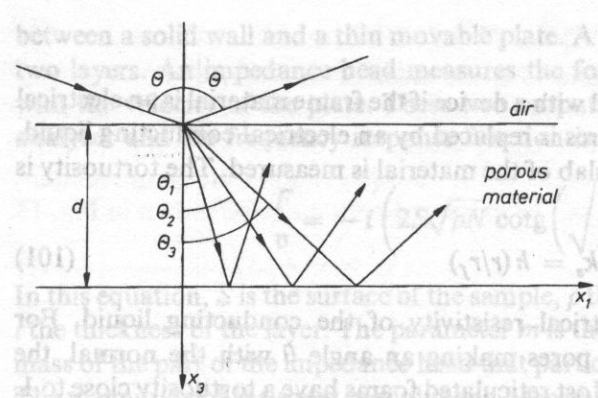


Fig. 16. A layer of foam with a thickness *d*, stuck on a hard backing.

The three incident waves can be represented with three displacement potentials:

$$\varphi_j' = A_j' \, e^{-jk_j(x_1 w_{j1} + x_3 w_{j3}) + j\omega t} \quad i = 1,2,3, \tag{102}$$

For the reflected waves we have:

$$\varphi_j'' = A_j'' \, e^{-jk_j(x_1 w_{j1} + x_3 w_{j3}) + j\omega t} \quad i = 1,2,3, \tag{103}$$

Since the wave numbers of the P_1, P_2 and S wave are complex, the w_{ji} are complex too. As a result the waves represented in Eqs. (102) and (103) are inhomogeneous waves. The planes with constant amplitude do not coincide with planes of constant phase. The total sound field in the layer is given by:

$$\varphi_j = \varphi_j' + \varphi_j'' = \bar{\varphi}_j' \, e^{-j\alpha_j x_3} + \bar{\varphi}_j'' \, e^{j\alpha_j x_3}, \tag{104}$$

$$\bar{\varphi}_j' = A_j' \, e^{-j\xi x_1 + j\omega t}, \tag{105}$$

$$\bar{\varphi}_j'' = A_j'' \, e^{-j\xi x_1 + j\omega t}, \tag{106}$$

$$\alpha_j = k_j w_{j3}, \zeta = k_j w_{j1}. \tag{107}$$

The sound field in the layer is known if the six amplitudes A_j' and A_j'' are known. The amplitudes can be calculated when six independent stresses and displacements are known at one of the two phases of the layer. The six independent stresses and displacements which have been chosen are: u_1, u_3, U_3, $\tau_{33}{}^s$ $\tau_{13}{}^s$ $\tau_{33}{}^f$.
Let **V** be the vector:

$$\mathbf{V} = [u_1, u_3, U_3, \tau_{33}^s, \tau_{13}^s, \tau_{33}^f]^T. \tag{108}$$

The displacement and the stress components can be written as a function of the potentials with the aid of the Eqs. (83) to (88).

$$u_1 = \frac{\partial \varphi_1}{\partial x_1} + \frac{\partial \varphi_2}{\partial x_1} - \frac{\partial \varphi_3}{\partial x_3}, \tag{109}$$

$$u_3 = \frac{\partial \varphi_1}{\partial x_3} + \frac{\partial \varphi_2}{\partial x_3} - \frac{\partial \varphi_3}{\partial x_1}, \tag{110}$$

$$U_3 = \mu_1 \frac{\partial \varphi_1}{\partial x_3} + \mu_2 \frac{\partial \varphi_2}{\partial x_3} - \mu_3 \frac{\partial \varphi_3}{\partial x_1}, \tag{111}$$

$$\tau^s_{33} = (P-2N)\,\nabla \cdot u + 2N \frac{\partial \mu_3}{\partial x_3}, \tag{112}$$

$$\tau^s_{12} = N\left(\frac{\partial \mu_3}{\partial x_1} + \frac{\partial \mu_1}{\partial x_3}\right), \tag{113}$$

$$\tau^f_{33} = R\nabla \cdot U + Q\nabla \cdot u. \tag{114}$$

These equations can be written in matrix form:

$$\mathbf{V} = [G(x_3)]\,\Phi \tag{115}$$

the vector $\mathbf{\Phi}$ being given by:

$$\mathbf{\Phi} = \begin{bmatrix} \varphi''_j + \varphi'_j \\ \varphi''_j - \varphi'_j \\ \varphi''_j + \varphi'_j \\ \varphi''_j - \varphi'_j \\ \varphi''_j + \varphi'_j \\ \varphi''_j - \varphi'_j \end{bmatrix}, \tag{116}$$

