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NOISE REDUCTION PROBLEMS OF VIBRATORY MACHINES

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Several new methods of reduction of the noise produced by vibratory machines, such as conveyors, vibratory tables and sieves, shakeout grates etc. are proposed. Special attention is devoted to the problems of optimization of vibration parameters in the machines mentioned above, with restrictions imposed by the required production output and quality of the manufacturing process. The methods consisting in modification of the character of vibration of housing elements and application of low-tuned coverings in machines are outlined.

1. Introduction

Vibratory machines used in various branches of industry for compaction and moulding of precast concrete members, screening, grinding and transportation of loose materials, for knocking-out of the castings, crushing and other technological operations are often nuisant and reduction of noise emitted from such sources is often extremely difficult. Noise emitted from these sources often reaches 110 to 120 dB (for instance during compaction of ceiling members in concrete prefabrication plants or for knocking-out of the castings), which can lead to a permanent demaging of hearing ability in the case of 30 to 50% workers who have been working for a period of time longer than about five years.

Reduction of the noise emitted by those machines, achieved by means of traditional methods is especially difficult, since their operation is connected with generation of intensive mechanical vibration, whereas the passive methods of reduction of emission and propagation of acoustic waves meet serious difficulties. So, for instance, possible use of housings of various types and sound absorbing and silencing cabins is very limited, since numerous vibration processes (for instance moulding of concrete members) require incessant manipulations by the operators. A usually short duration of technological operations of an order of several minutes causes also that auxiliary times connected with opening and closing of doors and flaps lead to a major prolongation of the production cycle. Similarly, acoustic screens are also not so effective in the case of reduction of propagation of the noise emitted by machines. Because of a strong sound component, of an order or several scores of cycles per second, their effectiveness is limited by a strong diffraction at the screen edges, by reduction of the area of the acoustic shadow and necessity of application of large-size screens. Large dimensions of production rooms and the necessary presence of operators in immediate vicinity of the machines reduce, in turn, the effectiveness of acoustic adaptation as a method of reduction of noise at the work stands. Hence, new noise reduction methods must be devised. Some of them have been presented in this paper.

2. Method of optimization of vibration parameters

Let us consider a machine as a translating rigid body vibrating with frequency ω and vibration amplitude A in the direction normal to the main exterior surface of machine body and emitting sound as a vibrating piston. A major reduction of vibration parameters is in the case impossible, because of the required production capacity. Hence, we should search for such a combination of parameters which ensure, on the one hand, the required production capacity and the desired quality of the production process and, on the other hand, reduction of the sound level. To cope with those demands we should construct two models of the phenomenon under consideration, taking into account the vibration parameters, and namely:

model of the production output and

model of the acoustic emission.

Moreover, information must be available (usually in the form of constraints imposed on the range of variability of the vibration parameters) concerning the effect of those vibration parameters on the quality of the technological process. The information will enable us to determine the curves of constant productivity in the space of vibration parameters, within the parameter variability range allowable from the point of view of quality, and then to find the point of minimum acoustic emission at those curves. Consider the example of parameter optimization of the vibratory table used for compaction of the concrete mix.

In the case under consideration it has been assumed that the vibration intensity described by the following expression

$$N_B = A^2 \omega^3 \tag{1}$$

can be assumed as a measure of productivity and, because of the required quality of the compaction process, we can usually assume that $2 < N_B < 7.5$ m²/s³, $0.2 < A < 1.2$ mm, $25 < \omega < 75$ c/s.

For machines with a flat body installed above the floor level, the model of vibrating piston without an acoustic baffle may be assumed, and for $k < 1$ (what is valid for medium-size machines) the noise level on the surface of the measuring sphere may be assumed as a measure of the acoustic emission:

$$L_A = 10 \lg \frac{4\rho S^3 A^2 \omega^6}{27\pi^4 c^3 I_0 S_0} - \Delta L_A(\omega) \tag{2}$$

where ρ, c — density and speed of sound in air, S — area of the vibrating surface, S_0 — area of the surface of a sphere on which we evaluate measure the sound level (S_0 for a 1 m radius has been assumed hereinafter), I_0 — reference sound intensity, $I_0 = 10^{-12}$ W/m², $\Delta L_A(\omega)$ — scale correction factor A according to IEC 123 and 179, k — wave number, a — characteristic dimension of the machine.

