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EXPERIMENTAL RESTORATION OF ULTRASONIC TOMOGRAMS BY MODIFIED WIENER FILTERS

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On the basis of the experimentally identified imaging properties, esp. point-spread-function and image and noise spectra, Wiener-type filters have been designed and applied to ultrasonograms of artificial or tissue-phantom objects. The derived type of filters are advantageous in not needing the average power cross-or autospectrum involving the properties of the undistorted image that is difficult to provide experimentally. It has been found that the filters yield quite unacceptable images due to the amplification of noise but simple modifications of their transfer characteristics suppressing the influence of zeroes in the transfer function of the imaging system enable to obtain substantially better results.

Considerable improvement in resolution is possible in spite of the strong nonlinearities involved in the image forming process. Suppressing speckle textures on the basis of differences between average power spectrum of the noise and of the useful image structures has shown only a modest success possibly due to rather small extend of the used image-and spectra matrices and also because of low extent of data not providing good spectral estimates.

1. Introduction

The principle of echo-ultrasonography does not allow for very high resolution in the resulting images. Especially lateral resolution is rather low which is a consequence of wide directional characteristics of ultrasonic transducers. Although there is a lot of improvement in providing sharper images thanks to better and more sophisticated transducers (fixed and dynamic focusing), still it seems worth to consider digital post-processing of the resulting images to sharpen them. Many attempts in this direction have been published, e.g. [1, 2]. The basic obstacle is the nonlinearity of the imaging process, caused primarily by wave interference of echoes followed by strongly nonlinear signal envelope detection and also by logarithmic characteristics of the signal processing chain on the receiving side. In principle, it is possible to process radio frequency signals before the envelope detection, but it is technically very demanding and at the same time the reported results e.g. [3, 4] are not substantially better.

The present contribution deals with the video-(envelope detected) signals, but in their original form as provided by a typical wobbling crystal sector scanner, e.g. in polar coordinates. It seems that using this form of non-format-converted data is one of the conditions for restoration; it has been experimentally shown [8] that imaging process can be considered practically isoplanar on most of the image area if using this form of data.

The problem of noise, inherent in every restoration, is especially difficult with ultrasonography due to presence of s.c. speckles that are specific textures caused by interference of echoes from elementary microscopic reflectors or scatterers that are insonified coherently. As these textures are influenced primarily by the properties of the imaging system and only in a less extend by the tissue characteristics, it would be mostly desirable to remove them from the image. Unfortunately, the spectral components of the noise occupy frequency areas common with the components of the useful structures so that linear filtering can only limit but not remove them completely. Another possibility how to suppress speckles is to average several images (tomograms) depicting the same useful structures but with diversified speckles. As such a diversification based on special technical means [5, 6] is again quite demanding, a simple approach, based on diversification by small scanhead displacements, has been tried [11] with certain promising results. It seems that combination of averaging (or preliminary median filtering [7]) and Wiener filtering could suppress the speckles in a practically interesting extend.

The paper is directed towards deriving a simple form of the Wiener filter which could be designed on the basis of knowledge available from identification of imaging properties of an ultrasonic scanner, as published elsewhere [8–10]. Several questions should be answered: if there is a possibility to deconvolve the responses to "point" targets in spite of the effect of interference and nonlinearity which substantially change the image in comparison with purely additive image, how to modify the theoretical inverse filter in order to improve its originally not acceptable performance and finally to evaluate the performance of s.c. correcting filter which should take into account the influence of noise. Also for this filter a modification will be needed.

