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A ROLE OF AIR IN COMPLEX ELASTIC MODULUS MEASUREMENTS OF SOLID SAMPLES BY TRANSMISSIBILITY METHOD

W. ZIÓŁKOWSKI AND A. ŚLIWIŃSKI

Institute of Physics, University of Gdańsk
(80-952 Gdańsk, ul. Wita Stwosza 57)

Anomalous dependence of visco-elastic parameters against frequency for solid polymers measured both in conditions of surrounding air as well as in vacuum were examined. In theoretical consideration the single system of one degree of freedom for representing a sample was assumed together with the presence of friction force introduced by the air. The values of E and η were measured in the air (10^5 Pa) and in the vacuum (10^2 Pa) conditions against frequency f from $20-2\times10^3$ Hz. The results show essential differences for both conditions. A comparison between numerical and experimental curves are presented. Anomalous behaviour of $E(f)$ and $\eta(f)$ against frequency in the air is essential for polymers of small values of Young modulus up to 10^6 N/m². The damping influence of the air having essential contribution in measurements of complex elastic modulus in light polymers must be taken into account when using the E, and η in situations the knowledge of exact values of those quantities is required, for example when mechanisms of internal friction in polymers are evaluated.

1. Introduction

The influence of the damping effect of air on measuring results in transmissibility method used for determination of visco-elastic properties of solids against frequency [1] is usually neglected. It is assumed that the formulae derived for the elastic modulus E and the loss factor η of a material sample treated as a dynamical system of a single degree of freedom and strictly valid for vacuum, only, are also correct in air atmosphere conditions. However, many of experimental results for complex elastic modulus obtained by the transmissibility method [2–5] show evident anomalies in the dynamic characteristics of measured materials. T. Pritz has stated in his paper [3] that the anomalies come from "the inherent error of the transmissibility method" not explaining any physical background for them. On the other hand, it might be suspected from the analysis of measuring data of E and η that the reason of appearing of the anomalies is related to the damping effect of surrounding air on the vibrating sample. To make sure about the hypothesis we have performed theoretical analysis as well as experimental verification of the role of the effect in the transmissibility method of visco-elastic properties measurements. This has been described in that paper.

Most of the authors [3 – 7] obtained their results by measuring the transmissibility T and the phase angle ϕ for different materials in the air but applying determination of them the formulae valid in vacuum, only, so their dynamical characteristics are significantly charged by the systematic error.

There are, also other sources of systematic errors in the method coming from damping of the glue used for cementing the specimen to the shaker, of the mass effect of an accelerometer [3 – 8] and of a cable, however these influences were neglected in the theoretical consideration given below. G.W. LAIRD and H.B. KINGSBURY [4] described how these errors can be eliminated. The glue effect may be neglected when its dynamical characteristics (E, η) are close to the ones of the polymer specimen being examined.

2. The complex transmissibility of a linear single degree of freedom system

A linear system of one degree of freedom is characterized by one resonant frequency and comprises of a single element of mass and one or few elements of stiffness and damping. Example of such single system may be a sample of a visco-elastic material of dimensions of much smaller than the corresponding to the vibration frequency wavelength in the material.

The Fig. 1 presents an element of mass supported by a visco-elastic sample laying on a foundation which vibrates sinusoidally with the frequency $f = \omega/2\pi$. It is assumed that the mass is supported at its center of gravity. The system is situated in the surrounding air which contributes an extraneous damping in its vibrations movement.

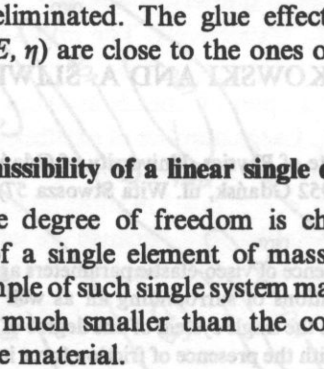


Fig. 1. The simple system representing the sample and loading mass.

This additional damping may be avoid by putting the system into the vacuum however it introduces in practice additional difficulties. Let F_0 represents the force of the air friction. For a small displacement of the mass M the force F_0 may be treated as a linear function of vibrational velocity, i.e. of the frequency as follows

$$F_0 = -\beta \dot{\tilde{x}}(t) = -\beta\omega \tilde{x}(t), \tag{1}$$

where β is the damping factor of the air and $\dot{\tilde{x}} = i\omega\tilde{x}$.

For greater vibrational velocities (but still less than the velocity of sound in the air) the force F_0 is a function of $\dot{x}^2$, however we shall consider only the linear case (1), here.

The equation of motion for the system may be written as

$$M\frac{d^2\tilde{x}_2(t)}{dt^2} = KE^*(\tilde{x}_1-\tilde{x}_2)-\beta(\dot{\tilde{x}}_1-\dot{\tilde{x}}_2) = KE^*\tilde{x}_1-KE^*\tilde{x}_2+\beta\dot{\tilde{x}}_2-\beta\dot{\tilde{x}}_1, \tag{2}$$

where the constant K is determined by the formula [6]:

$$K = (3A/L)\,(1+bs^2), \tag{3}$$

b is a numerical constant and for a rubber-like samples is equal 2, S is equal to the ratio of the loaded surface to the total force-free area. This is so called a shape factor and for rubber-like materials, for example for a cylinder of the diameter D and the height L the S is equal to $D/4L$.