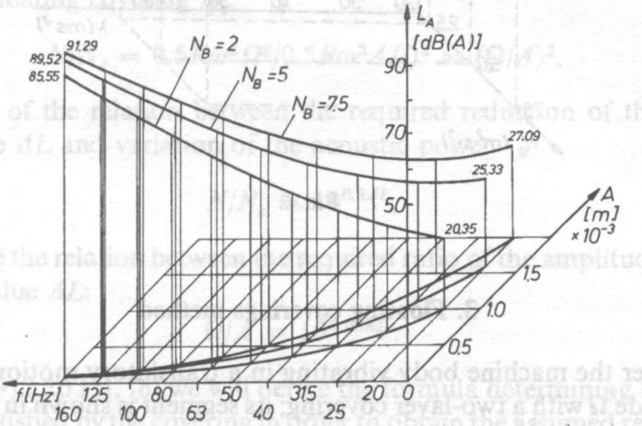

FIG. 1

Figure 1 presents curves of constant productivity in the plane of vibration parameters A, ω for $N_B = 2$, 5 and 7.5 m²/s², and the corresponding curves of the sound level. As it can be seen from these curves, in order to reduce of the sound level it is advisable to reduce the frequency of vibration to a level allowed by the quality of the compaction process and to compensate this reduction by the respective increasing of the amplitude of vibration according to relation (1).

When the procedure described above is used attention must be paid whether the translatory vibration of the machine body is really the main reason for the acoustic emission of the machine. So, for instance, in the case of compaction of concrete mix on a vibrating table which is not provided with a fixture for fastening the mould to the table, high-frequency bending vibration due to periodical pulling out of the mould from the table usually produces greater noise emission than the translatory vibration of the mould.

Model of acoustic emission will then differ from that described by the relationship (2). Taking advantage of ZABOROV estimate [4] of variation of the level of acoustic power due to the change of relative collision velocity and the interval between collisions, and relating the collision velocity to the vibration period of the machine, one may state [5] that in this case it is slightly better to reduce the amplitude and to increase the frequency; however, the results obtained do not exceed several decibels. The curve illustrating this relationship in coordinates N_B, λ (where $\lambda = A\omega^2$ is the vibration acceleration) is shown in Fig. 2.

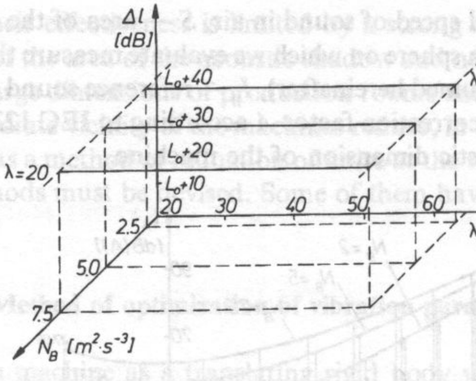

FIG. 2

3. Floating coverings method

Let us cover the machine body vibrating in a translatory motion with frequency ω and amplitude Ω with a two-layer covering; its segment is shown in Fig. 3. Layer 1 is made of a material with a low Young's modulus which, due to a considerable rigidity, will be considered to be undeformable and possess high density ρ and mass per unit

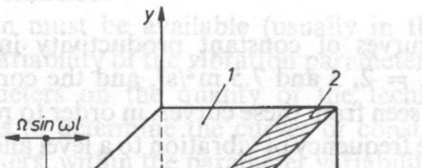

FIG. 3

area m. The equation of the motion of the system may then be written in the form [4, 5] as follows

$$\frac{\delta^2 w}{\delta t^2} = \frac{E^*}{\rho}\frac{\delta^2 w}{\delta x^2}, \quad \text{where } E^* = (1-\nu)\,E/1(1-\nu-2\nu^2), \tag{3}$$

$$w(0,t) = \Omega \sin \omega t, \tag{4}$$

$$\frac{\delta w(l,t)}{\delta x} E^* = -m \frac{\delta^2 w}{dt^2}(l,t). \tag{5}$$

