ARCHIVES OF ACOUSTICS
18, 2, 263–279 (1993)

BAND-LIMITED NOISE INTERFERENCE IN LOUDSPEAKER SYSTEMS

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It is often argued that the interference of sound from distant loudspeakers is negligible in the case of band-limited noise. Simple theory and experiment in a reflection-free environment with a pair of loudspeakers radiating a third-octave noise proves that in the case of coherent supply SPL variations are considerable. The variations disappear when non-coherent signal is applied indicating that non-coherent supply of loudspeakers is necessary wherever approximation of diffuse sound field is required.

1. Introduction

Diffuse sound field in test room is recommended in acoustic testing of hearing protectors and in some audiometric tests [1], [2]. In an ideal case the sound field has to be isotropic and homogeneous. Adequate approximation of ideal conditions of directional and spatial SPL uniformity is expected in the test site. Several loudspeakers have to be placed around the object to meet the demands of the directional distribution of the incident sound. However, the spatial uniformity of the SPL is destroyed by interference effects if the loudspeakers are fed coherently. The interference vanishes when non-coherent supply of loudspeakers is applied at the expense of increased complexity and cost of test equipment.

It has been argued in discussion of practical implementation of such measuring stand [3] that the interference effect of coherent supply of loudspeakers with third-octave noise is negligible because of stochastic phase of the signal. Investigation of that spatial phenomenon in the sound field of band-limited, constant-percentage noise is the subject of the present report.

2. Theoretical model of interference

A pair of directional, point sound sources, propagating band-limited noise of uniform spectral density W_0 is considered. The problem concerns calculation of mean-square value p^2_{rms} of the total sound pressure. The autocorrelation function $R(\tau)$ of the noise within bandwidth Δf in the frequency range $f_g - f_d$ is given by [4]:

$$R(\tau) = \frac{W_0}{\pi\tau} * \sin[\pi(f_g - f_d) * \tau] * \cos[\pi(f_g + f_d) * \tau]; \quad (1)$$

Let: $f = \sqrt{(f_g * f_d)}$ — center frequency of noise band, $a = \Delta f/f$ — constant percentage, relative bandwidth of the noise, ($a_{terc} = 0.23$, $a_{oct} = 0.707$), then,

$$R(\tau) = W_0 * \Delta f * \frac{1}{a\pi f\tau} * \sin[a\pi f\tau] * \cos[2\pi * \sqrt{a^2/4+1} * f\tau]; \quad (2)$$

RMS value p_i of the sound pressure from one directional source at distance r_i equals:

$$p_i = p * \frac{\Gamma_i}{r_i}; \quad (3)$$

where: p — mean-square value of the sound pressure at 1 m on the main axis of each source, $p^2 = W_0 * \Delta f$, Γ_i — directivity function of the source,

$$\Gamma_i = p(\theta)/p(0).$$

Total sound pressure of the two distant sources at any observation point in the space takes the form:

$$p^2_{rms} = \overline{(p_1 + p_2)^2} = p^2 * \left[\frac{\Gamma_1^2}{r_1^2} + \frac{\Gamma_2^2}{r_2^2} + 2 * \frac{\Gamma_1 \Gamma_2}{r_1 r_2} * \rho(kd)\right]; \quad (4)$$

The quantity $\rho(kd)$ in Eq. (4) is the normalized inter-correlation function of the two considered noise pressures at the observation point.

$$\rho(kd) = \frac{R(kd)}{R(0)} = \frac{2}{akd} * \sin\left(\frac{1}{2}akd\right) * \cos\left(\sqrt{a^2/4+1} * kd\right); \quad (5)$$

k — wave number, $k = 2\pi/l$, l — wavelength, d — path difference at observation point,

$$d = r_2 - r_1 \leqslant D;$$

D — distance between the two sources.

The function $\rho(kd)$ is shown in Fig. 1 for third-octave and octave noise bands.

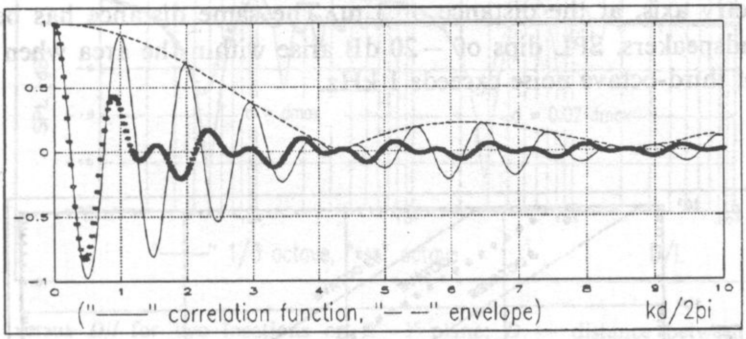


Fig. 1. Inter-correlation function of band limited noise from two sources: ——— third-octave noise, ***** octave noise.

The following may be observed from Eq. (4) and Fig. 1:

(i) Inter-correlation of band-limited noise from two coherent sources becomes negligible above some limiting value of d/l:
a) for third-octave noise, $d/l = 4$,
b) for octave noise, $d/l = 1.5$;
Corresponding SPL is approximately uniform and 3 dB higher than each component if d/l exceeds that limit.

(ii) Below the limiting value of d/l, i.e. for lower frequencies of noise, destructive interference arises with few dips and valleys of SPL. The largest SPL variations of 20 dB for third-octave noise (10 dB for octave noise) occur in the d/l range between 0.15 and 0.8.