The values of the vector $\mathbf{V}$ at the two faces of the layer can be obtained by putting x_3 resp. equal to 0 and $-d$:

$$\mathbf{V}(x_3 = -d) = [G(-d)]\,\Phi \tag{117}$$

$$\mathbf{V}(x_3 = 0) = [G(0)]\,\Phi, \tag{118}$$

Inverting $[G(0)]$ gives a relation between the values V and the two faces of the layer:

$$V(x_3 = -d) = [G(-d)]\,[G(0)]^{-1}\,V(x_3 = 0). \tag{119}$$

The matrix $[\gamma] = [G(-d)]\,[G(0)]^{-1}$ is an acoustic transfer matrix for the layer.

Transfer matrix of a layered system

Figure 17 represents two layers of porous material in contact with each other. M and M' are two points in each layer, close to the interface R. In the general case,

Fig. 17a. Two layers of foam in contact with each other.

characterizing the interface R requires the introduction of a coefficient of permeability for the interface [35].

However, for high porosity foams a simple transition matrix for the interface can be constructed. The continuity at the interface are:

$$u_1(M) = u_1(M'), \tag{120}$$

$$u_3(M) = u_3(M') \tag{121}$$

$$h_a[U_3(M) - u_3(M)] = h_b[U_3(M') - u_3(M')], \tag{122}$$

$$\tau^f_{33}(M)/h_a = \tau^f_{33}(M')/h_b, \tag{123}$$

$$\tau^s_{13}(M) = \tau^s_{13}(M), \tag{124}$$

$$\tau^s_{33}(M) + \tau^f_{33}(M') = \tau^f_{33}(M') + \tau^f_{33}(M'). \tag{125}$$

The Eqs. (120) and (121) are self explaining. The Eq. (122) expresses the continuity of the fluid across the interface. The Eq. (123) expresses the continuity of pressure in the pores at the two sides of the interface, while the Eqs. (124) and (125) express the continuity of the shear and total normal force at both sides of the interface. From these equations, we obtain a transfer matrix $[\gamma_{ab}]$. If the porosities of the two materials are equal, the transition matrix is unity. The transfer matrix for the layered material is given by:

$$[\gamma] = [\gamma_a]\,[\gamma_{ab}]\,[\gamma_b] \tag{126}$$

This method can be easily extended in the case of more than two layers.

3.3 Applications

<u>One layer of foam, stuck on a hard backing</u>

Figure 10 represents one layer of foam, stuck on a hard backing. The edge conditions at $x_3 = 0$ are:

$$u_1 = u_3 = U_3 = 0, \tag{127}$$

and at $x_3 = -d$:

$$\tau^s_{13} = 0, \tag{128}$$

$$\tau^s_{33} = -(1-h)\,p \tag{129}$$

$$\tau^f_{33} = -hp, \tag{130}$$

$$\omega\,[(1-h)\,u_3 - hU_3] = \frac{p}{Z} \tag{131}$$

p is the pressure in the air above the layer, close to the surface. The vector Eq. (108) can be written as:

$$\mathbf{V}(x_3 = -d) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tau^s_{33} \\ \tau^s_{13} \\ \tau^f_{33} \end{bmatrix}_{x_3=-d} . \tag{132}$$

These equations can be solved for the normal surface impedance Z:

$$Z = (\Delta_5\Delta_8 - \Delta_6\Delta_7)\times$$
$$\times\,[(1-h)^2\,(\Delta_2\Delta_7 - \Delta_1\Delta_8)\,h(1-h)\,(\Delta_6\Delta_1 - \Delta_2\Delta_5 - \Delta_8\Delta_3 + \Delta_4\Delta_7) + h^2(\Delta_3\Delta_6\Delta_4\Delta_5)]^{-1}, \tag{133}$$

$$\Delta_1 = \gamma_{24}\gamma_{55} - \gamma_{25}\gamma_{54},$$

$$\Delta_2 = \gamma_{26}\gamma_{55} - \gamma_{25}\gamma_{56},$$

$$\Delta_3 = \gamma_{34}\gamma_{55} - \gamma_{35}\gamma_{54},$$

$$\Delta_4 = \gamma_{36}\gamma_{55} - \gamma_{35}\gamma_{56},$$

$$\Delta_5 = \gamma_{44}\gamma_{55} - \gamma_{45}\gamma_{54},$$

$$\Delta_6 = \gamma_{46}\gamma_{55} - \gamma_{45}\gamma_{56},$$

$$\Delta_7 = \gamma_{64}\gamma_{55} - \gamma_{65}\gamma_{54},$$

$$\Delta_8 = \gamma_{66}\gamma_{55} - \gamma_{65}\gamma_{56}.$$

Figure 17b shows the normal surface impedance of a 2 cm thick partially reticulated foam layer at different angles of sound incidence. The parameters of the layer are given in the figure caption. It must be emphasized that, since the coupling between the