The complex modulus of elasticity E^* is defined by

$$E^* = E(1+i\eta), \tag{4}$$

where E is the dynamical Young modulus and η is the loss factor of the material.

For the sinusoidal displacement of the foundation

$$\tilde{x}_1(t) = x_1^* e^{i\omega t} = x_1 e^{i(\omega t+\phi_1)}, \tag{5}$$

the resulting displacement of the mass M is equal

$$\tilde{x}_2(t) = x_2^* e^{i(\omega t)} = x_2 e^{i(\omega t+\phi_2)} \tag{6}$$

x_1 and x_2 are amplitudes of the displacements,
$\phi=\phi_2-\phi_1$ — the phase shift between them, respectively.

From the equations (2), (5) and (6) one gets

$$\frac{x_2^*}{x_1^*} = \frac{KE^*-i\omega\beta}{-M\omega^2+KE^*-i\omega\beta} = \frac{1+i\left(\eta-\dfrac{\beta\omega}{KE}\right)}{\left(1-\dfrac{M\omega^2}{KE}\right)+i\left(\eta-\dfrac{\omega\beta}{KE}\right)} \tag{7}$$

and after some calculations one can derive for the transmissibility T and the phase angle ϕ, respectively

$$T = \left|\frac{x_2^*}{x_1^*}\right| = \frac{\left[1+\left(\eta-\dfrac{\beta}{KE}\omega\right)^2\right]^{1/2}}{\left[\left(1-\dfrac{M\omega^2}{KE}\right)^2+\left(\eta-\dfrac{\omega\beta}{KE}\right)^2\right]^{1/2}} \tag{8}$$

and

$$\operatorname{tg}\phi = \frac{-\frac{M\omega^2}{KE}\left[\eta - \frac{\omega\beta}{KE}\right]}{\left(1 - \frac{M\omega^2}{KE}\right) + \left(\eta - \frac{\omega\beta}{KE}\right)^2} \tag{9}$$

3. Determination of E and β

For the resonance of the simple system in vacuum the angular frequency $\omega = \omega_0$ and it is given by the relation

$$\omega_0^2 = \frac{KE_0}{M} \tag{10}$$

where E_0 is the Young modulus of the sample.

Introducing (10) into the equation (2) and (8) one gets

$$T^2 = \frac{1 + \left(\eta - \frac{\beta}{KE}\omega_0\right)^2}{\left(\eta - \frac{\beta}{KE_0}\omega_0\right)^2} \tag{11}$$

and after the transformation

$$\beta^2 - \frac{2KE_0\eta}{\omega_0}\beta + \frac{\eta^2K^2E_0^2}{\omega_0^2} - \frac{K^2E_0^2}{\omega_0^2(T^2-1)} = D \tag{12}$$

This square equation for β may be easily solved. On the other hand when the measurement of T and ω_0 are performed in vacuum $\beta = 0$ and then η and E may be easy calculated for the resonance frequency ω_0. Next the β value is determined using (12). The evaluated value for the case in the air was obtained as $\beta = 1800$ g/s and this value will be used to the numerical calculations described in the chapter 5.

4. Data for the numerical analyses

The formulae (8) and (9) were used for the numerical calculations of the transmissibility and the phase, respectively of a sample presenting the system of the single degree of freedom.

Different samples of polyurethane material were taken for measurements and for numerical analyses. The samples had the following geometry: the heights were $h = 0.01$ m or 0.02 m and the square cross-section of $A^2 = 10^{-4}$ m².

The preliminary values of the Young modulus E_0 and the loss factor η for the samples were determined in vacuum ($\beta = 0$, according to (10) and (11)) at resonances assuming they have presented single degree of freedom systems.

The samples were numbered from 1 – 4 and all data for them are collected in the Table 1.

Table 1. The data of the samples

Number	Height h [m]	Young's modulus $E\times10^{-7}$ [N/m²]	Loss factor η	Mass $m\times10^{3}$ [kg]	Mass on the sample $M\times10^{3}$ [kg]
1	0.019	1	0.35	1.14	22
2	0.02	0.133	0.6	1.05	22
3	0.01	0.154	0.59	0.56	45
4	0.01	0.197	0.78	0.56	22

5. Numerical results

The results of calculations are presented in the following Figures from 2a,b – 5a,b. In every Figure two curves are presented one for the vacuum ($\beta=0$) and the other for the air ($\beta=1800$ g/s) conditions; Figures (a) correspond to characteristics of the transmissibility T and (b) to characteristics of the phase.

In Fig. 2a the results of T for the sample 1 are given. It can be seen that the ratio $\Delta T/T(\beta=0)$ where

$$\Delta T = T(\beta=1800)-T(\beta=0),$$

has its maximum value at the resonance frequency. Also, one can deduce that in the case the loss factor η would be calculated from the formula (11) for $\beta=0$ and in air, then the value of η had been smaller than the real one.