(iii) SPL variations vanish again for $d/l < 0.15$. Here, SPL slowly falls down from the maximum to -1 dB. At maximum, which is located on symmetry axis of the loudspeaker system ($d = 0$), the 6 dB increase of SPL occurs.

The frequency dependence of interference effects is illustrated in Fig. 2 and Fig. 3.

The nature of spatial variations of SPL is similar to the frequency domain effects. In space the variable d changes from point to point in a regular manner as shown in Fig. 4. At fixed frequency SPL drops down from the maximum to the deepest minimum in close vicinity of main axis. Spatial fluctuations of SPL vanish beyond some angle corresponding to the condition (i) for d/l.

An example of SPL interference pattern on $X-Y$ plane for third-octave noise band centered at frequency $f = 1700/D$, $(D/l = 5)$, is shown in Fig. 5. As may be seen, the span of SPL variations increases proportionally with the distance from the origin of loudspeaker system along its main axis.

Another practical example is shown in Fig. 6. The diagram presents SPL variations over the small $0.3\ \text{m} \times 0.3\ \text{m}$ area, which may be considered as a test site for investigation of hearing protectors. The area is located in front of two loudspeakers on their symmetry axis, at the distance of 3 m. The same distance has been chosen between loudspeakers. SPL dips of -20 dB arise within the area when the center frequency of third-octave noise exceeds 1 kHz.

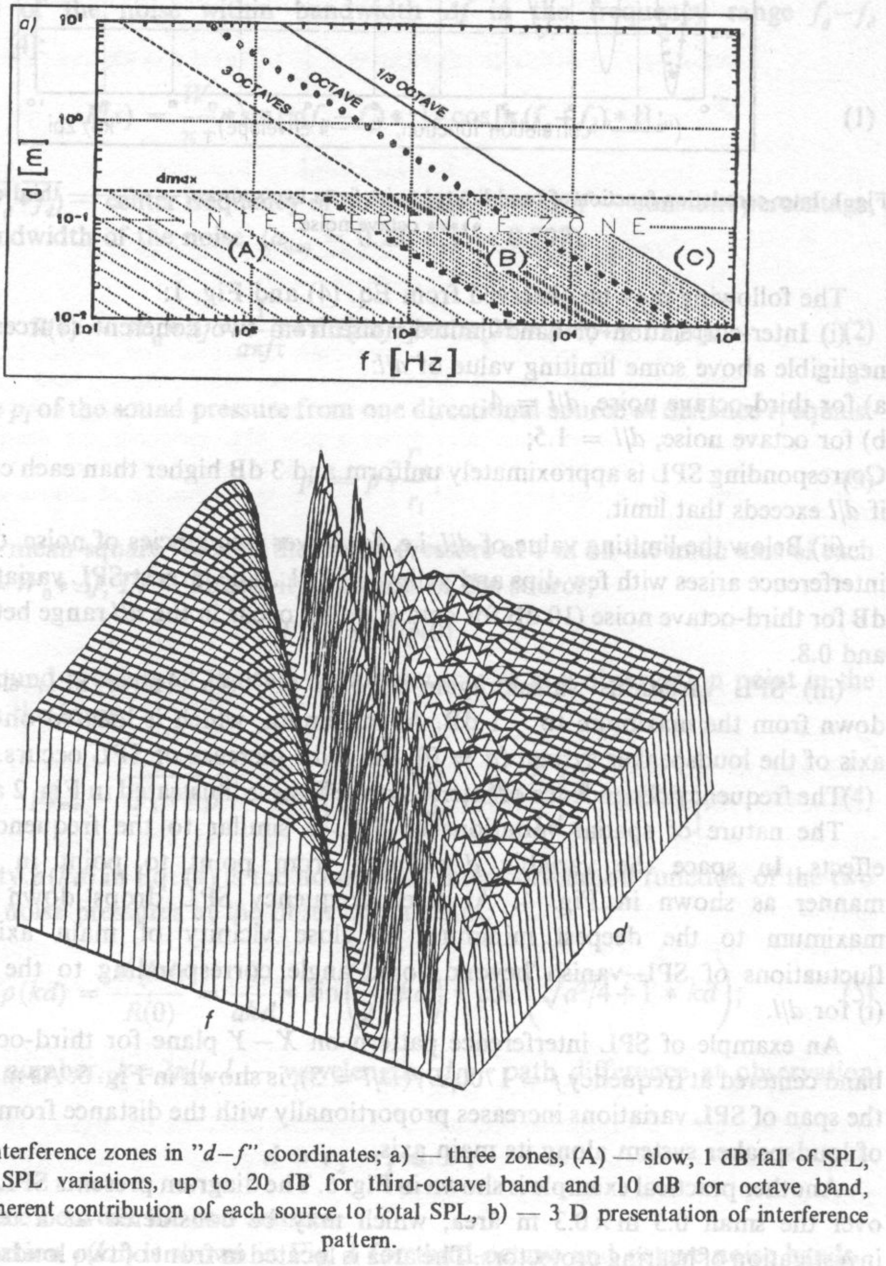


Fig. 2 a), b) Interference zones in "$d-f$" coordinates; a) — three zones, (A) — slow, 1 dB fall of SPL, (B) — rapid SPL variations, up to 20 dB for third-octave band and 10 dB for octave band, (C) — non-coherent contribution of each source to total SPL, b) — 3 D presentation of interference pattern.