Hence, the amplitude of vibration of the outer layer is

$$A(l) = \Omega/[\cos(\omega\sqrt{\rho l/E^*}) - (m\omega/\sqrt{E^*\rho})\sin(\omega\sqrt{\rho l/E^*})]. \tag{6}$$

Taking into consideration the relation between the average active acoustic powers in the case of absence of the covering N and in the case when the outer surface is covered with a floating covering N_c,

$$N/N_c = 0.5R\omega^2\Omega^2/0.5R\omega^2 A(l)^2 = (\Omega/A)^2, \tag{7}$$

and making use of the relation between the required reduction of the level of the acoustic pressure ΔL and variation of the acoustic power

$$N/N_c = 10^{0.1\Delta L}, \tag{8}$$

we can determine the relation between the required ratio of the amplitude of vibration to the desired value ΔL:

$$\Omega/A = 10^{0.05\Delta L}. \tag{9}$$

Subsituting (9) into Eq. (6) we will derive the formula determining the conditions which must be satisfied by the covering in order to obtain the assumed reduction of the level of the acoustic pressure

$$|\cos(\omega\sqrt{\rho l/E^*}) - m\omega/\sqrt{\rho E^*}\sin(\omega\sqrt{\rho l/E^*})| >= 10^{0.05\Delta L}. \tag{10}$$

It should be emphasized that, in spite of the fact that on the axis ω there is an numerable series of intervals where condition (10) can be satisfied, the covering for which the first frequency of free vibration is lower and the second frequency of free vibration is higher than the excitation frequency ω, is the most feasible one.

4. Method of flexural coverings

As early as in 1959, HECKL [2] proposed the plates subjected to kinematically induced vibrations by assuming the boundary conditions in the form of the assumed harmonic vibration to support at some points only. This leads in effect to excitation of vibration being the sum of translation and bending vibration, wherein sound radiation coefficient is slightly reduced.

The author of this paper has proposed such boundary conditions of plates with kinematically induced vibrations (for instance, covering plates of vibratory machine bodies) to obtain pure bending vibration of the plate, containing notranslation component, in spite of the assumed formula of the motion of the edge. This contributes to a mutual compensation of sound radiated by the regions of the plate vibrating in counterphase and enables us to use Wallace's integer [1] for optimization of parameters of the system on the basis of the condition of minimum of the radiation coefficient.