2. Modified filters for restoration

Supposed model of the image distortion has the general form

$$g(\overline{r}) = N\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\overline{r}-\overline{r}')\cdot f(\overline{r}')\,dx'dy' + n(\overline{r})\right] \tag{1}$$

where g means the distorted image, f is the original image, h is the point spread function of the imaging system and n is the noise component; $\overline{r} = (x, y)$. The linearly distorted image as described by the expression in square brackets is subject to a nonlinear transform N. The problem to solve the equation (1) for f is not an elementary one for

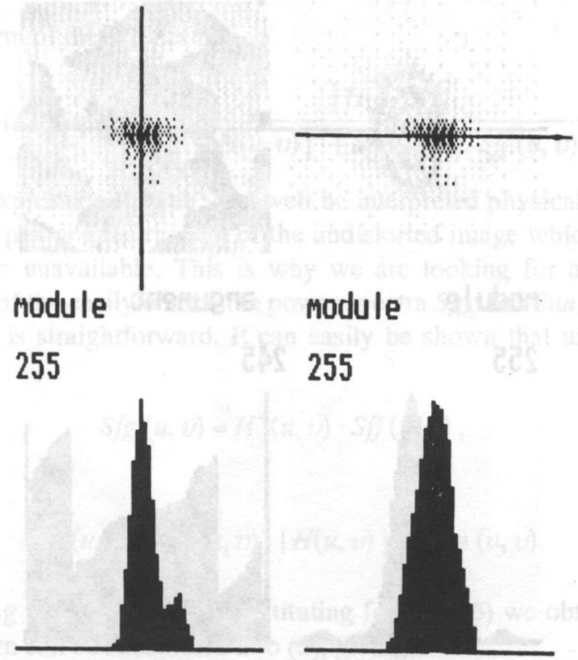

FIG. 1. Experimentally determined average point spread function of typical sector scanner.

two reasons: the noise n is not known explicitly-only some of its statistical characteristics can be experimentally determined – and the equation itself is "stiff", being too sensitive to small changes in its components. Also the nonlinearity complicates substantially the situation.

As for the nonlinearity, it can either be taken into account by an inverse transform N^{-1} which linearizes the problem – this leads to a homomorphic filter, or in some cases it can be neglected and linear restoration algorithm used directly. It has been shown experimentally several times that even this simplified approach can give acceptable results unless the nonlinearity is too severe.

The linearized version of (1) can be regarded as a family of equations where g, f and n are random fields while only h is a deterministic function; the solution $\hat{f}$ is then sought as an estimate minimizing on average the error function

$$e^2(\overline{r}) = E[(f(\overline{r}) - \hat{f}(\overline{r}))^2]$$

for a given g and a known statistics of n. The general solution, as well known, is then the conditional expectation of f given g but this is a complicated nonlinear function of g which depends on the joint probability density over the fields f and g that is practically impossible to obtain. This is why a simplified solution in the linear superposition form

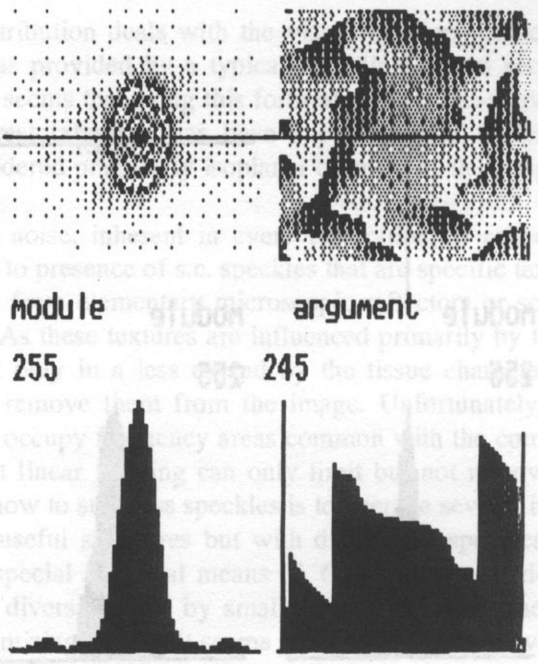

FIG. 2. Average transfer function of the sector scanner.