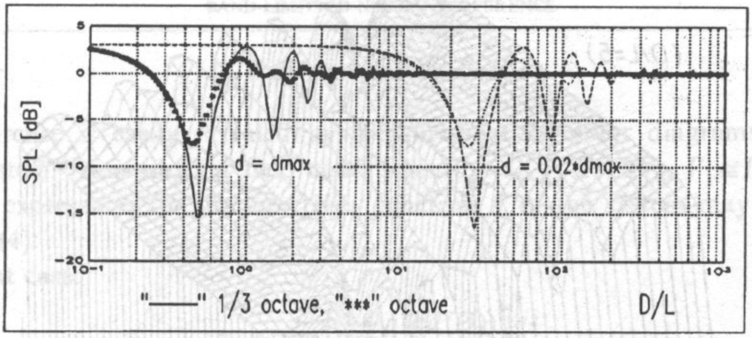


Fig. 3. SPL versus D/l for two locations on $X-Y$ plane; D — distance between loudspeakers, l — wavelength.

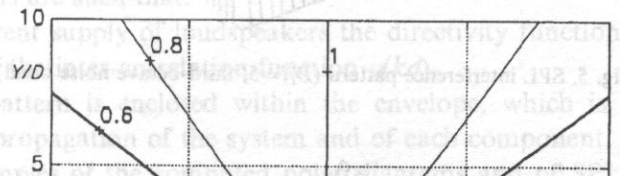


Fig. 4. Location of the path-difference d/D on $X-Y$ plane.

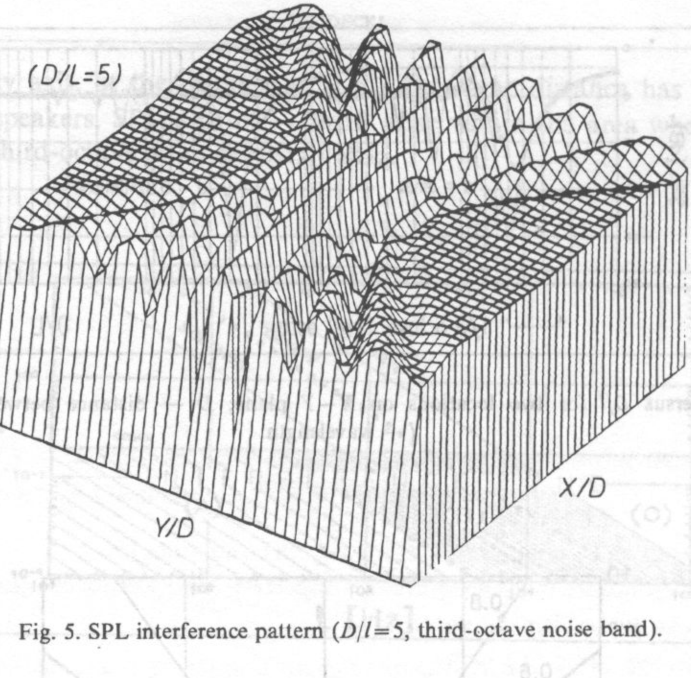


Fig. 5. SPL interference pattern ($D/l=5$, third-octave noise band).

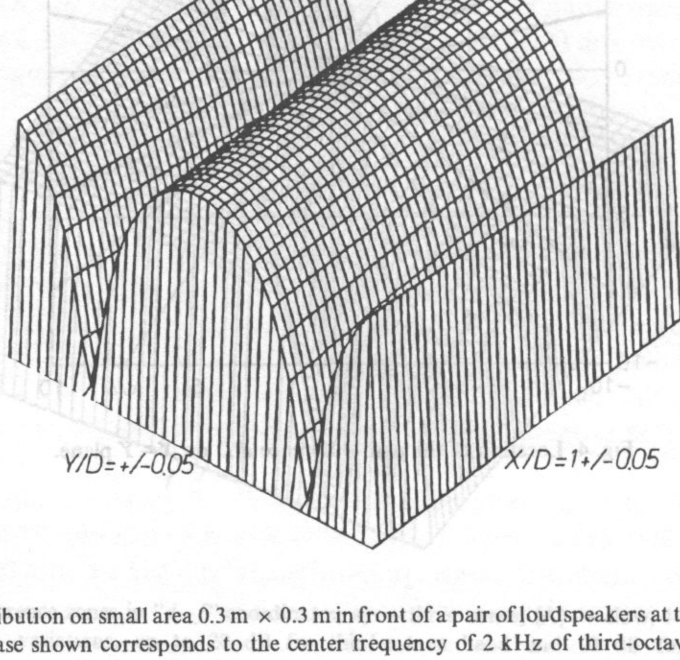


Fig. 6. SPL distribution on small area 0.3 m × 0.3 m in front of a pair of loudspeakers at the distance of 3 m. The case shown corresponds to the center frequency of 2 kHz of third-octave noise.

3. Experimental verification

Interference is easily revealed by measurement of polar diagrams of loudspeaker system. Assuming far field conditions ($r_0 \gg D$, $r_1 \cong r_2 \cong r_0$, $\Gamma_1 \cong \Gamma_2 \cong \Gamma_i$), the following expressions for the directivity function Γ of the system may be derived from Eq. (4):

a) coherent case:

$$\Gamma_c \cong \Gamma_i * \left[\frac{1+\rho(kd(\Theta))}{2}\right]^{1/2}; \tag{6}$$

b) non-coherent supply of each unit:

$$\Gamma_{nc} \cong \Gamma_i; \tag{7}$$

The conclusions are such that:

— in coherent supply of loudspeakers the directivity function Γ_c reflects initial fluctuations of the inter-correlation function $\rho(kd)$,

— polar pattern is enclosed within the envelope, which is the directivity of non-coherent propagation of the system and of each component.

