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NOISE GENERATED BY A MOVING LINE SOURCE IN A DISSIPATIVE ATMOSPHERE

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It is the purpose of this paper to present the resultant effect of geometrical spreading, air absorption, Doppler frequency shift and the distortion of the sound field due to the motion of a line source. It is assumed that the source is moving slowly along a straight line. The results obtained can be used for prediction of the *A*– weighed sound pressure level of railway noise.

Introduction

The sound field of any moving source is affected by the Doppler effect and convection effect. To give more accurate description, one has to take into account air absorption as well.

This paper will attempt to describe the combined effect of all these phenomena for a continuous line of dipole sources.

Well-known relations for a motionless line source have been used as the starting point (Section 1). Under the assumption of a low Mach number, explicit functions for mean-square sound pressure and its spectral density have been derived in Section 2, for both nondissipative and dissipative mediuma.

The results presented in this paper can be employed for railway noise prediction.

1. Source at rest

Some sources of noise, among them train noise, can be modeled by a continuous line of incoherent dipole sources. Hence the spectral density of the mean square sound pressure produced by a unit length is

$$\tilde{p}^2 = \frac{N(f)\cos^2\phi\rho c}{r^2}\exp\{-2\alpha(f)r\}, \tag{1}$$

where the characteristic impedance of air, ρc, equals 415 rayls (speed of sound

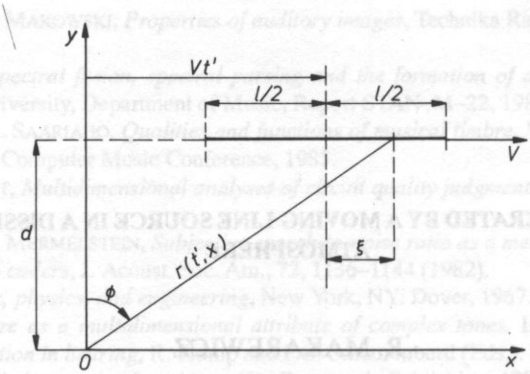

FIG. 1. Geometry of the moving line source with respect to the point of observation 0.

$c = 343$ m/s, air density $\rho = 1.21$ kg/m^3). The distance r and the angle ϕ are defined in Fig. 1.

Spectral characteristic

$N(f)$ in the above equation expresses the spectral characteristic of a source. There are many real sources for which $N(f)$ has one absolute maximum. Such a spectrum can be approximated by the function

$$N(f) = N^{(0)} f^m \exp(-\mu f), \quad m \geq 0. \tag{2}$$

The integration of this equation from $f = 0$ to $f = \infty$ gives (Ref. [1], integral 3.351.3):

$$N = N^{(0)} m! \, \mu^{-(m+1)}. \tag{3}$$

The spectrum described by Eq. (2) peaks at $f_{\max} = m/\mu$ and its maximal value is

$$N(f_{\max}) = \frac{N^{(0)} \mu}{m!} \left(\frac{m}{e} \right)^m \tag{4}$$

The set of sources of the same value of N and with $m = 0, 1, 2, \ldots$ can be described by the following equation, as illustrated in Fig. 2:

$$N(f) = \frac{N}{m!} \mu^{m+1} f^m \exp(-\mu f). \tag{5}$$

For a large value of μ low frequencies predominate. If the parameter μ decreases, then $N(f_{\max})$ (Eq. 4) declines and $f_{\max} = m/\mu$ shifts toward high frequencies.

The results of measurements of the sound pressure level with banpass filtres can be converted to the spectrum level. Then, making use of regression analysis, the numerical values of the parameters $N^{(0)}$, m and μ can be found. For example, the $A-$ frequency weighed railroad noise can be characterized by $N^{(0)} \approx 10^{-12}$, $m = 3$, $\mu = 2 \cdot 10^{-3}$.