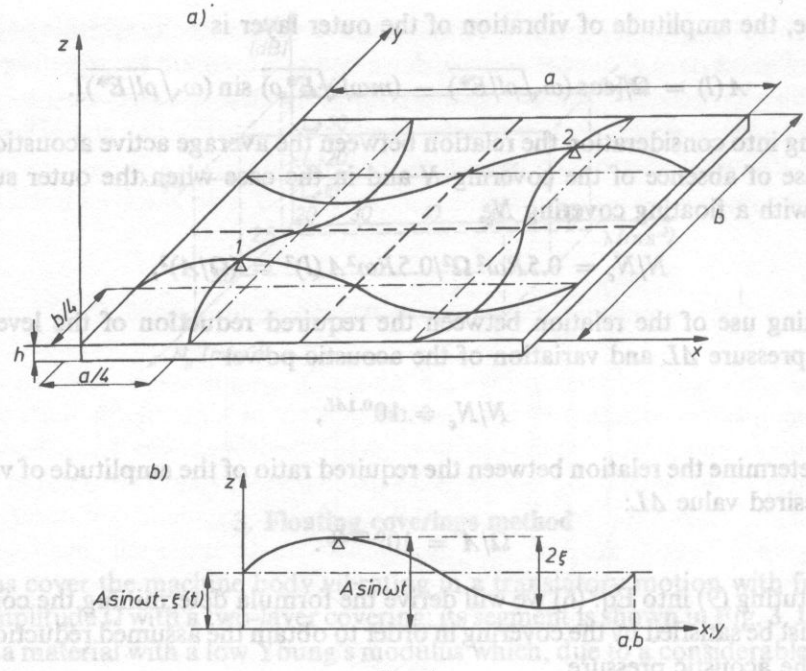

FIG. 4

In order to illustrate the proposed approach consider the case of a rectangular isotropic plate of constant thickness. We assume the boundary conditions as shown in Fig. 4. It shows a unit segment that is a "module" of the plate supported at two points situated on a diagonal and representing centers of two out of the four identical quarters into which the module can be divided. The mentioned points of support vibrate along the z-axis according to the formula

$$z_i = A \sin \omega t, \qquad i = 1, 2. \tag{11}$$

Let us assume that the steady state forced motion can be presented as a sum of a translatory convection motion $z_u\,(x, y, t)$ and flexural vibrations in the form analogical to the case of free vibration of an element freely supported at the edges. The equation of vibration of the plate will then have the form

$$\frac{\delta^4 z}{\delta x^4} + 2\,\frac{\delta^4 z}{\delta_z^2\,\delta_y^2} + \frac{\delta^4 z}{dy^4} + \frac{\rho h}{D} + \frac{\delta^2 z}{dt^2} = 0, \qquad D = Eh^3/12\,(1-\nu^2), \tag{12}$$

and in the case of a simply supported member the following formula holds:

$$\begin{gathered} z(0,y,t)=0, \quad z(x,0,t)=0, \quad z(a,y,t)=0, \quad z(x,b,t)=0 \\ \frac{\delta^2 z}{\delta x^2}(0,y,t)=0, \quad \frac{\delta^2 z}{\delta y^2}(x,0,t)=0, \quad \frac{\delta^2 z}{\delta x^2}(a,y,t)=0, \quad \frac{\delta^2 z}{\delta y^2}(x,b,t)=0. \end{gathered} \tag{13}$$

The form of free vibration of order 2.2 for the boundary conditions (13) has the form

$$z_g(x,y,t) = \xi \cdot \sin(2\Pi x/a) \cdot \sin(2\Pi y/b), \tag{14}$$

Let us note (see Fig. 4b) that in the case when the amplitude of vibration of points 1, 2 is equal to the amplitude of flexural vibration of the plate around a plane being in translatory motion, the resultant motion of the plate will then have the character of pure flexural vibration with respect to motionless a reference plane. In such a case the boundary conditions represent, in view of the neighbourhood of identical modules, a simply supported member (13). This confirms the validity of the formula describing the forced vibration as a sum $z(x,y,z) = z_u(t) + z_g(x,y,t)$, in conformity with Fig. 4b.

$$z(x,y,t) = A\sin\omega t + \xi(t)\,[\sin(2\Pi x/a)\sin(2\Pi y/b) - 1], \tag{15}$$

where $\xi = \xi(t)$ is now a generalized coordinate.

Analyse now the conditions which must be satisfied in order to ensure similarity of the form of forced vibration (15) of the plate of the form of flexural vibration of a plate with boundary conditions (13), that is to ensure the absence of a convection component in the plate motion. The equations of motion of the plate excited by vibration of the supports will be found by means of the Lagrange method.

Kinetic energy of the system is equal to

$$E = \iint\limits_{00}^{ba} \frac{1}{2}\rho h v^2(x,y,t)\,dx\,dy =$$

$$= \frac{1}{2}\rho h \iint\limits_{00}^{ba} \{A^2\omega^2\cos^2\omega t + 2A\omega\cos\omega t\;\xi\;[\sin(2\Pi x/a)\sin(2\Pi y/b) - 1] + \tag{16}$$

$$+ \dot{\xi}^2[\sin(2\Pi x/a)\sin(2\Pi y/b) - 1]^2\}\,dx\,dy =$$

$$= \frac{1}{2}m\left(A^2\omega^2\cos^2\omega t - 2A\omega\xi\cos\omega t + \frac{5}{4}\dot{\xi}^2\right).$$