$$\hat{f}(\overline{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(\overline{r},\overline{r}') \cdot g(\overline{r}')\, dS(\overline{r}'), \quad dS(\overline{r}') = dx'\,dy',$$

is looked for, which under the condition that the fields are homogeneous turns into convolution integral

$$\hat{f}(r) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(\overline{r}-\overline{r}') \cdot g(\overline{r}')\, dS(\overline{r}'). \tag{2}$$

where $m(\overline{r}-\overline{r}')$ can be regarded as a point spread function of a linear two-dimensional filter. Thus, linear filtering is used as the estimation procedure and the problem reduces into determining its frequency transfer function $M(u, v)$. The classical Wiener solution is

$$M(u, v) = Sfg(u, v)/Sgg(u, v), \tag{3}$$

where Sfg is power cross-spectrum of the fields f and g and Sgg is the auto power spectrum of g. Unfortunately, while Sgg can relatively easily be estimated by averaging spectra of individual realizations of g, the cross spectrum is very difficult, and in many cases impossible, to obtain.

This obstacle can be circumvented under the condition that f and n are uncorrelated and that at least one of them (usually noise) has zero mean. Then we arrive to the most commonly used form of the Wiener filter

$$M(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + Snn(u,v)/Sff(u,v)}. \tag{4}$$

Although the expression (4) can very well be interpreted physically, it relies on the knowledge of auto power spectrum *Sff* of the undistorted image which in certain cases (as in ours) is also unavailable. This is why we are looking for a form expressing $M(u, v)$ by means of the easily obtainable power spectra *Sgg* and *Snn*.

The derivation is straightforward. It can easily be shown that under given conditions,

$$Sfg(u,v) = H^*(u,v) \cdot Sff(u,v), \tag{5}$$

and

$$Sgg(u,v) = Sff(u,v) \cdot |H(u,v)|^2 + Snn(u,v). \tag{6}$$

Then expressing *Sff* from (6) and substituting for it in (5) we obtain an expression for *Sfg* which in turn can be substituted into (3), giving

$$M(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2} \cdot \frac{Sgg - Snn}{Sgg}. \tag{7}$$

This rather unusual form of the Wiener filter can be interpreted as cascade of two filters: the first one is the direct inverse filter while the second is a correction of the noise influence which is obviously unimportant in those areas of the frequency domain where *Snn* is negligible in comparison with *Sgg*.

When experimenting with a given class of images (with a common point spread function *h* and fixed statistical properties of noise) we can, using the form (7), benefit on dividing the problem into two parts: the first part being PSF identification and derivation of a direct inverse filter and the second one the estimation of the statistical properties of the distorted image and separate noise and derivation of the correcting filter. This is especially advantageous if there exists a possibility of providing almost noiseless images with "point", "line" or "edge" objects enabling to identify the PSF of the imaging process on one hand and pure noise images (or at least image areas) from which the noise power spectrum can be estimated on the other. There are usually no problems with providing good estimate of the spectrum of the distorted image class.

3. Modified inverse filter

The scanner average point spread function as determined experimentally [8] is depicted on Fig. 1, its spectrum on Fig. 2. The exact inverse of it (Fig. 3),

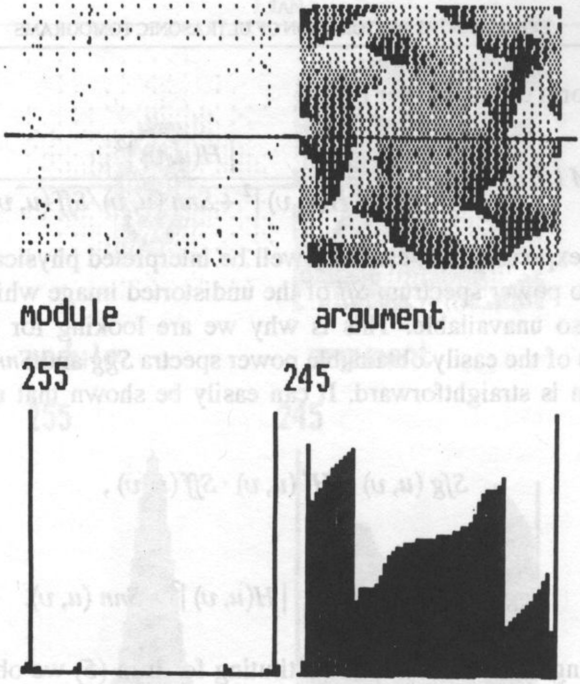

FIG. 3. Transfer function of the inverse filter.