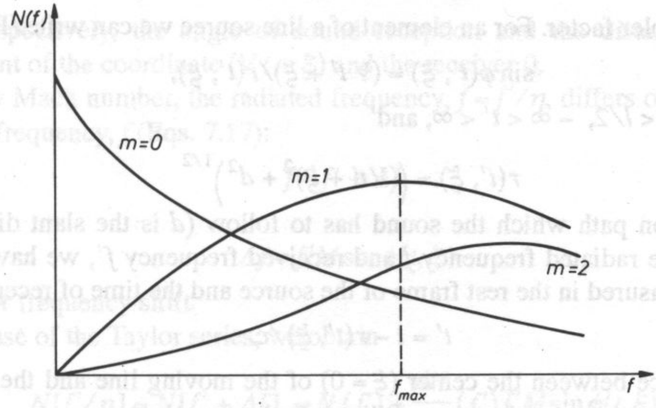

FIG. 2. Spectral characteristic of source (Eq. 2).

Atmospheric absorption

The absorption coefficient $\alpha(f)$ (Eq. 1) depends strongly on the frequency (f), and relative humidity (h_r), and less strongly on temperature (T). It also depends slightly on the ambient pressure. The explicit form of the function $\alpha(f, h_r, t)$ is rather complicated [2]. Under some conditions [3] the absorption coefficient α can be approximated by a linear function of frequency

$$\alpha(f, h_r, T) = \beta(h_r, T)f, \tag{6}$$

where

$$\beta = \alpha(f^*_{\max}, h_r, T)/f^*_{\max}.$$

Frequency f^* depends on the power spectrum of the source [3]. The numerical values of $\alpha(f^*, h_r, T)$ are given in international standards such as ANSI S1.26 or ISO 3891–1978 E.

2. Moving source

In this paper we restrict our attention to a line source of length l which moves with constant speed, V much smaller than the speed of sound c (Mach number $M = V/c << 1$), along a straight line at slant distance $d >> l/2$ (Fig. 1).

Due to the motion of the dipole, its radiation field, i.e., the spectral density $p^2(f')$ and the received frequency, f', become different from that produced when the source is at rest, $\tilde{p}^2(f)$, and f (Eq. 1). The convection and the Doppler effects are given by [4,5]:

$$\tilde{p}^2 = p^2\eta^4, \; f' = f\eta, \tag{7}$$

where

$$\eta(t', \xi) = (1 + M\sin\phi(t', \xi))^{-1} \tag{8}$$

denotes the Doppler factor. For an element of a line source we can write (Fig. 1)

$$\sin\phi(t',\xi) = (Vt' + \xi)/r(t',\xi), \tag{9}$$

where $-l/2 < \xi < l/2$, $-\infty < t' < \infty$, and

$$r(t',\xi) = \left((Vt' + \xi)^2 + d^2\right)^{1/2} \tag{10}$$

is the propagation path which the sound has to follow (d is the slant distance). Corresponding to the radiated frequency f and received frequency f', we have the time of emission (t') measured in the rest frame of the source and the time of reception (t) :

$$t' = t - r(t',\xi)/c. \tag{11}$$

Vt' is the distance between the center ($\xi = 0$) of the moving line and the y – axis (the line from the observer normal to the rectilinear path of motion).

From Eqs. (10) and (11), we get

$$Vt' = \left\{ Vt - M^2\xi - M\left[(Vt)^2 + (1-M^2)d^2 + (ct + M\xi)^2 - (ct)^2\right]^{1/2} \right\}/(1-M^2). \tag{12}$$

2.A. Nondissipative medium

A.1. Spectral density of mean square sound pressure. For a nondissipative medium we set $\alpha = 0$ and Eqs. (1,7) yield the spectral density of mean square sound pressure produced by a unit length:

$$p^2(f',\xi) = \frac{N(f'/\eta(\xi))\,d^2\rho c}{r^4(\xi)}\,\eta^4(\xi). \tag{13}$$

The total spectral density of mean square sound pressure is obtained by integration over the length of the line source (Eq. 13):

$$p^2(f') = d^2\rho c \int_{-l/2}^{l/2} \frac{N(f'/\eta(t',\xi))\,\eta^4(t',\xi)}{r^4(t',\xi)}\,d\xi. \tag{14}$$

where $\eta(t',x)$ and $r(t',\xi)$ are defined as above (Eqs. 8–10).