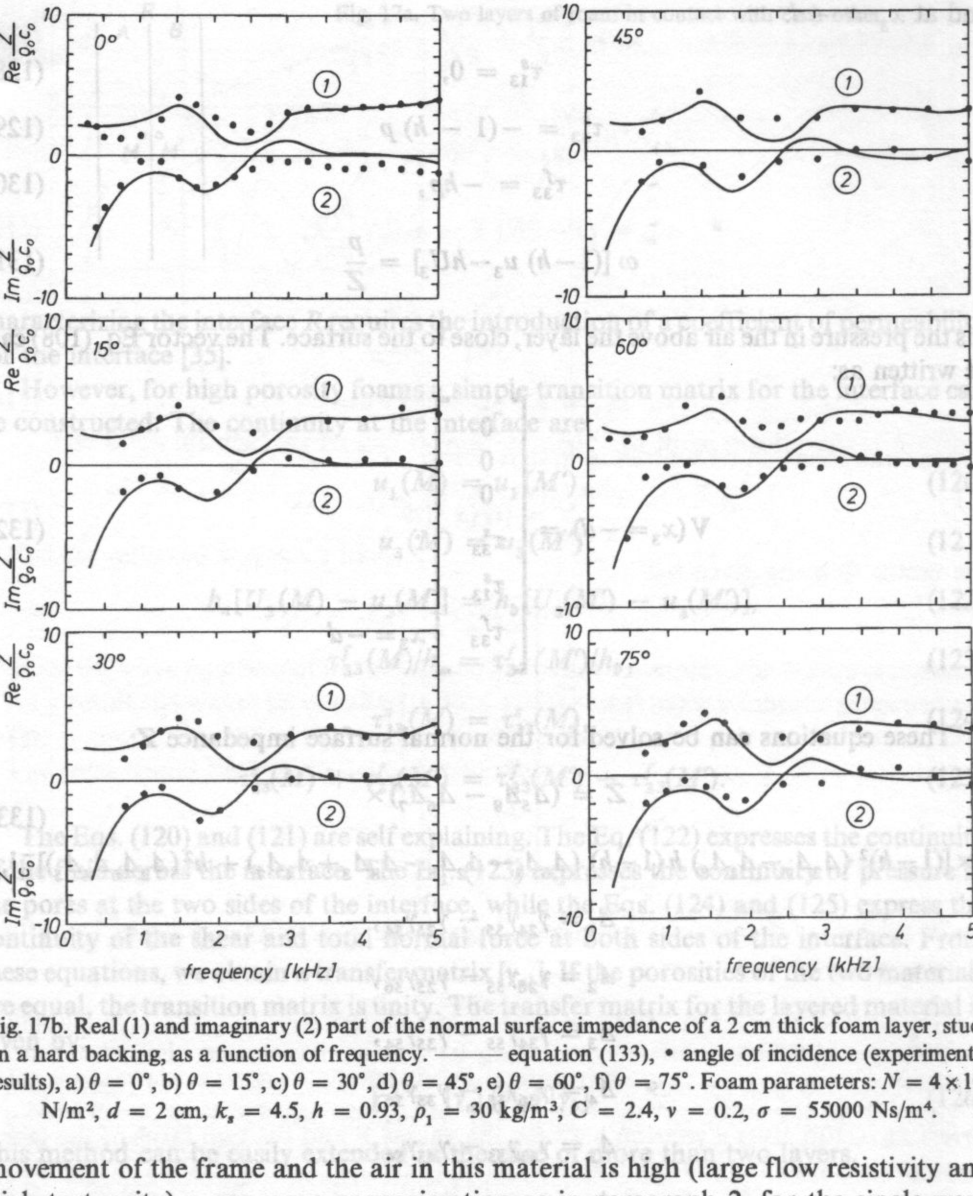


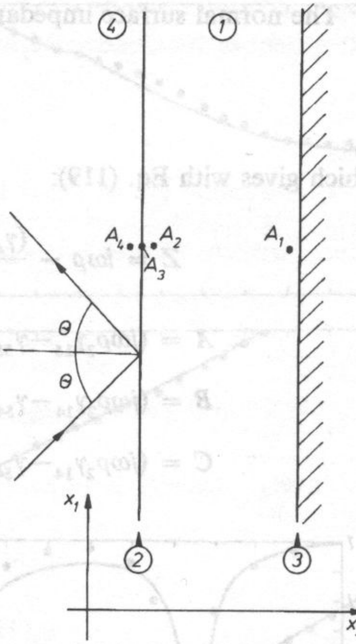
Fig. 17b. Real (1) and imaginary (2) part of the normal surface impedance of a 2 cm thick foam layer, stuck on a hard backing, as a function of frequency. ——— equation (133), • angle of incidence (experimental results), a) $\theta = 0°$, b) $\theta = 15°$, c) $\theta = 30°$, d) $\theta = 45°$, e) $\theta = 60°$, f) $\theta = 75°$. Foam parameters: $N = 4 \times 10^5$ N/m², $d = 2$ cm, $k_s = 4.5$, $h = 0.93$, $\rho_1 = 30$ kg/m³, $C = 2.4$, $\nu = 0.2$, $\sigma = 55000$ Ns/m⁴.

movement of the frame and the air in this material is high (large flow resistivity and high tortuosity) a one wave approximation as in paragraph 2, for the single wave propagation, is not valid.

A layer of porous material, covered with an impervious screen [36]

In a lot of applications, the foam has been covered with an impervious screen, in order to protect it from chemical agents or heat. The screen can also be applied to

Fig. 18. A layer of foam with an impervious screen. 1) layer of foam, 2) impervious screen, 3) hard backing, 4) air.



increase the acoustic absorption in the low frequency region. Figure 18 represents a foam layer, covered with an impervious membrane. A_3 is a point of the membrane. A_4 and A_2 are points in the air above the layer and in the porous layer, close to the membrane. The equations of motion of the membrane are:

