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THE USE OF HELMHOLTZ RESONATORS AND PANEL ABSORBERS IN THE DESIGN OF DUCT SILENCERS

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The use of Helmholtz resonators and panel absorbers in reducing the noise propagated in ducts is investigated both analytically and experimentally. Using a simple approach, the impedance of the resonators is calculated taking into account the presence of sound absorbing material in the cavity. Equations for the transmission loss are derived in both cases. Parameters such as dimensions, length of the opening and flow resistivity of the lining in case of Helmholtz resonators and panel thickness and dimensions in case of panel absorbers are systematically examined. Comparisons between experimental and predicted values are in good agreement showing that the model developed describes satisfactorily the behaviour of resonators in ducts and can be used for improved design of silencers incorporating resonators.

1. Introduction

There are many practical applications where the conventional types of dissipative mufflers can not be used. Particularly, in cases involving hot gaseous flow the pores of the sound absorbing lining are closed by oil or carbon particles, the fibres are blown out or thermal cracking of the lining occurs. A further disadvantage of dissipative mufflers is their poor attenuation at low frequencies.

In order to overcome the aforementioned disadvantages, silencers incorporating Helmholtz resonators or panel absorbers or a combination of both have been proved very useful [1, 2, 3]. The main advantages of such silencers are their low cost of construction and maintenance, their high resistivity and the good performance at low and mid-frequencies. Furthermore, these silencers can be "tuned" in order to perform in a predetermined frequency range.

Little analytical work has been done [4, 5, 6, 7] concerning the behaviour of resonators in ducts and the development of silencers using resonators has been based on rather empirical procedures.

It is the aim of the present paper to develop a simple model that describes the acoustical behaviour of Helmholtz resonators and panel absorbers in a flow duct. The model is based on the assumption that only wave propagation is allowed and that the flow velocities are low (<10 m/s), so that the behaviour of the resonators can be considered linear. Our approach is similar to that described by SULLIVAN for modelling perforated tube mufflers [5].

Based on these assumptions, the impedance of both the Helmholtz resonator and panel absorber are calculated taking into account the presence of sound absorbing material in the cavity. By using the calculated impedance, an expression for the transmission loss in both cases is obtained.

Finally, the predicted results are shown to be in good agreement with experimental findings giving thus confidence in the use of the model, at least for design purposes. The model allows useful parametric analyses that can lead to improved design of silencers incorporating panel absorbers and Helmholtz resonators, and can be easily extended to describe the noise reduction in ducts lined with an array of resonators. Further work in that direction is already under way and will be presented in a future publication.

2. Impedances

In the following we calculate the impedance of a Helmholtz resonator and a panel absorber with sound absorptive material in the cavity.

2.1. *Impedance of a Helmholtz resonator*

We start by analyzing first the resonator element shown in Fig. 1. The length of the resonator element is L, the width is b and its height is h. The length of the opening is l and the cavity is lined with a usual rock wool layer of thickness d. In order to take into account the additional losses due to the lining, we assume a complex propagation constant K_x within the resonator. To calculate K_x one needs to know the wall impedance of the lined side. It is very well known that in case of a plane wave incident

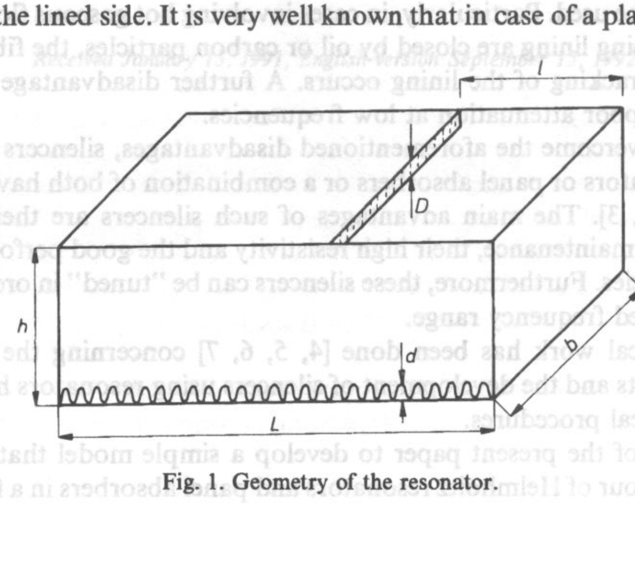

Fig. 1. Geometry of the resonator.

on a locally reacting lining of uniform thickness d backed by a rigid wall, the impedance encountered by the plane wave is given by

$$Z_w = -jY_w \cot(K_w d), \tag{1}$$

where Y_w and K_w are the complex characteristic impedance and propagation constant of the absorptive lining, respectively.