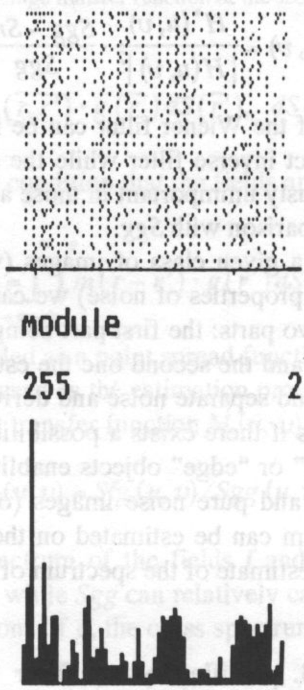

FIG. 4. Point spread function of the inverse filter.

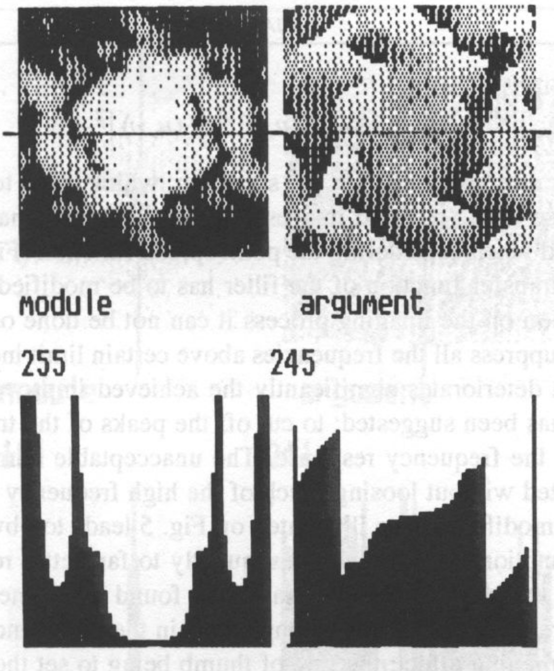

FIG. 5. Transfer function of the modified inverse filter.

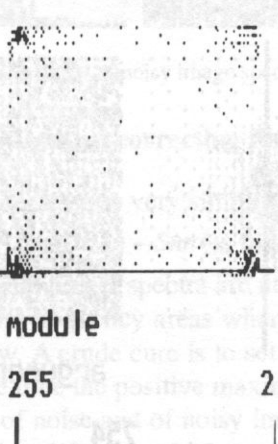

FIG. 6. Point spread function of the modified inverse filter.

$$M_1(u, v) = H^*(u, v)/|H(u, v)|^2 \tag{8}$$

has very high peaks due to “zeroes” in the spectrum, which leads to unacceptable level of narrow-band noise in the restored images even when the original noise is quite low. This can be expected when considering the point spread function (Fig. 4) of the filter.

Obviously, the transfer function of the filter has to be modified but due to the lack of precise information on the imaging process it can not be done on formal bases. The usual approach-to suppress all the frequencies above certain limit including the peaks-in not acceptable as it deteriorates significantly the achieved improvement in resolution. Another approach has been suggested: to cut off the peaks of the transfer function thus partially equalizing the frequency response. The unacceptable narrow-band amplification is then eliminated without loosing much of the high frequency bands important for the resolution. The modification as illustrated on Fig. 5 leads to obviously much smoother point spread function (Fig. 6) and consequently to far better restored images. The threshold value for the transfer function has to be found experimentally as a compromise between suppressing the spurious components in the result and preserving a significant extent of sharpening effect; the rule of thumb being to set the limit to only a few percent of the original maximum.