A few examples of the computed polar diagrams and of SPL distributions are shown in the following figures. Two cases, $D/l=1$ and $D/l=5$ are presented in Fig. 7 and Fig. 8 respectively. For simplicity, omnidirectional propagation was assumed in the computations. The diagrams were computed from the equal SPL contours, what is equivalent to polar diagrams of constant radius. Linear scale is used for the presentation of the results. Convenience of such presentation is obvious if the contour shapes are to be fitted to a particular area.

The diagrams in Fig. 9 are presented in log scale, for $D/l=1$, 2, 3. Here the directivities of experimental units are combined in the computations of resultant directivity of the system.

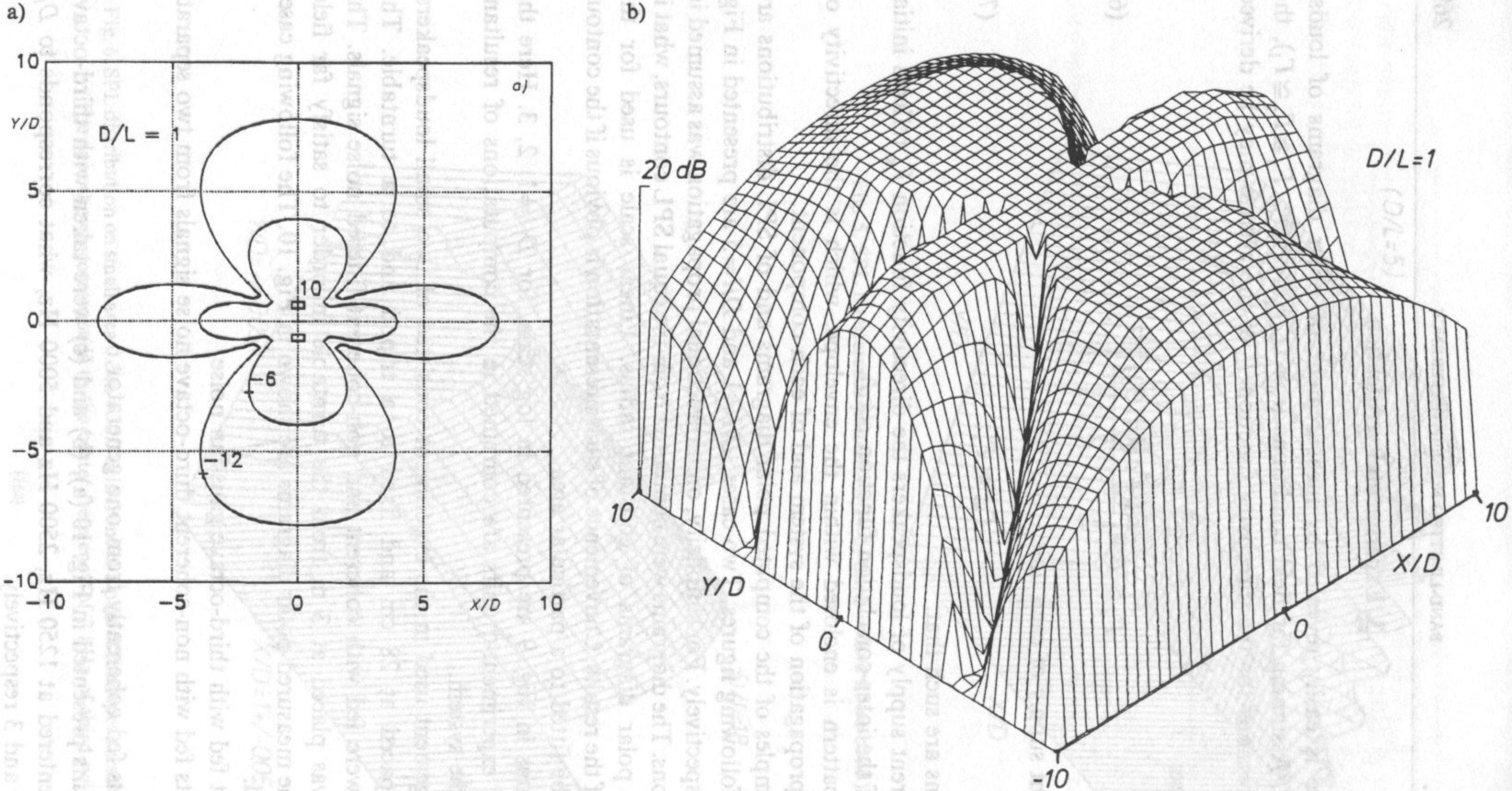
The arrangement used in the experiment consisted of two small loudspeakers, which were spaced at 28 cm and fixed to a tripod and to a turntable. The loudspeakers were fed with coherent and non-coherent filtered noise signals. The microphone was placed at 3 m from the turntable in order to satisfy far field conditions. The measured polar diagrams are shown in Fig. 10. The following cases were examined:

— one unit fed with third-octave gaussian noise,

— two units fed with non-coherent third-octave noise signals from two separate generators,

— two units fed coherently from one generator.