The Fig. 2 b presents the dependence of the phase angle ϕ against frequency for $\beta=0$ and $\beta=1800$ g/s (according to Eq. (9)). The influence of β on ϕ is evident when one have compared the two curves: $\phi(\beta=0)$ and $\phi(\beta=1800)$. The phase difference $\Delta\phi=\phi(\beta=1800)-\phi(\beta=0)$ increases against the frequency to achieve a maximum just before the resonance frequency. After a rapid drop of the phase difference down to the opposite sign in the region of the resonant frequency, the other extremum is achieved and next the gradual (decrease of the curve in the Figure) increase of the absolute value of the phase difference takes place.

It means that the Young modulus E as well as the loss factor η will be influenced by the air conditions when they are determined using Eqs. (8) and (9) valid for vacuum. So, in practice the error due to the neglecting of the air influence increases against the frequency, too.

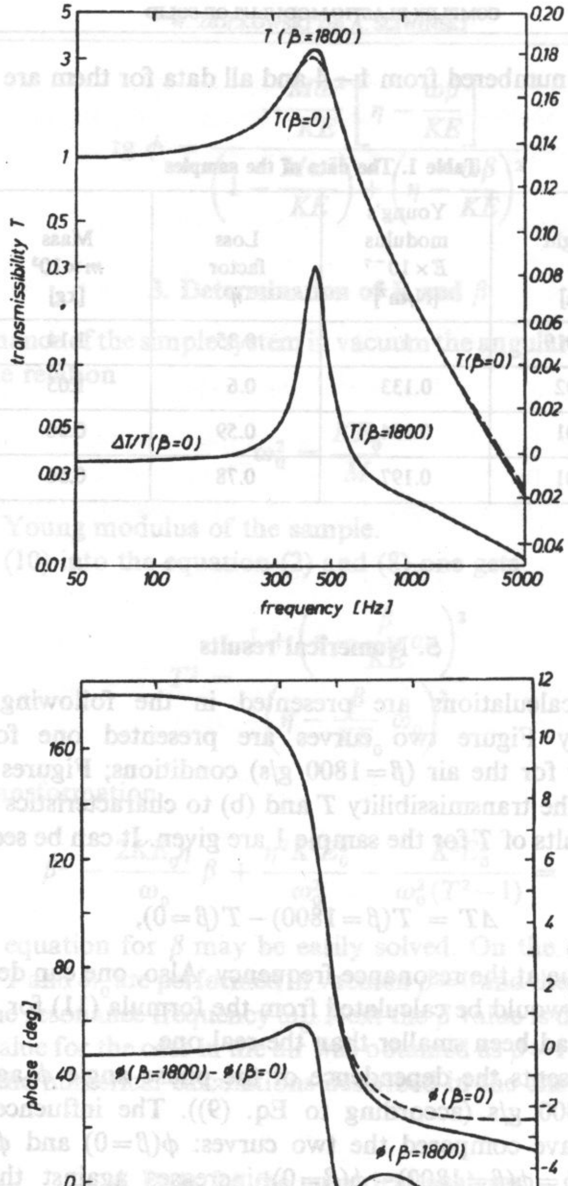


Fig. 2. Frequency dependence of (a) the transmissibility and (b) the phase angle for the sample 1.