Potential energy is expressed by the equation

$$U = \iint\limits_{00}^{ba} \frac{D}{2}\left[\left(\frac{\delta^2 z}{\delta x^2}\right)^2 + \left(\frac{\delta^2 z}{\delta y^2}\right)^2 + 2\nu\frac{\delta^2 z}{\delta x^2}\cdot\frac{\delta^2 z}{\delta y^2} + 2(1-\nu)\left(\frac{\delta^2 z}{\delta x\delta y}\right)\right]dx\,dy =$$

$$= D/2\iint\limits_{00}^{ba} [\xi^2(2\Pi/a)^4\sin^2(2\Pi x/a)\sin^2(2\Pi y/b) + \xi^2(2\Pi/b)^4\sin^2(2\Pi x/a)\cdot \tag{17}$$

$$\cdot\sin^2(2\Pi y/b) + 2\nu\xi^2(2\Pi/a)^2(2\Pi/b)^2\sin^2(2\Pi x/a)\sin^2(2\Pi y/b) + 2(1-\nu)\cdot$$

$$\cdot\,\xi^2(2\Pi/a)^2(2\Pi/b)^2\sin^2(2\Pi x/a)\sin^2(2\Pi y/b)]\,dx\,dy =$$

$$= 2\Pi^4 \mathrm{Dab}\,\xi^2(1/a^2 + 1/b^2)^2.$$

Substituting this relation into Lagrange's equations

$$\frac{d}{dt}\frac{\delta(E-v)}{\delta\dot{\xi}} - \frac{\delta(E-v)}{\delta\xi} = 0, \tag{18}$$

we obtain the equation of forced vibration of the plate

$$\ddot{\xi} + \frac{16}{5}\Pi^4\frac{Dab}{m}(1/a^2+1/b^2)^2\xi = -\frac{4}{5}A\omega^2\sin\omega t \tag{19}$$

Denoting by ω_n the frequency of free undamped vibration of the system described by the equation (19)

$$\omega_n = \sqrt{\frac{Dab}{5m}[(2\Pi/a)^2+(2\Pi/b)^2]^2}, \tag{20}$$

we can write the solution of the Eq. (19) in the following form

$$\xi(t) = -\frac{4}{5}A\frac{1}{(\omega_n/\omega)^2-1}\sin\cdot\omega t = \xi_0\sin\omega t \tag{21}$$

In order to determine the vibrations of the plate, corresponding (as far as their form, not frequency, is concerned) to free vibrations of order 2.2. of a plate of dimensions *axb*, simply supported at the edges we must satisfy the condition

$$\xi_0 = A. \tag{22}$$

This leads to the condition (23) which corresponds to the free vibration frequency of order 2.2 of the plate *axb* freely supported at the edges.

$$\omega = \omega_n\sqrt{5} \tag{23}$$

Hence, by supporting each of the "modules" of the plate at two points undergoing a harmonic motion with frequency ω and amplitude A we can obtain vibration of the module in the form corresponding to the vibration of order 2.2 of plate *axb* simply supported along the stationary edge. However the dimensions (for instance thickness) and material of the plate must be properly chosen: its frequency of order 2.2 in the case of a simply supported member should be equal to that of free vibration ω of that member.

Prior to proceeding to evaluate the variation of the radiated acoustic power, which can be achieved by the selection of the boundary conditions given above, considered now the caase frequently encountered in practice of a plate, which is a housing of the machine body, fastened for instance by welding to a framework vibrating together with the machine body (see Fig. 5a). Thick dashed lines show the location of that bracing framework. Cross-section through one of the quarters is shown in Fig. 5b.