The diagrams presented in Fig. 10 (a), (b) and (c) were taken with third-octave noise bands centered at 1250 Hz, 2500 Hz and 5000 Hz, what corresponds to D/l factors of 1, 2 and 3 respectively.



a)
Y/D
D/L = 1
a)
10
−6
−12
X/D
b)
20 dB
D/L=1
Y/D
X/D
10
0
−10

c)

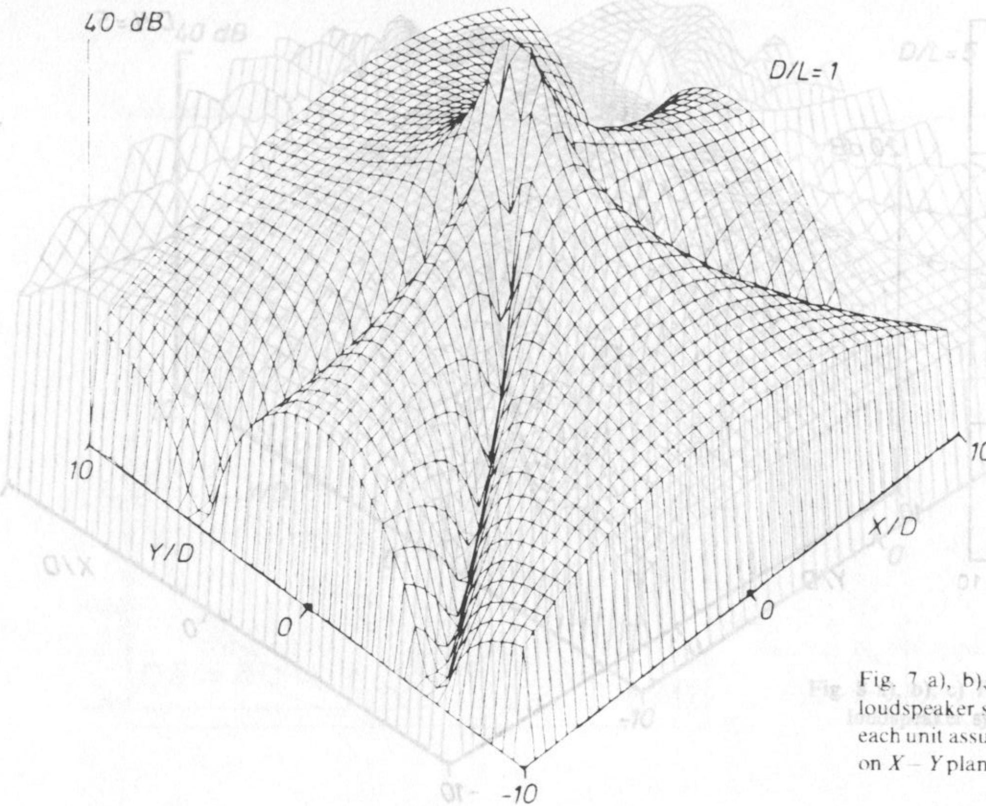
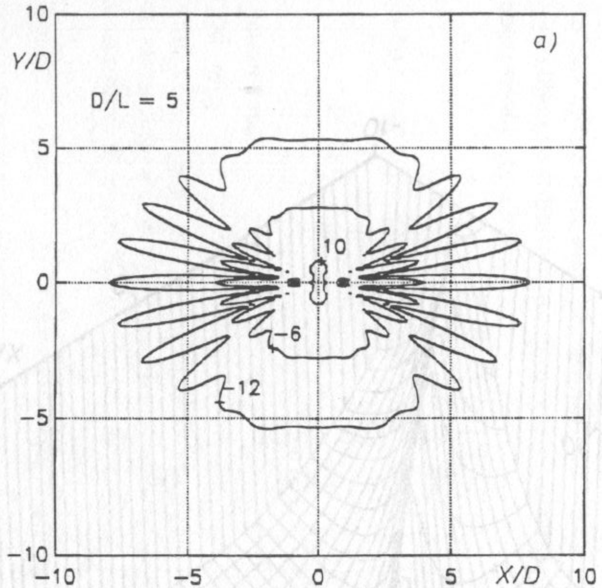
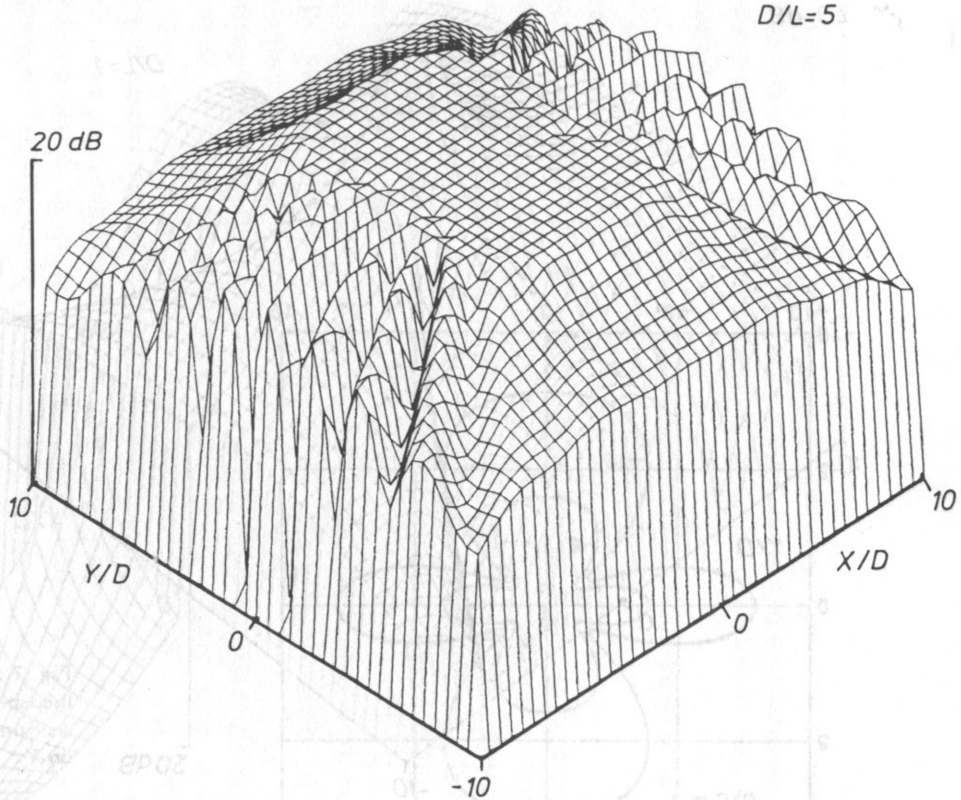