$$-\omega^2\rho_1 u_3(A_3) = \tau^s_{33}(A_2) + \tau^f_{33}(A_2) - \tau^s_{33}(A_4), \tag{134}$$

$$-\omega^2\rho_2(A_3) = \tau^s_{13}(A^2), \tag{135}$$

with:

$$\rho_1 = \rho - Tk^2\sin\theta/\omega^2 \tag{136}$$

$$\rho_2 = \rho - Sk^2\sin\theta/\omega^2 \tag{137}$$

T is the tension in the membrane and S is the stiffness of the membrane. The membrane causes the following edge conditions:

$$u_3(A_2) = U_3(A_2) = u_3(A_3) = U_3(A_4) \tag{138}$$

$$u_1(A_2) = u_1(A_3) \tag{139}$$

At A_1 we have

$$u_1(A_1) = u_3(A_1) = U_3(A_1) = 0 \tag{140}$$

The normal surface impedance is given by:

$$Z = \frac{-\tau^f_{33}(A_4)}{j\omega\, U_3(A_4)} \tag{141}$$

which gives with Eq. (119):

$$Z = i\omega\rho - \frac{(\gamma_{44}+\gamma_{64})A-(\gamma_{45}+\gamma_{65})B+(\gamma_{46}+\gamma_{66})C}{(\gamma_{24}A-\gamma_{25}B+\gamma_{26}C)} \tag{142}$$

$$A = (j\omega\rho_2\gamma_{15}-\gamma_{55})(\gamma_{26}-\gamma_{36}) - (j\omega\rho_2\gamma_{16}-\gamma_{56})(\gamma_{25}-\gamma_{35})$$

$$B = (j\omega\rho_2\gamma_{14}-\gamma_{54})(\gamma_{26}-\gamma_{36}) - (j\omega\rho_2\gamma_{16}-\gamma_{56})(\gamma_{24}-\gamma_{34})$$

$$C = (j\omega\rho_2\gamma_{14}-\gamma_{54})(\gamma_{25}-\gamma_{35}) - (j\omega\rho_2\gamma_{15}-\gamma_{55})(\gamma_{24}-\gamma_{34})$$