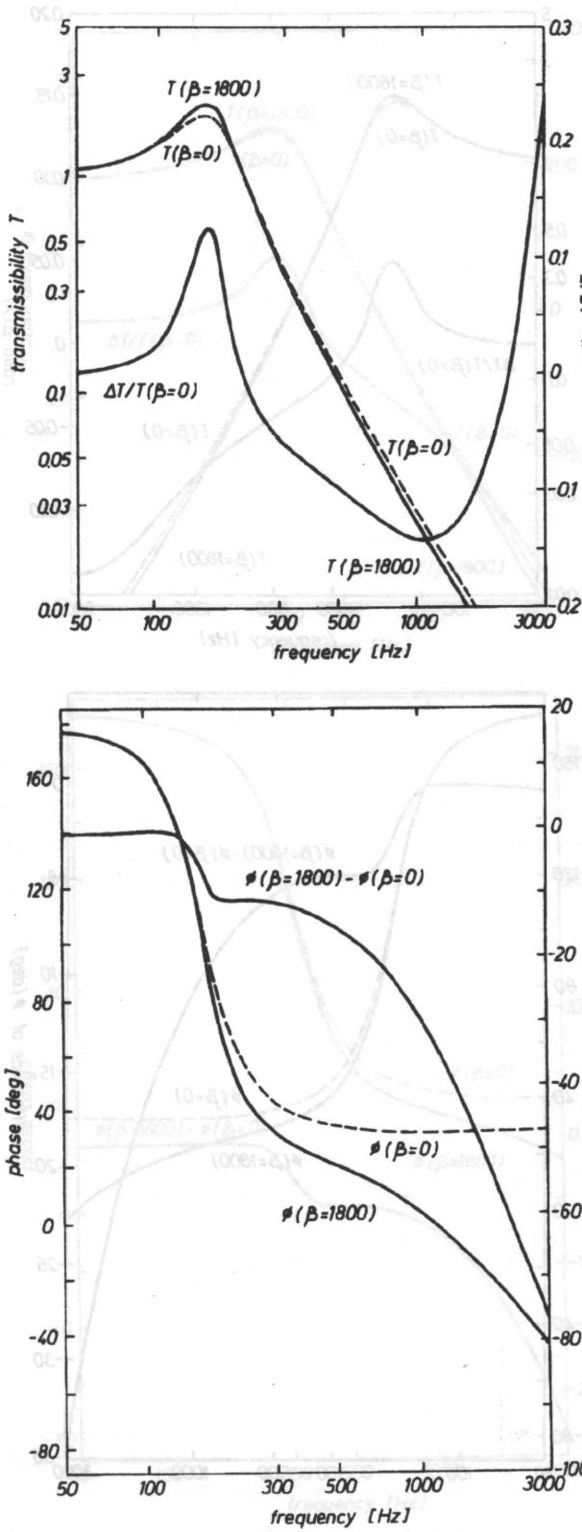
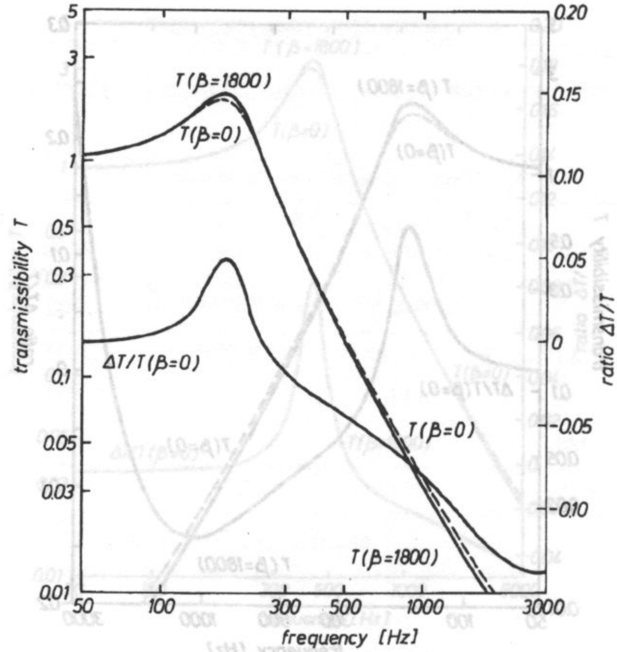


Fig. 3. Frequency dependence of (a) the transmissibility and (b) the phase angle for the sample 2.



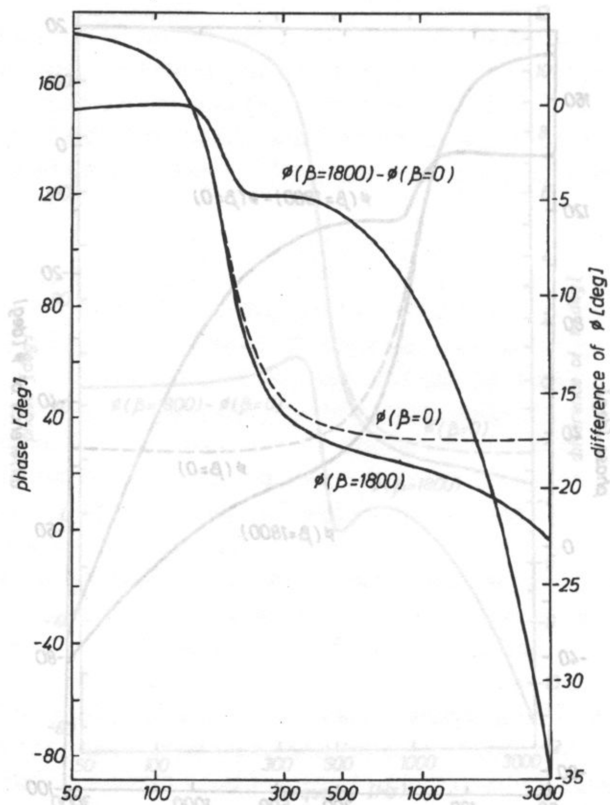


Fig. 4. Frequency dependence of (a) the transmissibility and (b) the phase angle for the sample 3.