Fig. 7 a), b), c) Third-octave directivity and SPL distribution of loudspeaker system computed for $D/L = 1$. Spherical propagation of each unit assumed: a) directivity pattern from equal SPL contours on $X - Y$ plane, b) SPL distribution within 20 dB range, c) SPL distribution within 40 dB range.

a)



b)



c)

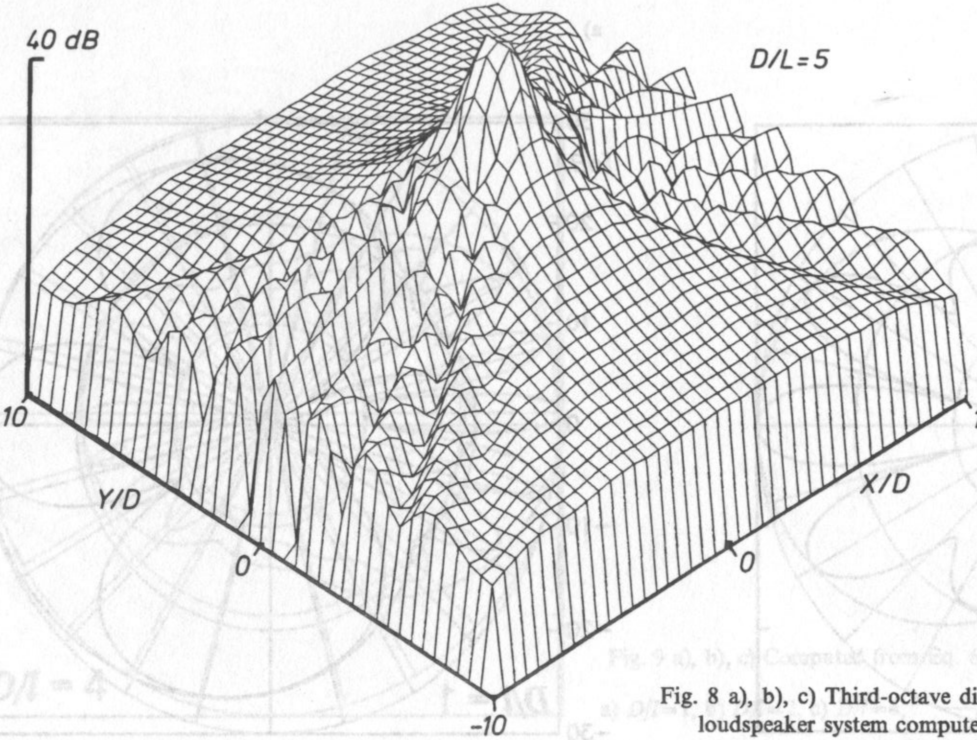
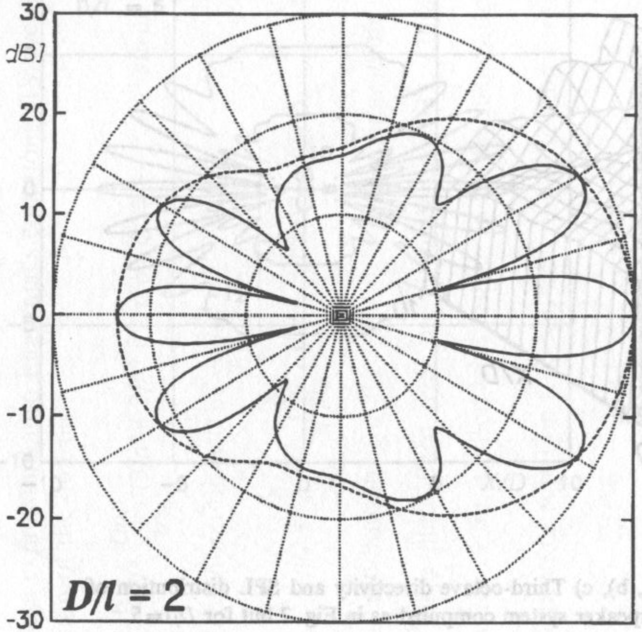
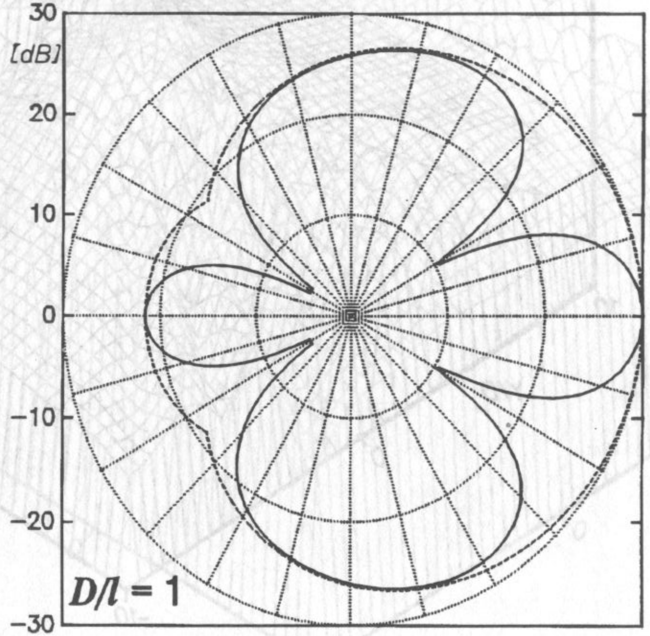