To get some insight into the physics of sound generation and propagation, we approximate the integral (14) under the assumption of a low Mach number: $M << 1$ and $l/2 << d$. Thus we can write (Eqs. 8–10, 12).

$$Vt' = Vt - Mr(t,0), \tag{15}$$

$$r(t',\xi) = r(t,\xi)\left[1 - M\sin\phi(t,\xi)\frac{r(t,0)}{r(t,\xi)}\right], \tag{16}$$

$$\eta(t',\xi) = 1 - M\sin\phi(t,\xi), \tag{17}$$

where

$$\sin\phi(t,\xi) = \frac{Vt + \xi}{r(t,\xi)} \quad \text{and} \quad r(t,\xi) = \left[(Vt+\xi)^2 + d^2\right]^{1/2} \tag{18}$$

represent, respectively, the angle of sound reception and the distance between the source element of the coordinate $(V t + \xi)$ and the receiver 0.

For a low Mach number, the radiated frequency, $f = f'/\eta$, differs only slightly from the received frequency, f (Eqs. 7.17):

$$f \approx f' + \Delta f, \tag{19}$$

where

$$\Delta f = f' M \sin \phi(t, \xi)$$

is the Doppler frequency shift.

Making use of the Taylor series, we obtain

$$N[f'/\eta] = N[f' + \Delta f] \approx N(f') + \frac{dN}{df'}(f') f' M \sin \phi(t, \xi). \tag{20}$$

Finally the expressions (14–17, 20) give the total spectral density of mean square sound pressure:

$$p^2(f', t) = p_S^2(f', t) + p_D^2(f', t) + p_C^2(f', t), \tag{21}$$

where

$$p_S^2(f', t) = d^2 \rho c N(f') \int_{-l/2}^{l/2} \frac{d\xi}{r^4(t, \xi)} \tag{22}$$

corresponds to a “quasi-stationary” source,

$$p_D^2\,(f', t) = M d^2 \rho c \frac{dN}{df'}(f') f' \int_{-l/2}^{l/2} \frac{\sin \phi(t, \xi)}{r^4(t, \xi)} d\xi \tag{23}$$

describes the influence of the Doppler effect, and

$$p_C^2\,(f', t) = 4 M d^2 \rho c N(f') \int_{-l/2}^{l/2} [\frac{r(t, 0)}{r\,(t, \xi)} - 1\,] \frac{\sin \phi(t, \xi)}{r^4(t, \xi)} d\xi, \tag{24}$$

can be related to the convection effect.

The influence of both effects (p_D^2, p_C^2) is proportional to the Mach number. However, we have assumed $M << 1$; therefore the quasi-stationary source $({p_S}^2)$ tends to dominate.

Making use of a table of integrals [1], we get the formulas (Eqs. 18, 22–24):

$$p_S^2(f', t) = \frac{1}{2} \frac{\rho c}{d} N(f') F_S(t), \tag{25}$$

$$p_D^2(f', t) = \frac{1}{3} M \frac{\rho c}{d} f' \frac{dN}{df'}(f')\, F_D(t), \tag{26}$$

$$p_C^2(f', t) = \frac{1}{3} M \frac{\rho c}{d} N(f') F_C(t), \tag{27}$$

with the time functions

$$F_S(t) = \sin\phi_2 \cos\phi_2 - \sin\phi_1 \cos\phi_1 + \phi_2 - \phi_1, \tag{28}$$

$$F_D(t) = \cos^3\phi_1 - \cos^3\phi_2, \tag{29}$$

$$F_C(t) = 3\,[1 + (Vt/d)^2]^{1/2}(\cos^4\phi_1 - \cos^4\phi_2) - 4(\cos^3\phi_1 - \cos^3\phi_2), \tag{30}$$

where the angles ϕ_2 and ϕ_1 are associated with the leading and trailing edge of the line source:

$$\sin\phi_2 = \frac{Vt + l/2}{\left((Vt + l/2)^2 + d^2\right)^{1/2}}, \quad \sin\phi_1 = \frac{Vt - l/2}{\left((Vt - l/2)^2 + d^2\right)^{1/2}}, \tag{31}$$

$$\cos\phi_2 = \frac{d}{\left((Vt + l/2)^2 + d^2\right)^{1/2}}, \quad \cos\phi_1 = \frac{d}{\left((Vt - l/2)^2 + d^2\right)^{1/2}}.$$

At $t = 0$, when the center of the line source ($\xi = 0$) is opposite to the point of observation, the above expressions yield: $p_D^2(f, 0) = 0$ and $p_C^2(f, 0) = 0$. Hence the contributions of the Doppler and convection effects vanish at the moment of passing by.