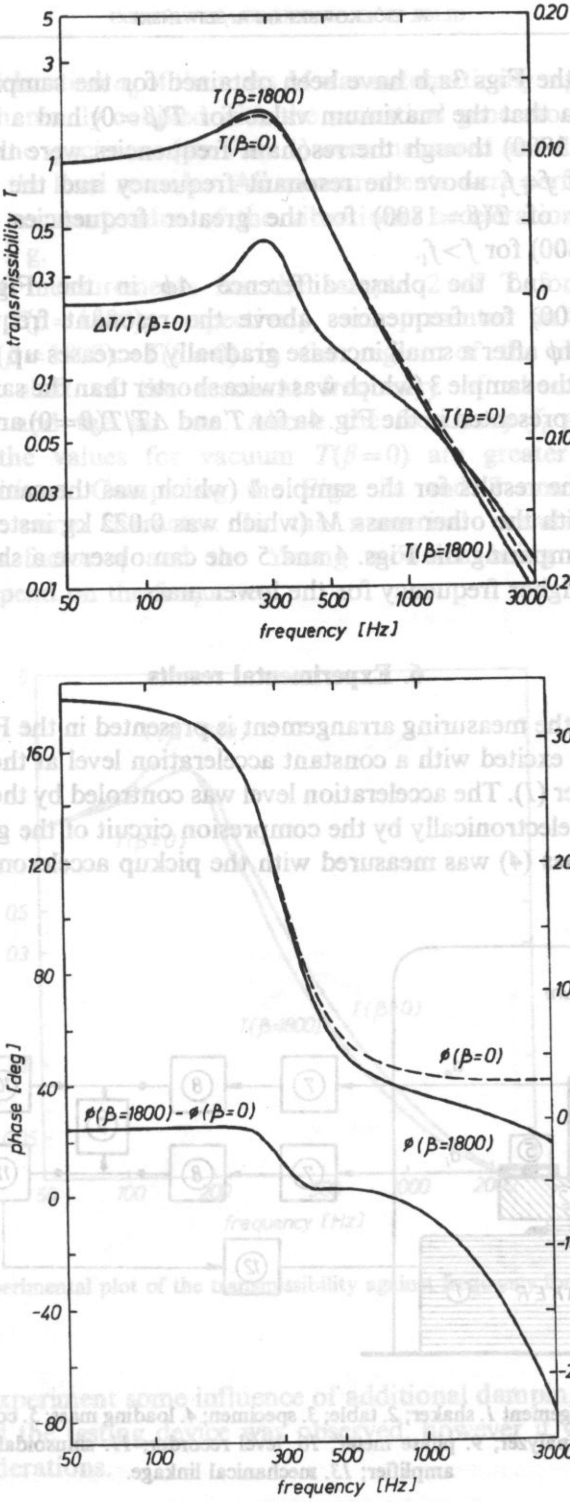


Fig. 5. Frequency dependence of (a) T, $\Delta T/T$ and (b) ϕ, $\Delta\phi$ for the sample 4.

The curves in the Figs 3a,b have been obtained for the sample 2. It is evidently seen in the Fig. 3a that the maximum value for $T(\beta=0)$ had a smaller value than the one for $T(\beta=1800)$ though the resonant frequencies were the same. The both curves intersect at $f=f_i$ above the resonant frequency and the curve for $T(\beta=0)$ lies over the one of $T(\beta=1800)$ for the greater frequencies i.e. the values of $T(\beta=0)>T(\beta=1800)$ for $f>f_i$.

The phase ϕ and the phase difference $\Delta\phi$ in the Fig. 3b, show that $\phi(\beta=0)>\phi(\beta=1800)$ for frequencies above the resonant frequency. Above the resonance region $\Delta\phi$ after a small increase gradually decreases up to the frequency f_i.

The results for the sample 3 (which was twice shorter than the sample 2 but with the some material) are presented in the Fig. 4a for T and $\Delta T/T(\beta=0)$ and in the Fig. 4b for ϕ and $\Delta\phi$.

In the Fig. 5 the results for the sample 4 (which was the same as the sample 3, however loaded with the other mass M (which was 0.022 kg instead of the 0.045 kg — see Tab. 1). Comparing the Figs. 4 and 5 one can observe a shift of the resonant frequency to the higher frequency for the lower mass.

6. Experimental results

The scheme of the measuring arrangement is presented in the Fig. 6. The samples were harmonically excited with a constant acceleration level at the shaking table (*2*) driven by the shaker (*1*). The acceleration level was controled by the accelerometer (*5*) and kept constant electronically by the compresion circuit of the generator (*11*). The response of the mass (*4*) was measured with the pickup accelerometer (*6*).

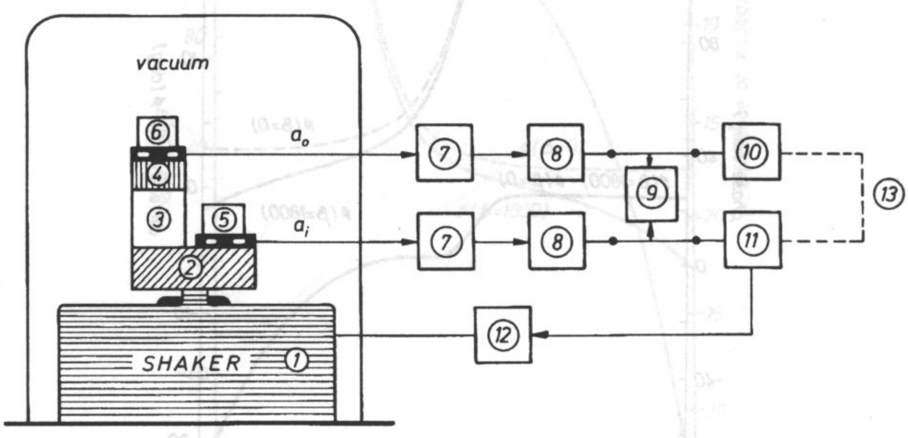


Fig. 6. Measuring arrangement *1*. shaker; *2*. table; *3*. specimen; *4*. loading mass; *5*. control accelerometer; *6*. pickup accelerometer analyzer; *9*. phase meter; *10*. level recorder; *11*. sinusoidal generator; *12*. power amplifier; *13*. mechanical linkage.