Fig. 8 a), b), c) Third-octave directivity and SPL distribution of loudspeaker system computed as in Fig. 7 but for $D/l=5$.

b)



a)



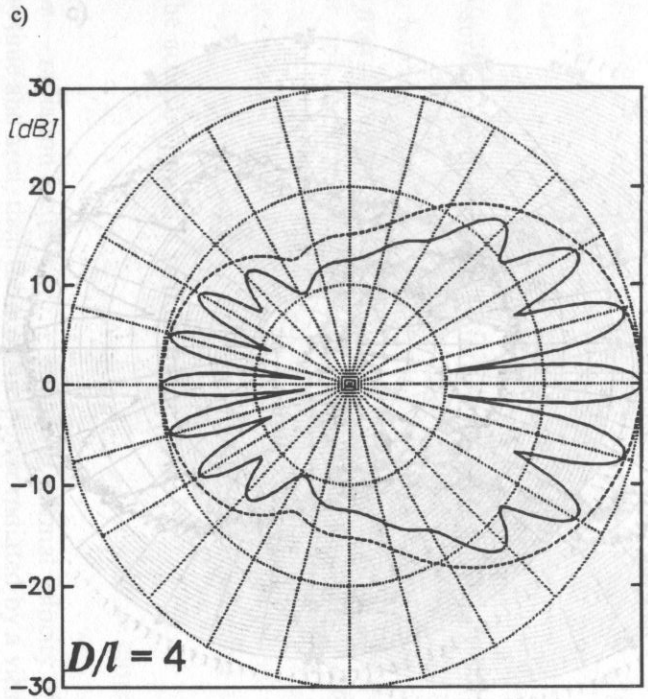
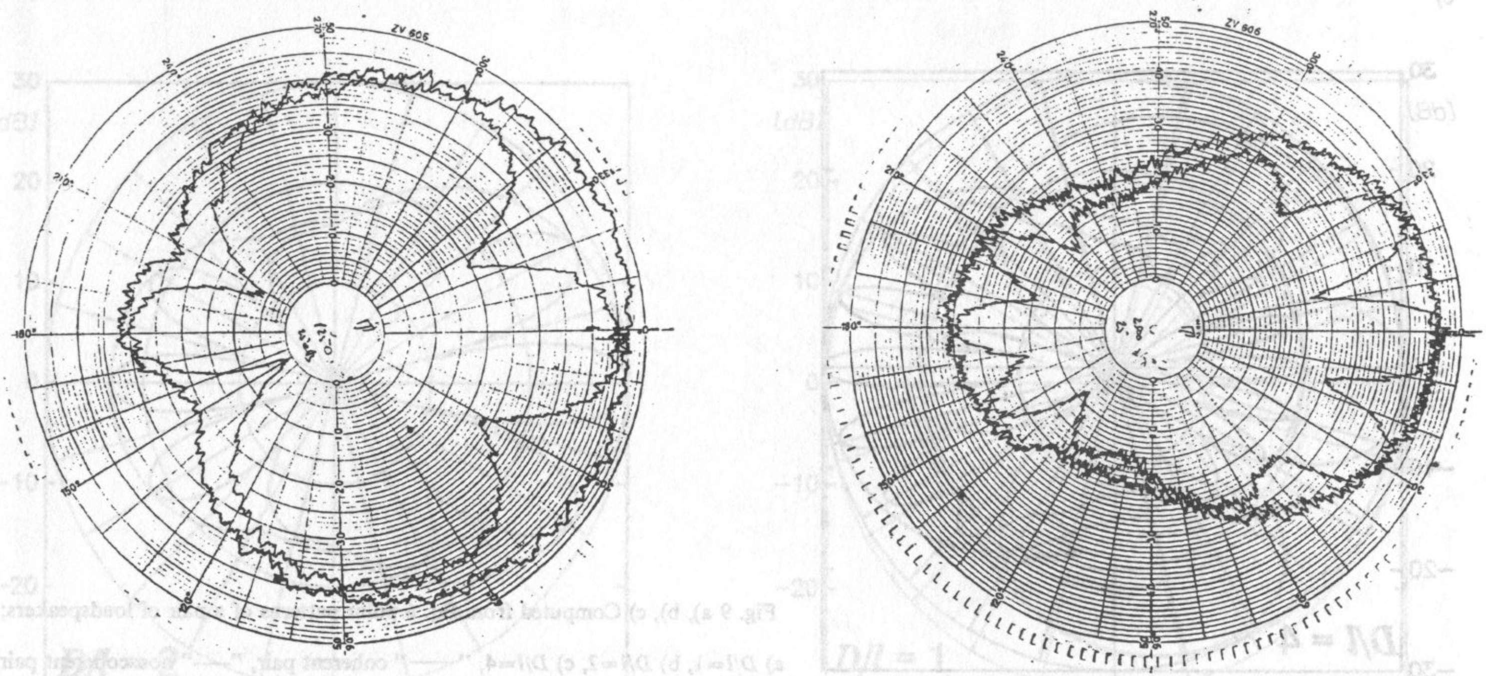