A.2. Time history of mean square sound pressure. The relation between the mean square sound pressure and its spectral density is

$$p^2(t) = \int_0^\infty p^2(f', t)\, df', \tag{32}$$

where f' is the observed frequency.

Thus (Eqs. 21, 25–27, 32)

$$p^2(t) = p_S^2(t) + p_D^2(t) + p_C^2(t), \tag{33}$$

where the contributions of the "quasi-stationary" source, Doppler effects, and the convection effect are given by

$$p_S^2(t) = \frac{N\rho c F_S(t)}{2d}, \tag{34}$$

$$p_D^2(t) = M\frac{P\rho c F_D(t)}{3d}, \tag{35}$$

$$p_C^2(t) = M\frac{N\rho c F_C(t)}{3d}. \tag{36}$$

Here, parameters N and P are related to the power spectral density of a line source:

$$N = \int_0^\infty N(f')\, df', \quad P = \int_0^\infty \frac{dN}{df'}(f') f'\, df'. \tag{37}$$

Assuming that $N(f')$ has the form given by Eq. (2), we obtain N from Eq. (3) and $P = -N$ [1]. Thus Eqs. (33–36) combine to yield

$$p^2(t) = \frac{N\rho c}{2d}\left\{F_S(t) - \frac{2}{3}M[F_D(t) - F_C(t)]\right\}, \tag{38}$$

with $F_S(t)$, $F_D(t)$, and $F_C(t)$ defined by Eqs. (28–31).

Equation (38) described the time variations of the mean-square sound pressure for a line source in motion.

Expression (28) shows that the function $F_S(t)$ is even: $F_S(-t) = F_S(t)$. In contrast $F_D(-t) = -F_D(t)$ and $F_C(-t) = -F_C(t)$, i.e., both functions are odd. Thus, introducing the measure of $p^2(t)$ asymmetry (Fig. 3)

$$\Delta p^2 = p^2(t^*) - p^2(-t^*),$$

one gets (Eq. 38):

$$\Delta p^2(t^*) = -\frac{2N\rho c}{3d} M[F_D(t^*) - F_C(t^*)]. \tag{39}$$

It follows that the asymmetry of the mean square sound pressure is proportional to the Mach number and decreases with the slant distance (d),

2B. *Dissipative medium.*

B.1. Spectral density of mean square sound pressure. Including air absorption, one obtains more general expressions for the mean square sound pressure, p^2, and its spectral density, $p^2(f)$.

From Eqs. (1,14) and Fig. 1, we get

$$p^2(f') = d^2\rho c\int_{-l/2}^{l/2} N\frac{[f'/\eta]\,\eta^4}{r^4(t',\xi)}\exp(-2\alpha(f')r(t',\xi))\,d\xi, \tag{40}$$

where $r(t', \xi)$ and $\eta(t', \xi)$ are determined by Eqs. (8–10).

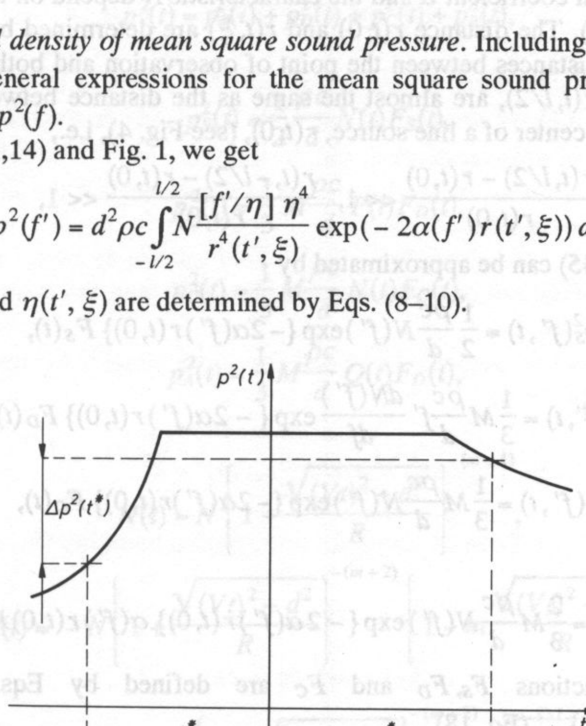

FIG. 3. Measure of $p^2(t)$ asymmetry, $\Delta p^2(t^*)$ (Eq. 39).