The output acceleration a_0 of the mass M was automatically registed with the level recorder (*10*) mechanically coupled with the acoustical generator (*11*).

The polyurethane specimens (see Tab. 1) were measured and the transmissibilities were registed with the level recorder. All measurements were performed at the room temperature. The constant value of the vibrational acceleration amplitude of the shaking table $a_i = 1$ g.

The results of measurements for the sample 2 of T for vacuum $T(\beta=0)$ and in the air $T(\beta=1800)$, respectively, are presented in the Fig. 7. One can see, that $T(\beta=1800) > T(\beta=0)$ in the region of the resonant frequency f_r. There was a shift of the resonant frequency of about 12 Hz between the vacuum case and the air one. Above the frequency f_i at which the both curves intersect the values for vacuum $T(\beta=0)$ are greater than for the air $T(\beta=1800)$ conditions. Comparing the Figs. 3a and 7 one can say that the curves have the same character. In the numerical calculations, there were assumed the loss factor η and the Young modulus being constant, however the real values depend on the frequency.

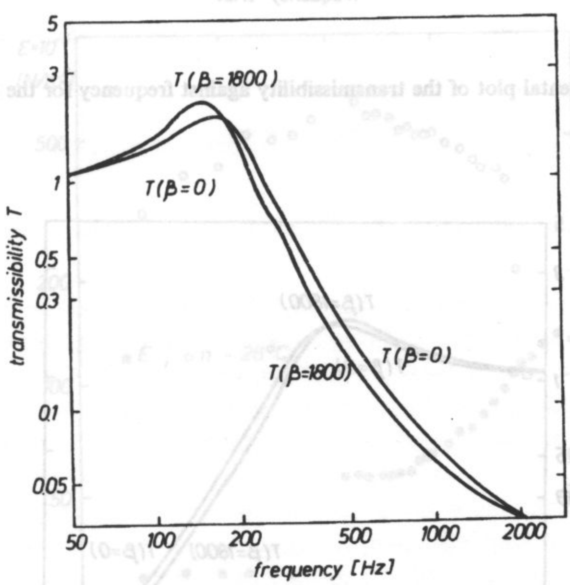


Fig. 7. Experimental plot of the transmissibility against frequency for the sample 2.

Also, in the experiment some influence of additional damping of the glue joining the specimen and the testing device was observed, however it was neglected in the theoretical considerations.

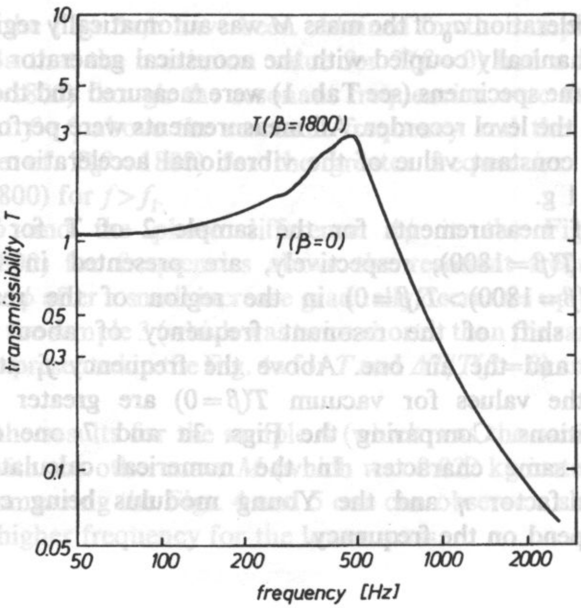


Fig. 8. Experimental plot of the transmissibility against frequency for the sample 1.

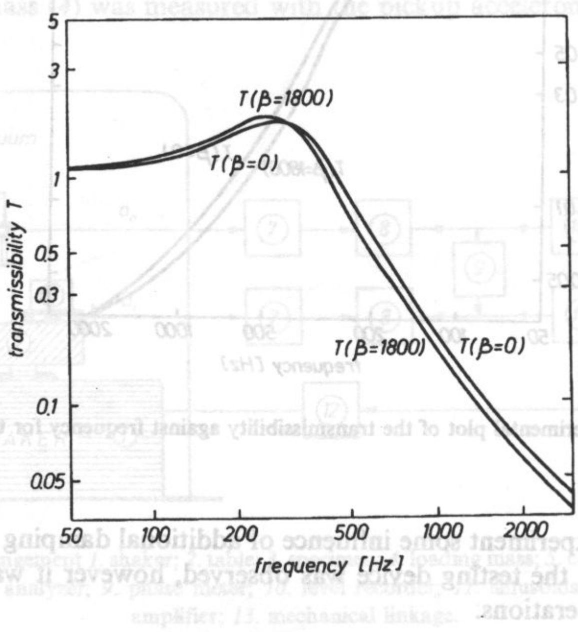


Fig. 9. Experimental plot of the transmissibility against frequency for the sample 4.