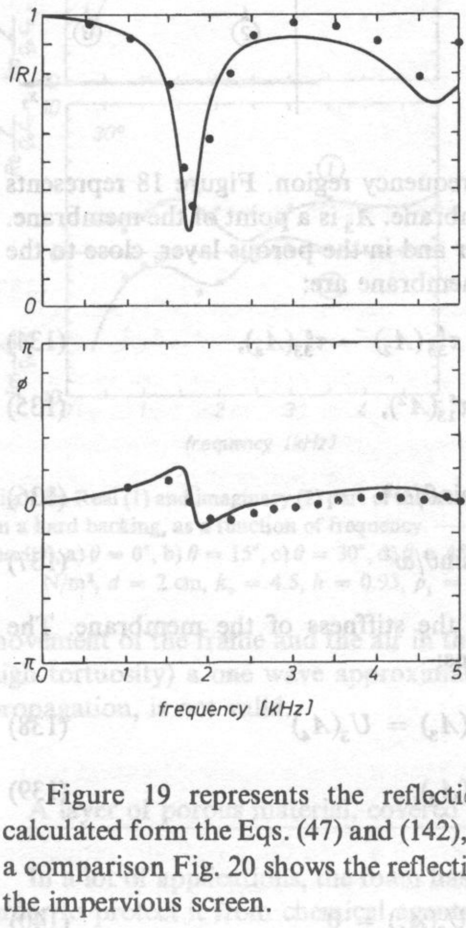


Fig. 19. Magnitude and phase of the reflection coefficient of a plastic foam, covered with an impervious screen. Material parameters: $T = S = 0$, $\rho = 0.02$ kg/m³, thickness = 25×10^{-6} m. —— equations 47 and 142, • experimental results.

Figure 19 represents the reflection coefficient at normal sound incidence, calculated form the Eqs. (47) and (142), of a foam layer with an impervious screen. As a comparison Fig. 20 shows the reflection coefficient of the same foam layer without the impervious screen.

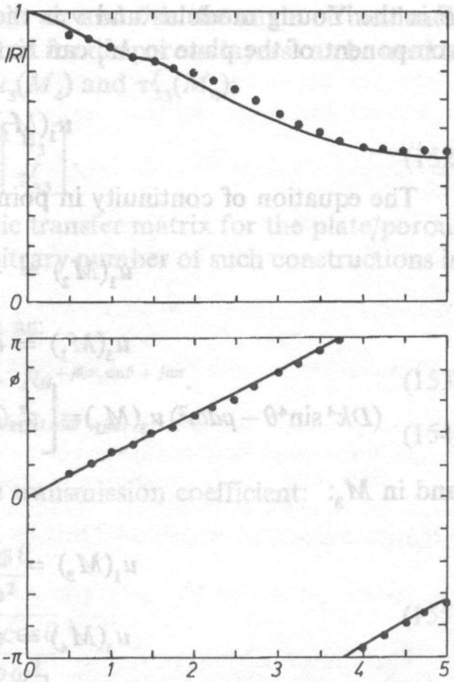


Fig. 20. Magnitude and phase of the reflection coefficient of the foam layer as in Fig. 19, but without the screen.

The acoustic transmission through layered systems

The acoustic insulation properties of foam layers, sandwiched between elastic solid plates has a wide range of application, including buildings, cars, airplanes and industrial plants. A reliable theoretical model to predict the acoustic insulation of the partitions can save a lot of development time. We can use the matrix formalism, developed in the preceding paragraph, to calculate the sound transmission through such layered systems. Figure 21 represents a plate/porous layer/plate system. Let u be the displacement of the plate and let M_1 and M_4 be two points at the left side, respectively the right side of the plates. The equation of motion of the plate, glued on a layer is given by [37]:

$$D\,\frac{\partial^4 u_3}{\partial x_1^4} + \rho d\,\frac{\partial^2 u_3}{\partial t^2} = [\tau^s_{33}(M_2)+\tau^f_{33}(M_2)-\tau^f_{33}(M_1)] + \frac{d}{2}\left[\frac{\partial \tau^s_{13}}{\partial x_1}(M_2)\right]. \quad (143)$$

In this equation, ρ is the density and d the thickness of the plate, D, the bending stiffness of the plate is given by:

$$D = \frac{Ed^2}{12\,(1-\nu^2)}. \quad (144)$$

E is the Young modulus and ν is the Poisson ratio of the plate material. The x_1 component of the plate in M_1 can be written as:

$$u_1(M_1) = -\frac{d}{2}\frac{\partial u_3}{\partial x_1}. \tag{145}$$

The equation of continuity in point M_1 are:

$$u_1(M_2) = -j\frac{d}{2}k\sin\theta\, u_3(M_1), \tag{146}$$

$$u_3(M_1) = u_3(M_2) = U_3(M_2), \tag{147}$$

$$(Dk^4\sin^4\theta - \rho d\omega^2)\, u_3(M_1) = \left[\tau^s_{33}(M_2) - \tau^f_{33}(M_2) - \tau^f_{33}(M_1) + \frac{d}{2}\frac{\partial\tau_{13}}{\partial x_1}(M_1)\right], \tag{148}$$

and in M_3:

$$u_1(M_3) = +j\frac{d'}{2}k\sin\theta\, u_3(M_4), \tag{149}$$

$$u_3(M_4) = u_3(M_3) = U_3(M_3), \tag{150}$$

$$(D'k^4\sin^4\theta - \rho' d'\omega^2)\, u_3(M_4) = \left[\tau^f_{33}(M_4) + \tau^f_{33}(M_3) - \tau^s_{33}(M_3) + \frac{d'}{2}\frac{\partial\tau_{13}}{\partial x_1}(M_3)\right]. \tag{151}$$