Fig. 9 a), b), c) Computed from Eq. 6 polar patterns of a pair of loudspeakers;

a) $D/l=1$, b) $D/l=2$, c) $D/l=4$, " —— " coherent pair, "-----" non-coherent pair.

a) b)



c)

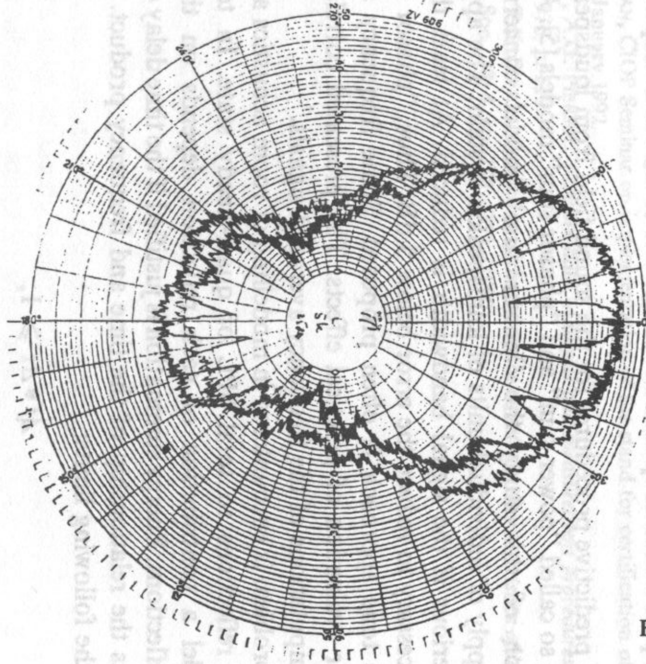


Fig. 10a), b), c) Measured with third-octave noise polar patterns of a pair of loudspeakers: a) $D/l=1$, b) $D/l=2$, c) $D/l=4$.

Comparison of the diagrams in Fig. 9 and Fig. 10 evidently confirms the predicted interference patterns in the sound pressure field of loudspeaker system radiating band-limited noise coherently. Close agreement of the experimental data with the theory confirms also the usefulness of the presented interference model for further investigation of more complex systems.

4. Final remarks

(i) Recommended conditions for acoustic testing of hearing protectors in reflection-free environments may not be fulfilled with simple, coherently fed loudspeaker system.
High level, spatial variations of SPL shall be expected at the frequency range above 1 kHz.

(ii) Promising results of the present study of stationary sound field of two sound sources may be effectively applied to extended number of sources. The most interesting applications of the correlation model are as follows:
— prediction of narrow- or wide-band directivity patterns of sophisticated loudspeaker arrays,
— more accurate predictions of spatial distribution of SPL from multiple loudspeaker arrangements,
Commonly used predictive procedures for the sound field from loudspeaker systems are based on the so called "power sum" and "phasor sum" models [5]. According to the interpretation of this study the "power sum" model concerns cases of non-coherent supply of individual units of loudspeaker system. The approximation error may be serious if the signal bandwidth is reduced. At least three octaves bandwidth is necessary to reduce the error below 5 dB. Thus, "phasor sum" model built on a pure tone approach is often proposed. Some frequency averaging is indispensable for evaluation of practical effects. The presented correlation model allows direct computations of the frequency averaged signals.

(iii) Similar problem is encountered in modelling of room acoustics and concerns contribution of reflections to total SPL of quasi-steady state. In terms of the correlation model a contribution of each reflection depends on the temporal distribution of reflections. Energy summation is justified if the time delay of successive reflections fulfills the relationship of the time and frequency product. Correlation model suggests the following limits:

$$\Delta f * \Delta\tau \geqslant 1;$$

or in frequency-spatial variables:

$$\Delta f * \Delta r \geqslant c;$$

c — sound velocity.

The delays of the order of at least ten milliseconds are necessary to satisfy the above relations in case of octave noise bands in the frequency range above 100 Hz. It means that the early reflections are the energy contributors of the summation process. Probably, such approximation may also be extended to the diffuse part of reverberation because of multiple and random phase shifts of delayed reflections.

Acknowledgments

This report was sponsored by grant No 4 4160 91 02 from the State Committee for Scientific Research. Part of the material was presented to the 39th Polish Open Seminar on Acoustics, Cracow, September 1992.

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