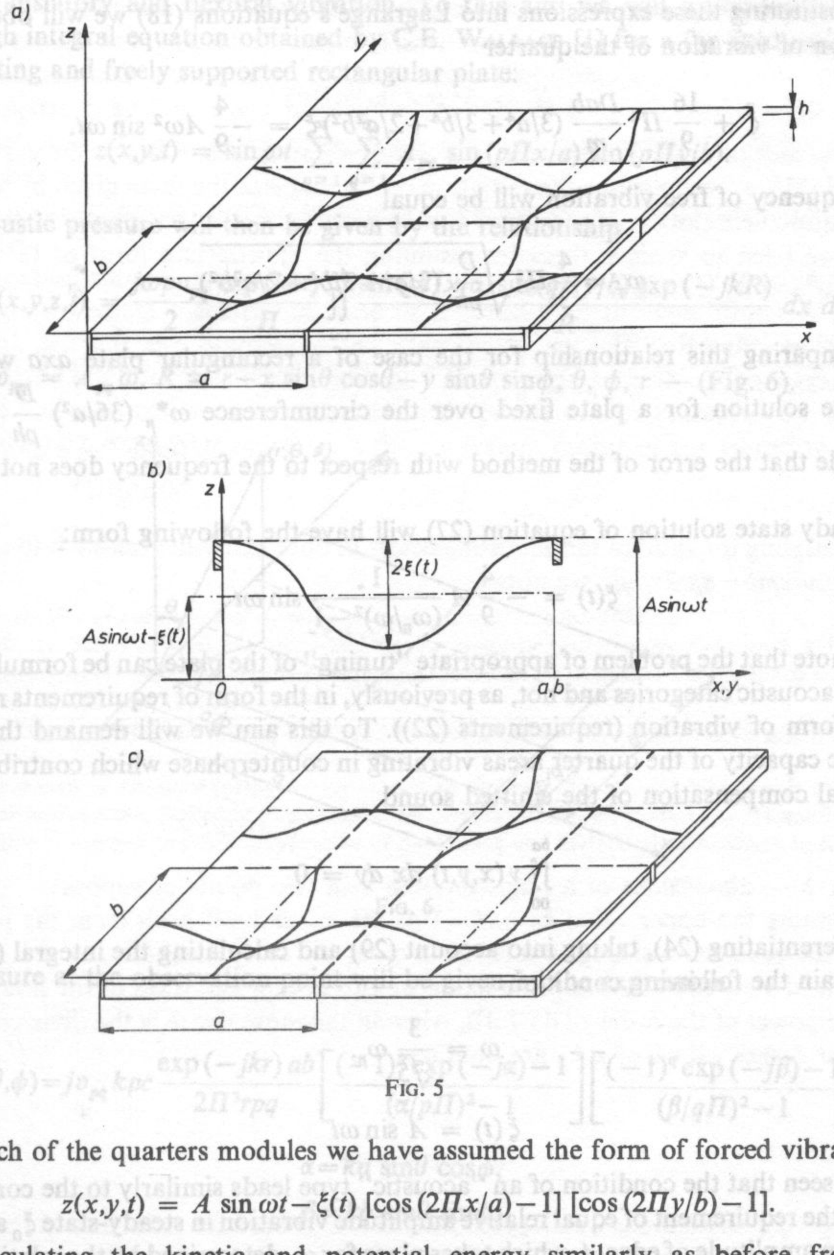

FIG. 5

For each of the quarters modules we have assumed the form of forced vibration

$$z(x,y,t) = A \sin \omega t - \xi(t) [\cos(2\Pi x/a) - 1] [\cos(2\Pi y/b) - 1]. \tag{24}$$

Calculating the kinetic and potential energy similarly as before from the relationships (16) and (17), we will obtain

$$E = \frac{1}{2} m [A^2\omega^2 \cos^2\omega t - 2A\omega\dot{\xi}(t) \cos \omega t + \frac{9}{4} \dot{\xi}^2(t)]. \tag{25}$$

$$U = 2\Pi^4 abD\xi^2(3/a^2 + 3/b^2 + 2/a^2b^2). \tag{26}$$

Substituting these expressions into Lagrange's equations (18) we will obtain the equation of vibration of the quarter

$$\ddot{\xi} + \frac{16}{9} \Pi^4 \frac{Dab}{m} (3/a^4+3/b^4+2/a^2b^2)\,\xi = -\frac{4}{9} A\omega^2 \sin \omega t. \tag{27}$$