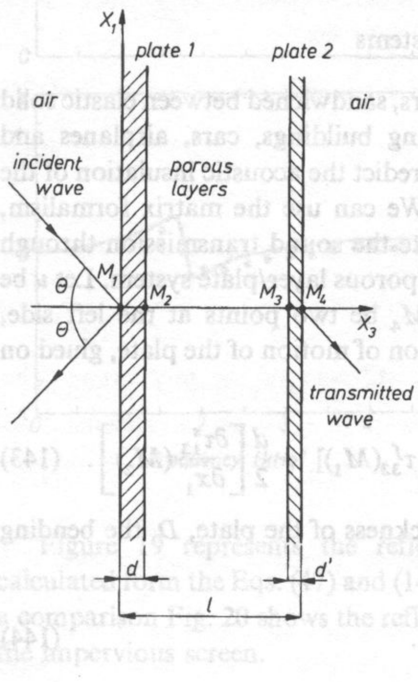


Fig. 21. A porous layer placed between two plates.

In these equations, k is the wave number in air and θ is the angle of incidence. These equations, together with the transfer matrix for the porous layers can be used to express $u_3(M_1)$ and $\tau^f_{33}(M_3)$ as a function of $u_3(M_4)$ and $\tau^f_{33}(M_4)$:

$$\begin{bmatrix} \mathbf{u}_3 \\ \tau^f_{33} \end{bmatrix}_{M_1} = [\boldsymbol{\beta}] \begin{bmatrix} \mathbf{u}_3 \\ \tau^f_{33} \end{bmatrix} . \tag{152}$$

The matrix [β] can be considered as an acoustic transfer matrix for the plate/porous material/plate system. The extension to an arbitrary number of such constructions in series is straightforward.

The sound field in M_1 and M_4 can be written as:

$$\varphi(M_1) = e^{+jkx_1\sin\theta + j\omega t} + Re^{+jkx_1\sin\theta + j\omega t} . \tag{153}$$

$$\varphi(M_4) = Te^{+jkx_1\sin\theta - jkl\cos\theta + j\omega t} . \tag{154}$$

With the Eqs. (40) and (41) we obtain for the transmission coefficient:

$$T = \frac{2j\dfrac{k\cos\theta}{\rho\omega^2}}{X + Y\dfrac{jk\cos\theta}{\rho\omega^2}} . \tag{155}$$

$$X = \left(\beta_{11}\frac{jk\cos\theta}{\rho\omega^2} - \beta_{12}\right)e^{-jkl\cos\theta} . \tag{156}$$

$$Y = \left(\beta_{21}\frac{jk\cos\theta}{\rho\omega^2} - \beta_{22}\right)e^{-jkl\cos\theta} . \tag{157}$$

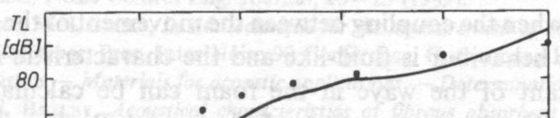


Fig. 22. Transmission loss factor of a steel plate/foam/steel plate system. Steel data: $d = 0.001$ m, $D = 16.3$ $(1+0.01j)$ Nm. Foam parameter: $d = 0.05$ m.

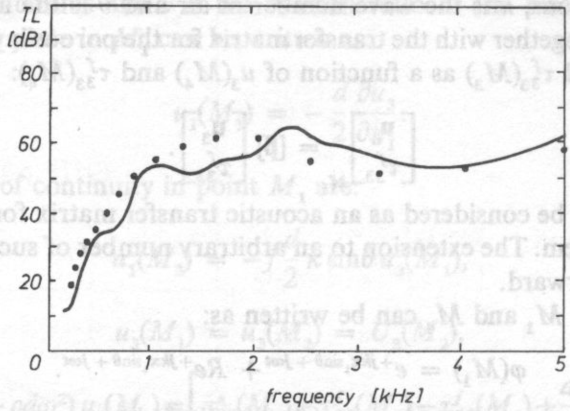


Fig. 23. Transmission loss factor of a plywood/foam/plywood system. Plywood data: $d = 0.085$ m, $D = 390$ $(1+0.04j)$ Nm. Foam parameter: $d = 0.05$ m.