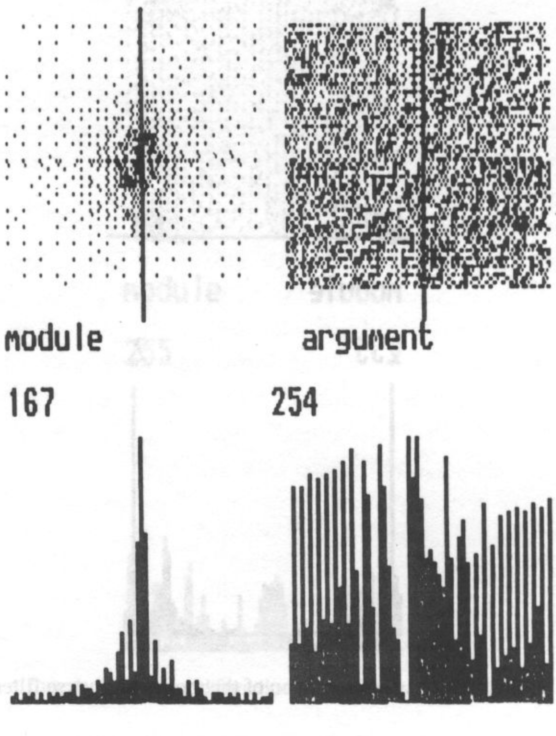

FIG. 7. Example of speckle noise spectrum.

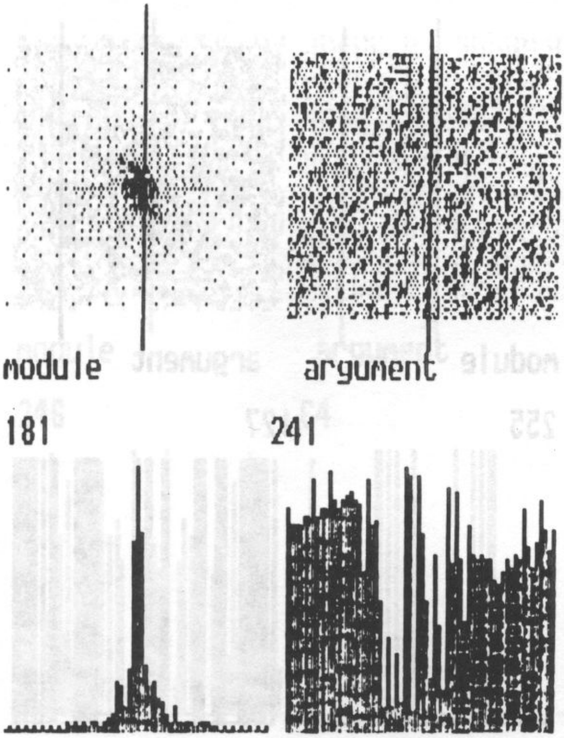

FIG. 8. Example of noisy image spectrum.

4. Modified correcting filter

The theoretical correcting filter has the very simple form

$$M_2(u, v) = [\,Sgg(u, v) - Snn(u, v)\,]/Sgg(u, v) \tag{9}$$

in which good estimates of the auto power spectra are needed. It may happen that due to imperfect estimates, there will be frequency areas where $Snn > Sgg$ which is obviously false from physical point of view. A crude cure is to set the numerator in (9) to zero for frequencies, limiting at the same time the positive maxima of $M(u, v)$ to a certain value. Examples of individual spectra of noise and of noisy image are on Fig. 7 and 8, respectively. The two spectra are visibly different which promises the possibility of deriving the correcting filter. Nevertheless, the derived filter as shown on Fig. 9 does not seem very reasonable, obviously due to the imperfect spectral estimates based on the averages of several power spectra. Thus the modification as described above was necessary yielding the filter depicted on Fig. 10.