To evaluate the above integral, the analysis can be carried out (with $M << 1$) in a manner quite parallel to that in Section A2. The final result is

$$p^2(f',t) = p_S^2(f',t) + p_D^2(f',t) + p_C^2(f',t) + p_A^2(f',t), \tag{41}$$

where

$$p_S^2(f',t) = d^2\rho c N \int_{-l/2}^{l/2} \frac{\exp(-2\alpha r(t,\xi))}{r^4(t,\xi)}\, d\xi, \tag{42}$$

$$p_D^2(f',t) = M d^2 \rho c f' \frac{dN}{df'} \int_{-l/2}^{l/2} \frac{\sin\phi \exp(-2\alpha r(t,\xi))}{r^4(t,\xi)}\, d\xi, \tag{43}$$

$$p_C^2(f',t) = 4 M d^2 \rho c N \int_{-l/2}^{l/2} \left[\frac{r(t,0)}{r(t,\xi)} - 1\right] \frac{\sin\phi \exp(-2\alpha r(t,\xi))}{r^4(t,\xi)}\, d\xi, \tag{44}$$

and

$$p_A^2(f',t) = 2 M d^2 \rho c N \alpha r(t,0) \int_{-l/2}^{l/2} \frac{\sin\phi \exp(-2\alpha r(t,\xi))}{r^4(t,\xi)}\, d\xi, \tag{45}$$

where the absorption coefficient α and the characteristic N depend on the observed frequency f' (Eqs. 2,6). The distance $r(t,0)$ and $r(t,\xi)$ are determined by Eq. 18. When $l/2 << d$, then the distances between the point of observation and both edges of a line source, $r(t,-l/2)$, $r(t,l/2)$, are almost the same as the distance between the point of observation and the center of a line source, $r(t,0)$, (see Fig. 4), i.e.,

$$\frac{r(t,l/2) - r(t,0)}{r(t,0)} << 1, \quad \frac{r(t,-l/2) - r(t,0)}{r(t,0)} << 1, \tag{46}$$

Thus Eqs. (42–45) can be approximated by

$$p_S^2(f',t) = \frac{1}{2}\frac{\rho c}{d} N(f') \exp\{-2\alpha(f') r(t,0)\} F_S(t), \tag{47}$$

$$p_D^2(f',t) = \frac{1}{3} M \frac{\rho c}{d} f' \frac{dN(f')}{df} \exp\{-2\alpha(f') r(t,0)\} F_D(t), \tag{48}$$

$$p_C^2(f',t) = \frac{1}{3} M \frac{\rho c}{d} N(f') \exp\{-2\alpha(f') r(t,0)\} F_C(t), \tag{49}$$

and

$$p_A^2(f',t) = \frac{2}{3} M \frac{\rho c}{d} N(f') \exp\{-2\alpha(f') r(t,0)\}\, \alpha(f') r(t,0) F_D(t). \tag{50}$$

The time functions F_S, F_D and F_C are defined by Eqs. (28–30) and $r(t,0) = \{(Vt)^2 + d^2\}^{1/2}$ (Eq. 18).