The transmission loss factor TL is equal to (38):

$$TL = -10\log\left(2\int_{0}^{\pi/2} |T|^2 \sin\theta \cos\theta \, d\theta\right). \tag{158}$$

The Figure 22 shows the transmission loss factor of a 5 cm thick foam layer, sandwiched between two steel plates. Figure 23 shows the transmission loss of a parition made of a plywood panel, a 5 cm thick polyurethane foam layer and a second plywood panel. The plate and foam parameters are given in the figure caption.

Conclusions

The acoustic characteristics (sound absorption and transmission) of absorbing materials can be calculated from independent measurable parameters. In the case of reticulated foams, when the coupling between the movement of the frame and the air is weak, the acoustic behaviour is fluid-like and the characteristic impedance and the propagation constant of the wave in the foam can be calculated from the flow resistivity and empirical equation from Delany and Bazley. The reflection and transmission coefficient of layers of reticulated foam can be calculated easily with a matrix formalism.

In the case of non-reticulated foams or when the foam is covered with a membrane, plate of facing with a high flow resistivity, the acoustic behaviour is more complicated. Two compressional and one shear wave can propagate in these materials. The propagation constants and the relative amplitudes of these waves can be calculated from the porosity, the complex shear modulus, the Poisson ratio, the tortuosity and the flow resistivity using the Biot theory. Frequency dependent viscous and thermal effects have to be taken into account. A layer of material can be characterized acoustically with a 6×6 matrix. The matrix formalism allows for a straightforward calculation of the reflection and transmission coefficient of a variety of layered systems.

References

[1] L.L. BERANEK, *Acoustic impedance of porous materials*, J. Acoust. Soc. Am., **13**, p. 248–260 (1942).

[2] L.L. BERANEK, *Acoustical properties of homogeneous, isotropic rigid tiles and flexible blankets*, J. Acoust. Soc. Am., **19**, 4, 556–568 (1947).

[3] P.M. MORSE, R.H. BOLT, R.L. BROWN, *Acoustical impedance and sound absorption*, J. Acoust. Soc. Am., **12**, 2 (1940)

[4] C. ZWIKKER, C.W. KOSTEN, *Sound absorbing materials*, Elsevier Publ. Comp. New York 1949.

[5] K. ATTENBOROUGH, *Acoustical characteristics of porous materials*, Physics Reports, **82**, 3, 179–227 (1982)

[6] *International Standard ISO 354, Measurement of sound absorption in a reverberant room, incl. annex D, Test specimen mountings for sound absorbing tests*, 1985, 1991 (ISO/TC 43/SC 2/WG 7).

[7] *International Standard Document ISO/CD 11654 — Acoustics — Rating of sound absorption*, 1992 (ISO/TC 43/SC 2/WG 20).

[8] *International Standard Document ISO/CD 10534 — Determination of sound absorption coefficient and impedance or admittance by the impedance tube method* 1991.

[9] *Brüel and Kjaer Document, Two microphone impedance measurement tube — type* 4206, 1991.

[10] A. COPS, W. LAURIKS, *Application of new two-microphone techniques to measure the sound absorption characteristics of acoustic materials*, Proc. Second Int. Congress on Acoustic Intensity, p. 511–518, 1985.

[11] J.F. ALLARD, B. SIEBEN, *Measurement of acoustic impedance in a free field with two microphones and a spectrum analyzer*, J. Acoust. Soc. Am., **77**, 1617–1618 (1985).

[12] M. MINTEN, A. COPS, W. LAURIKS, *Absorption characteristics of an acoustic material at oblique incidence measured with the two-microphone technique*, J. Sound and Vibration, **120**, 3, 499–510 (1988).

[13] J.Y. CHUNG, D.A. BLASER, *Transfer function method of measuring induct acoustic properties*, I. *Theory*, J. Acoust. Soc., **68**, 3, 907–913 (1980).

[14] J.Y. CHUNG, D.A. BLASER, *Transfer function of measuring induct acoustic properties*, II. *Experiment*, J. Acoust. Soc. Am., **68**, 3, 914–921 (1980).

[15] W.T. CHU, *Transfer function technique for impedance and absorption measurements in an impedance tube using a single microphone*, J. Acoust. Soc. Am., **80**, 2, 555–560 (1992).