5. Discrete approximations

The frequency domain procedures as described above have to be practically realized by discrete approximations, e.g. using discrete Fourier transform (DFT) instead of the

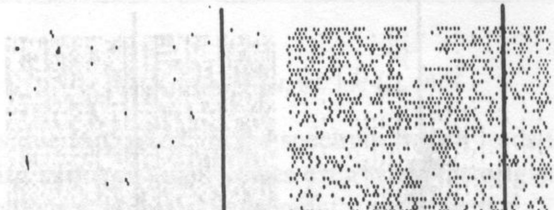

FIG. 9. Transfer function of the theoretical correcting filter.

continuous one. To prevent undesirable effects of circular convolution that consequently replaces the linear one, substantially bigger matrices then just necessary for keeping the needed amount of image and spectral data must be used which increases the computational and memory demands to the equipment used. Though it is obvious that the size of spectral matrices in case of providing convolution via frequency domain must be four times bigger then the image data and PSF matrices in order to enable appropriate padding by zeroes, the extent of the PSF of the restoration filter is a priori unknown and may well be bigger than the available size of matrices. When working with rather small matrices, it must be realized that the marginal effects caused by circular convolution may influence substantial part of the resulting image.

The presented figures have been derived using original image matrices of the size 32×32 ; frequency domain operations were carried out in matrices 64×64 big. While this seems to be quite sufficient for deriving the direct inverse filter as the extent of the original point spread function is rather small, the inverse filtering itself should have been done in bigger matrices, at least 96×96, as the PSF of the filter has in principal size 64×64. Also, such small matrices can describe the content of the distorted images or even of image noise only very roughly. This is probably one of the reasons of not very good estimates of the power spectra and consequently of obviously imprecise correcting filter (Fig. 9).

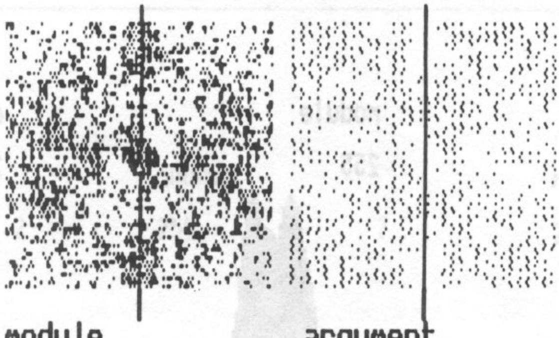

FIG. 10. Transfer function of the modified correcting filter

6. Discussion of results

First part of the results consists of restorations of images of couples of “point” targets that were provided by using the modified direct inverse filter ($M_1(u, v)$). The first example (Fig. 11) (a) depicts the response to a couple with negligible distance of the targets, the second (b) corresponds to the case of a distance just at the limit of resolution and the third one (c) is a case under the resolution limit of the imaging system. Fig. 12 a, b, c shows the restorations by the unmodified filter while on Fig. 13 we see the same objects but restored by the heavily modified filter. Obviously the simple modification of the filter removed most of the spurious signals in the restored images. On the other hand, the sharpness of the peaks visible on line profiles, which corresponds to obtainable resolution is practically the same in Fig. 12 as in Fig. 11. This is rather surprising as the high frequency peaks in the original filter have been suppressed in the modification as much as to only 2 per cent of their original value. The example (c) proves that substantial resolution improvement is possible and preserved even by using a heavily modified filter.