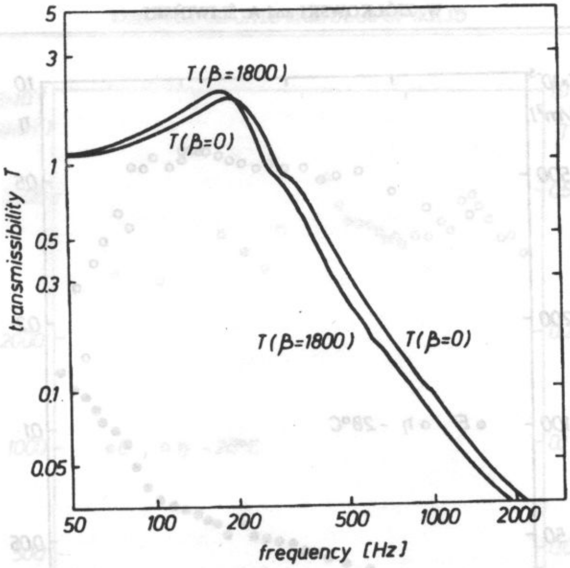


Fig. 10. Experimental plot of the transmissibility against frequency for the sample 3.

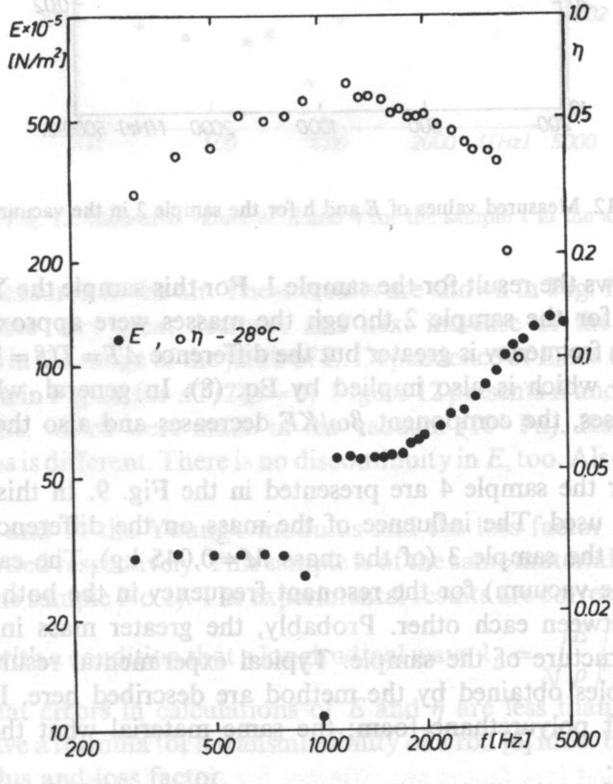


Fig. 11. Dynamic Young's modulus and loss factor of the soft polyurethane from (the same material as the sample 2) plotted against frequency.

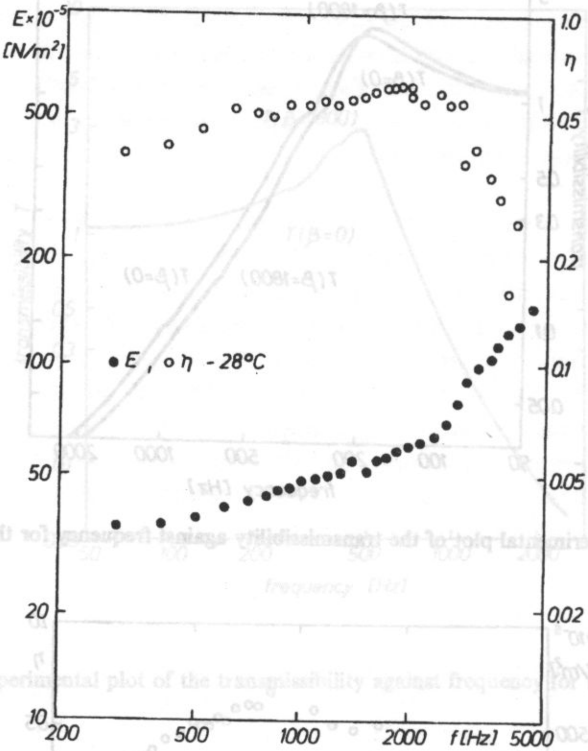


Fig. 12. Measured values of E and h for the sample 2 in the vacuum.

The Fig. 8 shows the result for the sample 1. For this sample the Young modulus was greater than for the sample 2 though the masses were approximatelly equal. Now, the resonant frequency is greater but the difference $\Delta T = T(\beta = 1800) - T(\beta = 0)$ is practically zero which is also implied by Eq. (8). In general, when the Young modulus E increases, the component $\beta\omega/KE$ decreases and also the difference ΔT decreases.

The results for the sample 4 are presented in the Fig. 9. In this case the mass $M = 0.022$ kg was used. The influence of the mass on the difference ΔT is shown in the Fig. 10 for the sample 3 (of the mass $M = 0.045$ kg). The calculated values for η and E (in the vacuum) for the resonant frequency in the both samples 3 and 4 are different between each other. Probably, the greater mass introduced some changes in the structure of the sample. Typical experimental results E and η for the analysed samples obtained by the method are described here. It was used the sample of the soft polyurethane foam; the same material what the specimens 1, 3 and 4 was made.