For fiber-based sound absorbing materials often used in mufflers K_w and Y_w are given by [8]

$$\frac{K_w}{K_0} = (\chi)^{1/2}\left\{1 - j\frac{\xi}{\rho_0\omega\chi}\right\}^{1/2} \tag{2}$$

$$\frac{Y_w}{Y_0} = \frac{1}{\sigma}\frac{K_w}{K_0}, \tag{3}$$

where ρ_0 is the air density, ξ is the flow resistance of the unit thickness of the porous bulk material, σ is the porosity, χ is the structural factor and K_0 is the wave number in air. By considering the resonator as an infinite duct of height h lined on one side (Fig. 2), the wavenumber K_x is given as a solution of the well-known transcendental equation [9]

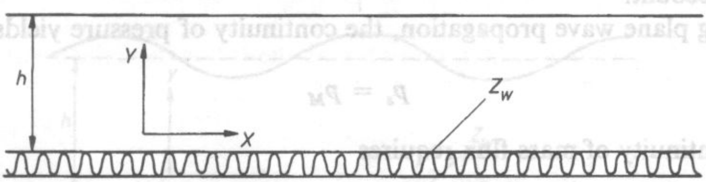

Fig. 2. Resonator element as infinite duct.

$$-j\sqrt{E}\tan\sqrt{E} = \beta,$$

$$\text{with} \quad E = (K_0 h)^2 - (K_x h)^2 \quad \text{and} \quad \beta = K_0 h\, \rho_0 c_0\, 1/Z_w. \tag{4}$$

The real and imaginary parts of the first root of Eq. (4) may be obtained by nomograms or numerically. A very reasonable and convenient approximation can be obtained by writing (see ref. [9]).

$$E = \frac{105 + 45j\beta \pm \sqrt{11025 + 5250j\beta - 1605\beta^2}}{20 \pm j\beta}. \tag{5}$$

with

$$\beta = K_0 h \rho_0 c_0 \frac{1}{Z_w} = jh\frac{K_0^2 \sin(K_w d)}{K_w \cos(K_w d)}. \tag{6}$$

Equations (5) and (6) give two complex values for K_x; the one that gives lower attenuation is of particular importance. The impedance of the resonator consists of the neck inertance and cavity compliance with damping in series. The compliance can be expressed in terms of K_x as

$$Z_s = \frac{p_s}{u_s} = -j\rho_0 c_0 \left(\frac{K_x}{K_0}\right) \frac{\cos K_x (L-l)}{\sin K_x (L-l)}, \tag{7}$$

where p_s and u_s are the acoustic pressure and velocity in the resonator.

The inertance of the neck, i.e. the opening can be expressed as

$$Z_M = \frac{F_M}{u_M} = \frac{P_M S_0}{u_M} = j\omega m_s, \tag{8}$$

where F_M is the driving force, u_M is the volume velocity at the opening and $S_0 = lb$ is the area of the opening. Hence m_s is the effective oscillatory mass taking into account the mass correction. In other words, the opening is considered to be circular with an equivalent radius S_0/π, so that one can approximately write $m_s \approx \rho_0 S_0 \sqrt{S_0/\pi}$. By adding the impedance given by Eqs. (7) and (8), the combined impedance of the resonator can be obtained. However, by adding the impedances the geometry must be taken into account.

Assuming plane wave propagation, the continuity of pressure yields

$$p_s = p_M \tag{9}$$

while the continuity of mass flux requires

$$S_0 u_M = S u_S. \tag{10}$$

Here, S_0 and S are the areas of the opening and of the free cross-section of the duct respectively.