Frequency of free vibration will be equal

$$\omega_n = \frac{4}{3} \Pi^2 \sqrt{\frac{D}{\rho h} (3/a^4+3/b^4+2/a^2b^2)}. \tag{28}$$

Comparing this relationship for the case of a rectangular plate axa with the accurate solution for a plate fixed over the circumference $\omega^*_n\ (36/a^2)\ \frac{D}{\rho h}$ we can conclude that the error of the method with respect to the frequency does not exceed 3.39%.

Steady state solution of equation (27) will have the following form:

$$\xi(t) = -\frac{4}{9} A \frac{1}{(\omega_n/\omega)^2-1} \sin \omega t. \tag{29}$$

Let us note that the problem of appropriate "tuning" of the plate can be formulated at once in acoustic categories and not, as previously, in the form of requirements relating to the form of vibration (requirements (22)). To this aim we will demand the same acoustic capacity of the quarter areas vibrating in counterphase which contributes to a mutual compensation of the emitted sound

$$\int_0^b\!\!\int_0^a v(x,y,t)\, dx\, dy = 0 \tag{30}$$

Differentiating (24), taking into account (29) and calculating the integral (30) we will obtain the following condition

$$\omega = \frac{3}{\sqrt{5}}\, \omega_n, \tag{31}$$

$$\xi(t) = A \sin \omega t \tag{32}$$

It is seen that the condition of an "acoustic" type leads similarly to the condition (22), to the requirement of equal relative amplitude vibration in steady-state ξ_0 and the vibration amplitude of edge A, which takes place for ω_n determined by the relationship (31). The value ω_n can be adjusted again to the existing requirements by assuming proper elastic and inertia properties of the plate.

Let us now proceed to estimate the emission limit which may be obtained if a plate of dimensions $naxmb$ (n, m — natural numbers, a, b — dimensions of a single module) vibrating like a piston in an acoustic baffle is replaced by a plate subject simultaneously to translatory and flexural vibration. To this aim we will use the solution of Rayleigh integral equation obtained by C.E. WALLACE [1] for a far field emitted by a vibrating and freely supported rectangular plate:

$$z(x,y,t) = \sin \omega t \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} A_{pq} \sin (p\Pi x/a) \sin (p\Pi y/b). \tag{33}$$

Acoustic pressure will then be given by the relationship

$$p(x,y,z,t) = \frac{j\omega\rho v_{pq} \exp(j\omega t)}{2} \int_0^a \int_0^b \frac{\sin(p\Pi x/a) \sin(q\Pi y/b) \exp(-jkR)}{\Pi R} dx\, dy \tag{34}$$

where: $v_{pq} = A_{pq}\,\omega$, $R \cong r - x\sin\theta\cos\theta - y\sin\theta\sin\phi$, θ, ϕ, r — (Fig. 6).

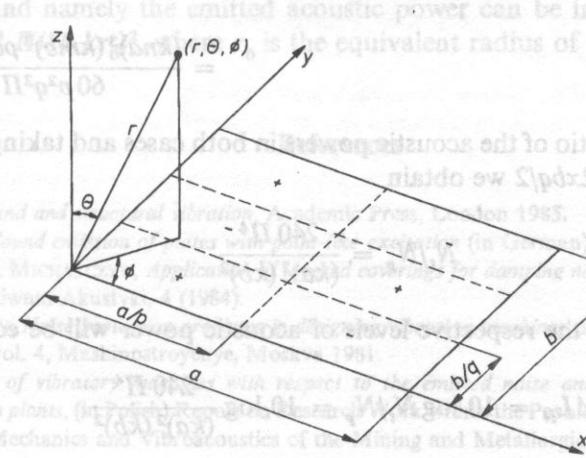

FIG. 6

Pressure at the observation point will be given by the expression

$$p(r,\theta,\phi) = jv_{pq} k\rho c \frac{\exp(-jkr)\, ab}{2\Pi^3 rpq} \left[\frac{(-1)^p \exp(-j\alpha) - 1}{(\alpha/p\Pi)^2 - 1}\right] \left[\frac{(-1)^q \exp(-j\beta) - 1}{(\beta/q\Pi)^2 - 1}\right] \tag{35}$$