The dynamic Young's modulus E and the loss factor η were calculated from the system of the equations (8) and (9). In calculations β were neglected, whereas the values

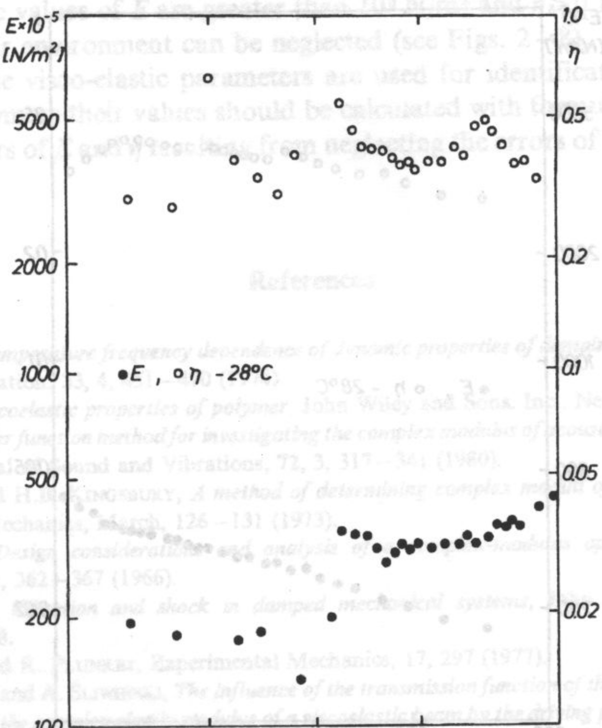


Fig. 13. Measured values of E and η for the sample 1 in the air.

T and ϕ were measured in the air. These results are shown in Fig. 11 for temperature $T_1 = 28°C$. We see very great decrease and next increase of the value E and the maximum value in the range of the jumb of E. Dependence of E has the same character as have been seen in Fig. 2a for $\Delta T/T(\beta=0)$. Figure 12 presents E and η values obtained by measurements, which were made in the vacuum (10^2 Pa). Here the plot of the Youngs modulus is different. There is no discontinuity in E, too. Also, the values E and η are smaller.

At Figs. 13 and 14 the Young's modulus and the loss factor in the air and the vacuum, are plotted respectively. This sample is of the same material as used to the plot of T at Fig. 8 (the sample No 1). The experimental results are correct up to about 2000 Hz; this agrees with a condition that a longitudinal wave $\lambda_L = \sqrt{\frac{E}{\rho}}\Big/ f$ is less than $h/8$. It is equivalent that errors in calculations of E and η are less than 10%. Above this longitudinal wave a formula for a transmissibility of a rod [6] must be used to calculate Young's modulus and loss factor.

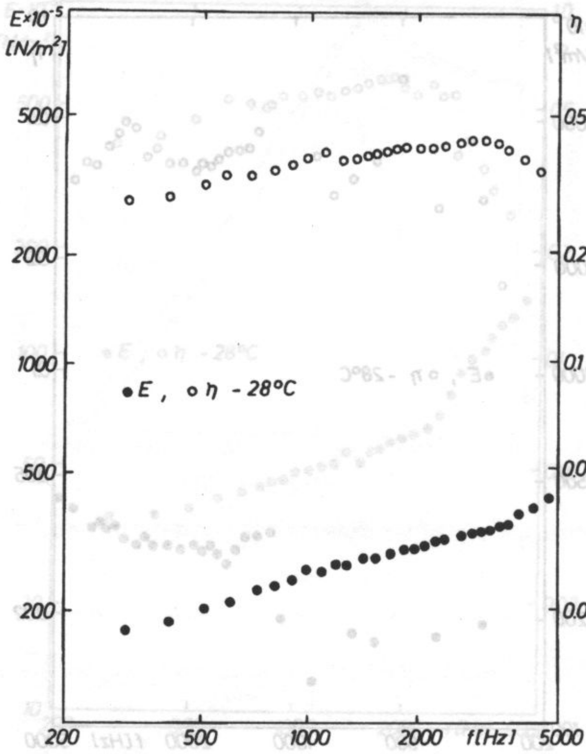


Fig. 14. Frequency dependence of E and h for the vacuum for the sample 1.

7. Conclusions

Theoretical considerations, numerical analysis and some experimental examination leads to the following conclusions:

1. The equations (8) and (9) show that for the determined values of η, E and ω characterizing the visco-elastic properties, the influence of the air damping can be essential or nonessential contribution to the calculation of complex transmissiblity; the situation depends on those values.

2. The essential influence of the air damping on the measuring of the complex transmissibility is observed for samples of small densities, $\rho < 6\ 10^2$ kg/m³, Young modulus 10^6 N/m² and loss factor equal 0.6. It is seen from the Figs 3a–5a presenting numerical curves as well as from experimental transmittance dependences for normal conditions and for vacuum (10^2 Pa) that the measured values of Young modulus and loss factor are different for this different conditions (see Figs. 11–14).

3. In case the values of E are greater than 10^8 N/m^2 and $\eta > 0.1$ the contribution from losses of air environment can be neglected (see Figs. 2–8).

4. In case the visco-elastic parameters are used for identification of relaxation processes in polymers their values should be calculated with formulae (8) and (9). In practice the errors of E and η resulting from neglecting the errors of air friction should be determined.

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