Thus, one can write

$$\frac{p_s}{u_s} = \frac{p_M S}{u_M S_0} = \frac{F_M}{u_M}\frac{S}{S_0^2}, \tag{11}$$

or alternatively

$$\frac{F_M}{u_M} = \frac{p_s}{u_s}\frac{S_0^2}{S}. \tag{12}$$

By using Eqs. (9) through (12) the total impedance of the resonator can be expressed as

$$Z_t = j\omega m_s + \frac{S_0^2}{S}\left[-j\rho_0 c_0 \frac{K_x}{K_0}\frac{\cos K_x (L-l)}{\sin K_x (L-l)}\right]. \tag{13}$$

2.2. Impedance of a panel absorber

The panel abosrber under consideration is shown in Fig. 3. It consists of a flexible panel backed by an air cavity of depth h. The bottom of the cavity is lined with a usual

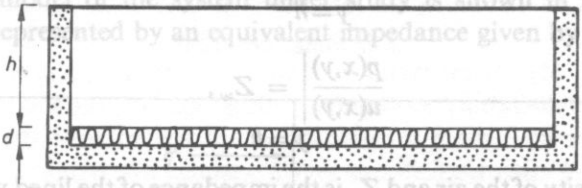

Fig. 3. Geometry of panel absorber.

rock wool layer of thickness d. The impedance of the panel absorber can be considered as the inertance of the vibrating panel and the cavity compliance with damping. In order to take into account the losses in the cavity due to the lining, we have to assume a complex propagation constant K_x in the x-direction. To calculate K_x in a convenient and simple way, we consider the absorber element as infinite, see Fig. 4. By assuming a pressure of the form $p(x, y) = p(y)\, e^{-jK_x x}$, the two-dimensional acoustic wave equation yields

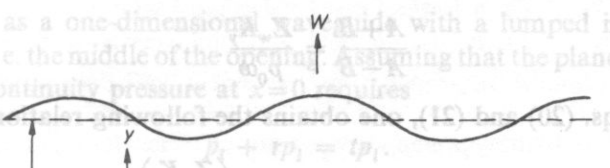

Fig. 4. Idealized model of panel absorber.

$$\frac{\partial^2 p(y)}{\partial y^2} + (K_0^2 - K_x^2)\, p(y) = 0. \tag{14}$$

The solution for $p(y)$ is

$$p(y) = A\, e^{-jK_y y} + B\, e^{+jK_y y}, \tag{15}$$

with

$$K_y^2 = K_0^2 - K_x^2. \tag{16}$$

The equation governing the vibratory motion of the plate is

$$(DK_x^4 - \omega m'')\, w = p(x,y)\,|_{y=h}\,, \tag{17}$$

where w is the plate displacement in the y-direction, D is the bending stiffness, and m'' is the mass per unit area of the plate.

The boundary conditions for the acoustic field are

$$\left.\frac{\partial p}{\partial y}\right|_{y=h} = \rho_0 \omega^2 w, \tag{18}$$

$$\left.\frac{p(x,y)}{u(x,y)}\right|_{y=h} = Z_w, \tag{19}$$

where ρ_0 is the density of the air and Z_w is the impedance of the lined wall. By using Eqs. (15) and (18), Eq. (17) becomes

$$D\,(K_x^4 - K_p^4)\left(-j\frac{K_y}{\rho_0 \omega^2}\right) = \frac{\dfrac{A+B}{A-B} - j\tan K_y h}{1 - j\dfrac{A+B}{A-B}\tan K_y h}, \tag{20}$$

where K_p is the in vacuo free flexural wavenumber of the plate. Furthermore, by using Eq. (15) the boundary condition of the lining expressed by Eq. (19) can be written as

$$\frac{A+B}{A-B} = \frac{Z_w K_y}{\rho_0 \omega}. \tag{21}$$

By combining Eqs. (20) and (21), one obtains the following relationship for K_y

$$D\,[(K_0^2 - K_y^2)^2 - K_p^4]\left(-j\frac{K_y}{\omega^2 \rho_0}\right) = \frac{\left(\dfrac{Z_w K_y}{\omega \rho_0}\right) - j\tan(K_y h)}{1 - j\left(\dfrac{Z_w K_y}{\omega \rho_0}\right)\tan(K_y h)}. \tag{22}$$