$$\alpha = ka \sin\theta \cos\phi,$$

$$\beta = kb \sin\theta \sin\phi.$$

If the acoustic power is written in the form

$$N = \sigma\rho\, c\, S \langle \bar{v}^2 \rangle \tag{36}$$

where σ — radiation factor, $\langle \bar{v}^2 \rangle$ — square of normal velocity of the plate averaged over the time and area, then the acoustic power radiated by a vibrating plate, treated as a rigid body in the case of low frequencies ($k<1$), can be calculated by substituting in the expression (36) $\langle \bar{v}^2 \rangle = v^2$, $\sigma \cong k^2 S/2\Pi$, where

$$N_t = S^2 \rho c A^2 \omega^4 / 4\Pi c^2 \tag{37}$$

In the case of a plat of the same area S but vibrating according to the expression (15) and (21), we will assume the radiation coefficient σ in the form given by Wallace for vibration being an even function of p and q (such forms give the minimum of σ and, hence, we tried to achieve them by assuming the appropriate form of the forced vibration and by a suitable choice of the boundary conditions

$$N_p \cong \frac{2\,(kna)^2 (kmb)^3}{15p^2q^2\Pi^5}\,\rho c \int_0^{mb}\int_0^{na}\left[\frac{\omega}{2\Pi}\int_0^{2\Pi/\omega} A^2\omega^2\cos^2\omega t \sin^2(2\Pi x/a)\sin^2(2\Pi y/b)\,dt\right]dx\,dy = \frac{(kna)^3 (kmb)^3\,\rho c A^2\omega^2 S}{60\,p^2q^2\Pi^5} \tag{38}$$

Calculating the ratio of the acoustic powers in both cases and taking into account that $S = naxmb = ap/2xbq/2$ we obtain

$$N_t/N_p = \frac{240\,\Pi^4}{(ka)^2 (kb)^2} \tag{39}$$

and the difference of the respective levels of acoustic power will be equal

$$\Delta L_N = 10 \log N_t/N_p = 10 \log \frac{240\,\Pi^4}{(ka)^2 (kb)^2}. \tag{40}$$

where a, b — dimensions of a single module with two points of support.

Assuming the upper value $ka = kb = 1$ as the upper limit both from the point of view of the practical working conditions of most vibratory machines, as well as from the point of view of the assumption made we can obtain variation of the level of the acoustic power of the order of 43.7 dB, wherein the more dense is the division of the plate, the larger values of ΔL_N are obtained.

5. Conclusions

The proposed methods which principally are to be used for reduction of the sound level radiated by the surfaces of vibratory machine bodies in translatory motion do not exhaust all the possibilities of reduction of noise of such machines. There exist numerous traditional methods, which can be used simulatneously with the methods described above. The following methods should be mentioned:

— rationalization of the time of vibration for instance, in the case of the machines for compaction of concrete mix, a delayed engagement of the vibrating drive at a moment when the mould has already been partly filled: such procedure ensures a considerable lowering of the emitted noise level,

— elimination of the sources of impact noise (for instance by fixing the moulds to the support beams in the vibrating tables, application of tension and support counterflexure of sieves in vibrating sieves, fixing of loose elements, such as washers and cotter pins,

— maintenance of proper technical condition of the equipment, particularly vibrators.

Attention should also be paid to proper housing of vibratory machines. These machines are often installed, because of ingorance of the conditions of acoustic emission so that the bottom of the machine is cut off from the environment (foundation in a socket with side sealing). The effect thus obtained is contrary to the desired one and namely the emitted acoustic power can be increased by the value $L = 10 \log (27 \Pi / 16 kr_0)^2$, where r_0 is the equivalent radius of the vibrating surface.

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