[16] J.F. ALLARD, Y. CHAMPOUX, *In situ two microphone technique for measurement of the acoustic surface impedance of materials*, Noise Control Eng. Journal, 15–23 (1989).

[17] J.F. ALLARD, A. COPS, W. LAURIKS, *In situ technique for the measurement of the surface impedance at normal and oblique incidence*, Proc. Inter-Noise 90 Conference, Gothenburg 1990.

[18] *ISO/DIS 9053 Acoustics — Materials for acoustic applications — Determination of airflow resistance.*

[19] M.E. DELANY, E.N. BAZLEY, *Acoustical characteristics of fibrous absorbent materials*, NPL Aero Report Ac37 1969.

[20] L.P. DUNN, W.A. DAVERN, *Calculation of acoustic impedance of multi-layered absorbers*, Applied Acoustics, **19**, 321–334 1986.

[21] F.P. MECHEL, *Absorption cross section of absorber cilinders*, J. Sound. Vib. **107**, 1, 131–148 (1986).

[22] J.F. ALLARD, A. AKNINE, C. DEPOLLIER, *Acoustic properties of partially reticulated foams with high and medium flow resistivity*, J. Acoust. Soc. Am., **79**, 6, 667–670 (1986).

[23] J.F. ALLARD, C. DEPOLLIER, A. L'ESPERANCE, *Observation of the Biot slow wave in a plastic foam of high flow resistance at acoustical frequencies*, J. Applied Physics **59**, 10, 3367–3370 (1986).

[24] J.F. ALLARD, C. DEPOLLIER, W. LAURIKS, *Measurements and prediction of surface impedance at oblique incidence of a plastic foam of high flow resistivity*, J. Sound. Vib. **132**, 1, 51–60 (1989).

[25] M.A. BIOT, *Theory of propagation of elastic waves in a fluid saturated porous solid, I. Low frequency range*, J. Acoust. Soc. Am., **28**, 1, 168–178 (1956).

[26] M.A. BIOT, *Theory of propagation of elastic waves in a fluid saturated porous solid, II. High frequency range*, J. Acoust. Soc. Am., **28**, 1, 179–191 (1956).

[27] D.L. Johnson, *Recent developments in the acoustic properties of porous media,* Proc. Int. School of Physics Enrico Fermi, Course XCIII ed. D. Sette, North Holland Phys. Pub. 1986.

[28] M.A. Biot, D.G. Willis, *The elastic coefficients of the theory of consolidation,* J. Appl. Mech., 594–601 (1957).

[29] C. Depollier, J.F. Allard, W. Lauriks, *Biot theory and stress-strain relations in porous sound absorbing materials,* J. Acoust. Soc. Am., **84**, 6, 2277–2279 (1988).

[30] W.M. Ewing, W.S. Jardetzky, F. Press, *Elastic waves in layered media,* Mc Graw–Hill Book Co New York 1957.

[31] H. Deresiewicz, *The effect of boundaries on wave propagation in liquid filled porous solids. I. Reflection of plane waves at a free plane boundary non-dissipative case.* Bull. Seism. Soc. Am., **50**, 4, 599–607 (1960).

[32] H. Deresiewicz, J.T. Rice, *The effect of boundaries on wave propagation in liquid filled porous solids. III. Reflection of plane waves at a free plane boundary general case,* Bull. Seism. Soc. Am., **52**, 3, 595–625 (1962).

[33] T. Pritz, *Transfer function method for investigating the complex modulus of acoustic materials: rod-like specimen,* J. Sound. Vib., **81**, 3, 359–376 (1982).

[34] J.F. Allard, C. Depollier, P. Rebillard, W. Lauriks, A. Cops, *Inhomogeneous Biot waves in layered media,* J. Appl. Phys., **66**, 6, 2279–2284 (1989).

[35] H. Deresiewicz, R. Skalak, *On uniqueness of dynamic poroelasticity,* Bull. Seism. Soc. Am., **52**, 4, 783–788 (1963).

[36] W. Lauriks, A. Cops, J.F. Allard, C. Depollier, P. Rebillard, *Modelization at oblique incidence of layered porous materials with impervious screens,* J. Acoust. Soc. Am., **87**, 3, 1200–1206 (1990).

[37] L. Landau, E. Lifchitz, *Théorie d'élasticité,* Edition MIR 1967.

[38] A.D. Pierce, *Acoustics, an introduction to its physical principles and applications,* Mc Graw Hill Book Co., New York 1986.