For the sake of simplicity and for the situation examined here (low frequencies and small height) it is reasonable to assume $K_y h \leqslant 1$ so that $\tan(K_y h) \approx K_y h$. By using the above approximation Eq. (22) becomes

$$\left(\frac{Z_w}{\omega \rho_0} - jh\right) = -j\frac{D}{\omega \rho_0}\,[(K_0^2 - K_y^2)^2 - K_p^4]\left[1 - j\,K_y^2\left(\frac{Z_w h}{\omega \rho_0}\right)\right]. \tag{23}$$

The above equation is of the third order in K_y^2 and thus can lead to three types of free motion. Two of them are closely related to the bending waves on the plate and the third to that of the air waves in the cavity. Equation (23) can be numerically solved for the transverse wave number K_y and by substitution in Eq. (16) the desired wavenumber K_x can be obtained. Finally, the impedance of the panel absorber can be expressed in terms of K_x as

$$Z = j\omega m'' - j\rho_0 c_0\left(\frac{K_x}{K_0}\right)\frac{\cos(K_x h)}{\sin(K_x h)}. \tag{24}$$

3. Transmission loss

3.1. Helmholtz resonator

The idealized model of the system under study is shown in Fig. 5. Since the resonator can be represented by an equivalent impedance given by Eq. (13) the duct

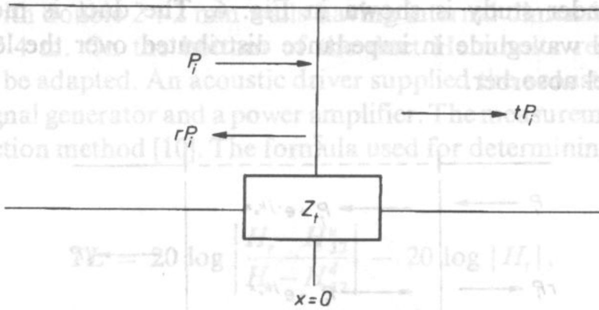

Fig. 5. Idealized model of duct-resonator.

can be modeled as a one-dimensional waveguide with a lumped impedance Z_t at a discrete point, i.e. the middle of the opening. Assuming that the plane wave condition is fulfilled, the continuity pressure at $x=0$ requires

$$p_i + rp_i = tp_i \,. \tag{25}$$

where p_i is incident pressure and r and t are the reflection and transmission coefficients, respectively. Furthermore, the continuity of mass flux on the opening $x=0$ yields

$$Su_n(x) = S_0 u_r(x) + Su_{n+1}(x), \tag{26}$$

where $u_n(x)$, $u_{n+1}(x)$ are the volume velocities before and after the opening, respectively, and $u_r(x)$ is the velocity at the opening. Equation (26) can be written in terms of the incident pressure as

$$S\frac{p_i - rp_i}{\rho_0 c_0} = \frac{tp_i}{Z_t} S_0 + S\frac{tp_i}{\rho_0 c_0}, \tag{27}$$

or alternatively

$$p_i - rp_i = tp_i \frac{\rho_0 c_0}{Z_t}\frac{S_0}{S} + tp_i \,. \tag{28}$$

Eqs. (25) and (28) can be easily solved for t (assuming $p_i = 1$) yielding

$$t = \frac{1}{1 + \dfrac{\rho_0 c_0}{Z_t}\dfrac{S_0}{S}} \,. \tag{29}$$

Finally, the transmission loss can expressed as

$$TL = 20 \log (1/t). \tag{30}$$

3.2. Panel absorber

The system under study is shown in Fig. 6. The duct is modeled again as a one-dimensional waveguide in impedance distributed over the length L, i.e. the length of the panel absorber.

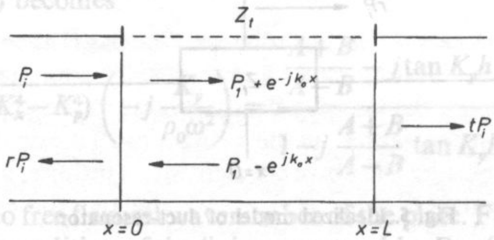

Fig. 6. Idealized model of duct-panel absorber.