Expressions (48) and (49) describe the Doppler effect and convection effect modified by air absorption. To find the physical meaning of Eq. (50), let us note that $F_D(t)$ is

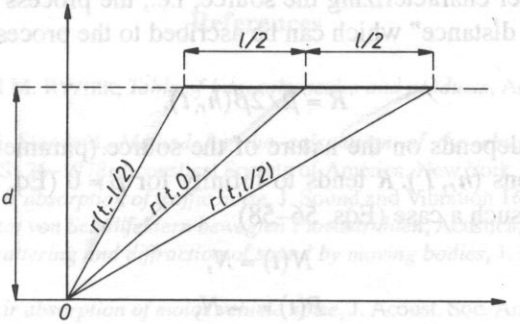

FIG. 4. Definition of the distances $r(t, -l/2)$, $r(t, l/2)$, and $r(t, 0)$ (Eqs. 46).

negative for $t < 0$ and positive for $t > 0$ (Eqs. 29,31). Thus, we have $p_A{}^2 < 0$ and $p_A{}^2 > 0$ for $t < 0$ and $t > 0$ respectively. Such an increase of the mean square sound pressure can be explained by the decrease of air absorption caused by the Doppler frequency shift.

B.2. Time history of mean square sound pressure. From Eqs. (32, 47–50) we can obtain the mean square sound pressure as follows:

$$p^2(t) = p_S^2(t) + p_D^2(t) + p_C^2(t) + p_A^2(t), \tag{51}$$

where

$$p_S^2(t) = \frac{1}{2}\frac{\rho c}{d} N(t) F_S(t), \tag{52}$$

$$p_D^2(t) = \frac{1}{3} M \frac{\rho c}{d} P(t) F_D(t), \tag{53}$$

$$p_C^2(t) = \frac{1}{3} M \frac{\rho c}{d} N(t) F_C(t), \tag{54}$$

$$p_A^2(t) = \frac{1}{3} M \frac{\rho c}{d} Q(t) F_D(t), \tag{55}$$

with

$$N(t) = N \left\{ 1 + \frac{\sqrt{(Vt)^2 + d^2}}{R} \right\}^{-(m+1)}, \tag{56}$$

$$P(t) = -N \left\{ 1 + \frac{\sqrt{(Vt)^2 + d^2}}{R} \right\}^{-(m+2)} \cdot \left\{ 1 - m \frac{\sqrt{(Vt)^2 + d^2}}{R} \right\}, \tag{57}$$

and

$$Q(t) = (m+1) N \left\{ 1 + \frac{\sqrt{(Vt)^2 + d^2}}{R} \right\}^{-(m+2)} \cdot \frac{\sqrt{(Vt)^2 + d^2}}{R}. \tag{58}$$

N is the parameter characterizing the source, i.e., the process of generation (Eq. 3) and R is the "critical distance" which can be ascribed to the process of propagation, i.e., air absorption [6]:

$$R = \mu/2\beta(h_r, t). \tag{59}$$

Its numerical value depends on the nature of the source (parameter μ, see Eq. 2) and the weather conditions (h_r, T). R tends to infinity for $\beta = 0$ (Eq. 6), i.e., for a nondissipative medium. In such a case (Eqs. 56–58)

$$N(t) = N,$$
$$P(t) = -N, \tag{60}$$
$$Q(t) = 0,$$

and the expression (52–54) simplifies to the form given by Eqs. (34–36) with $P = -N$.

3. Conclusions

The final result of geometrical spreading, air absorption, Doppler, and convection effects due to the motion of a line source have been described in terms of means square sound pressure (p^2). Making use of the definition, $L_p = 10 \lg (p^2/p_0^2)$ one can predict the sound pressure $(p_0 = 2 \times 10^{-5}\ \mathrm{N/m^2})$. The spectrum of a line source, frequency-weighed or not, is characterized by the function $N(f)$ (Eq. 2). In the case of A-frequency weighing, $N = N_A(f)$ we are able to calculate the A-weighed sound pressure level, $L_{pA} = 10 \lg (p_A^2/p_0^2)$, of noise generated, e.g., by a train.

The formulae have been derived under the assumption that the noise is generated by a continuous line of dipole sources.

The most general expressions (Eqs. 51–55) hold true for $M << 1$, linear dependence of the absorption coefficient on frequency (Section 1.B), and the condition determined by the inequality $l/2 << d$.

Air absorption can be neglected and one can use a simpler expression (38), when $r(t, 0) << R$, i.e. (Eqs. 51–59)

$$|t| << \frac{1}{V}\sqrt{[\mu/2\beta(h_r, T)]^2 - d^2}. \tag{61}$$

Numerical values of β can be found for any relative humidity (h_r) and air temperature (T) (Eq. 6).

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