By assuming plane wave propagation, the continuity of pressure at $x=0$ requires

$$p_i + rp_i = p_{1+} + p_{1-}\ e^{-jK_0L}, \tag{31}$$

while at $x=L$

$$p_{1+}\ e^{-jK_0L} + p_{1-} = tp_i, \tag{32}$$

where p_{1+} and p_{1-} are the complex amplitudes of the incident and reflected waves between $x=0$ and $x=L$. On the other hand, the continuity of mass flux at $x=0$ can be expressed as

$$\frac{p_i - rp_i}{\rho_0 c_0} = \frac{p_{1+} - p_{1-}\ e^{-jK_0L}}{Z_t}, \tag{33}$$

and at $x=0$ similarly

$$\frac{p_{1+}\ e^{-jK_0L} - p_{1-}}{Z_t} = \frac{tp_i}{\rho_0 c_0}. \tag{34}$$

The system of Eq. (31) through (34) can be easily solved for t and the transmission loss can be expressed as

$$TL = 20 \log (1/t) \tag{35}$$

4. Experimental results

In order to verify the theoretical predictions, a series of transmission loss measurements on Helmholtz resonators and panel absorbers was made. During the measurements the main parameters determining the geometry of the elements were systematically varied.

The experimental arrangement consisted of a square section steel duct (anechoically terminated) with double 2+2 mm walls having internal dimensions 500 × 500 mm and a length of 4 m. On the bottom of the duct Helmholtz resonators or panel absorbers could be adapted. An acoustic driver supplied the acoustic signal fed to the source from a signal generator and a power amplifier. The measurements were made by the transfer function method [10]. The formula used for determining the transmission loss was

$$TL = 20\log\left|\frac{H_r - H_{12}^u}{H_r - H_{12}^d}\right| - 20\log|H_t|, \quad (36)$$

where H_{12}^u and H_{12}^d are the transfer functions measured at the upstream and downstream directions respectively, $H_t = \left|\frac{S_{dd}}{S_{uu}}\right|^{1/2}$ with S_{dd}, S_{uu} denoting the autospectra at the downstream and upstream measurement locations. Finally, $H_r = e^{jK_0\lambda}$, where λ is the microphone spacing. To measure the transfer functions, four 1/4 in. condenser microphones, two at either side were used, the outputs of which were fed to a four-channel analyzer. The experimental arrangement is shown in Fig. 7.

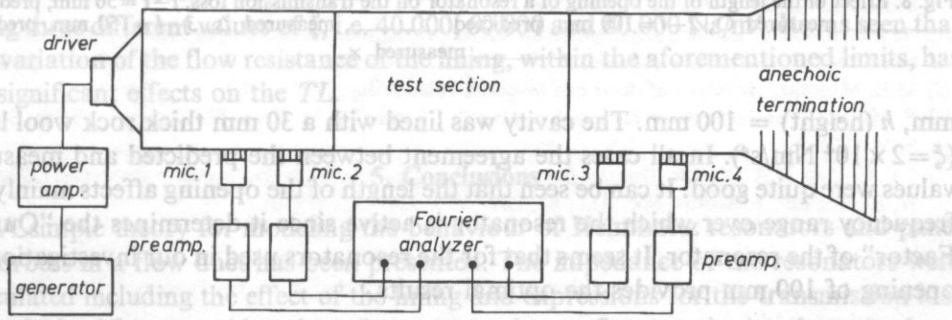

Fig. 7. Experimental arrangement.

The results are presented in form of curves of transmission loss in decibels plotted against frequency in Hz. The theoretical curves were calculated using the procedure described in the previous sections.

To investigate the effect of the main parameters involved on the transmission loss, a series of parametric studies were made, where the length of the opening in case of system. It seems that for the resonators used in our investigation, an opening of 100 mm provides the optimal results.

2. The variation of the flow resistance of the lining in both cases within the extreme values used (40.000 – 80.000 Ns/m⁴) has no significant effect on the results.

3. The mounting conditions of the panel in case of a panel absorber affects the *TL*. Consideration, however, of a finite panel results in a complicated analysis, too difficult to be used for design purposes.

The model developed can be easily extended to describe the sound transmission in ducts lined with an array of Helmholtz resonators or panel absorbers or a combination of both. After the derivation of the impedances for a lined resonator and panel absorber, a duct silencer can be treated as a one-dimensional waveguide with lumped impedances in discrete positions and an expression for the TL of the silencer can be obtained. These aspects will be discussed in a separate publication.

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