The second part of the results concerns the use of the correcting filter ($M_2(u, v)$). An example of the input image data with marked area for further processing is shown in Fig. 14. Clearly, the used areas are very small. The results of restoration are shown on

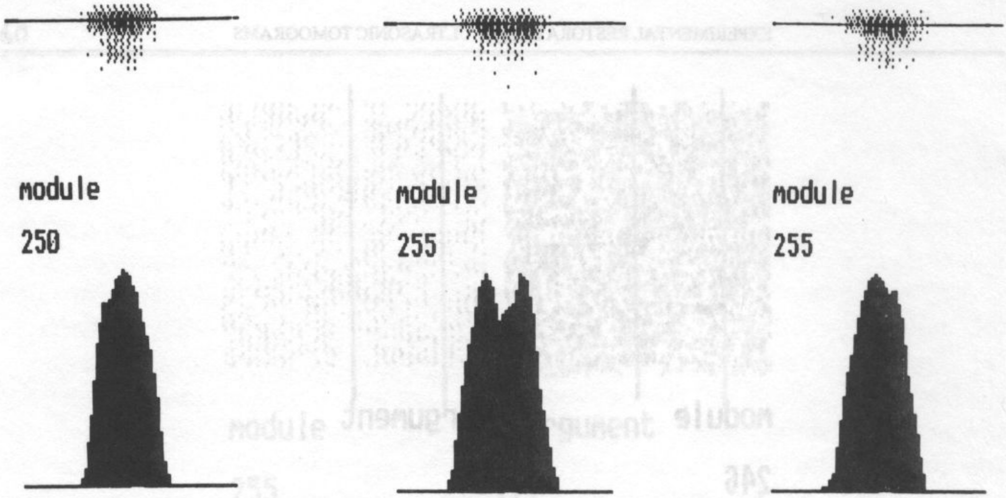

FIG. 11. Original responses to the couples of „point" targets horizontal distances: *a* …0 mm, *b* … 3.9 mm, *c* … 2.3 mm.

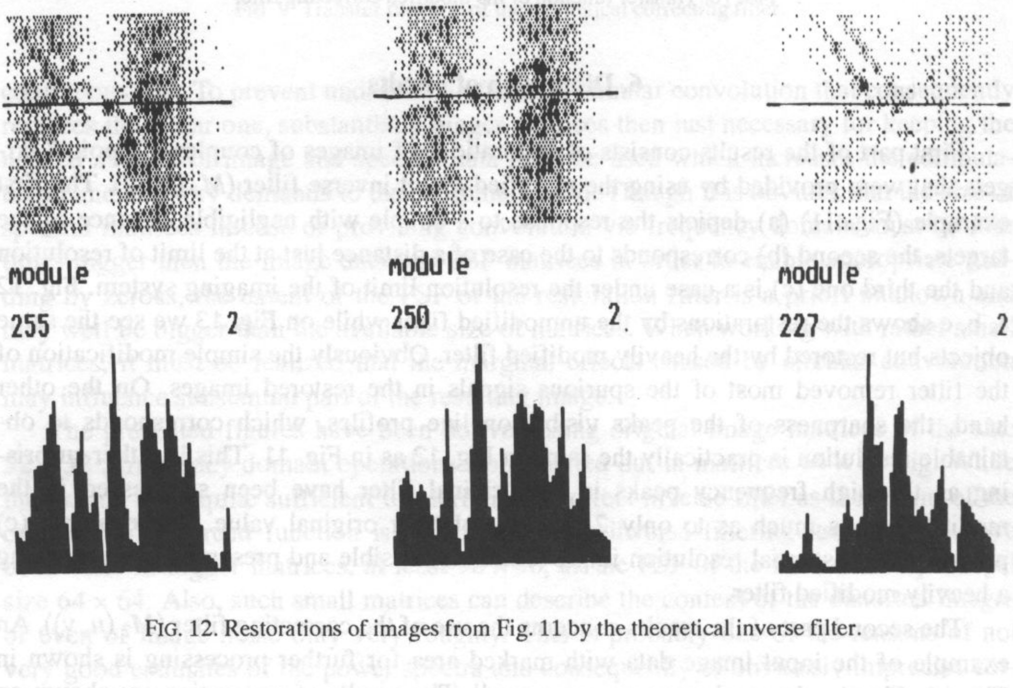

FIG. 12. Restorations of images from Fig. 11 by the theoretical inverse filter.

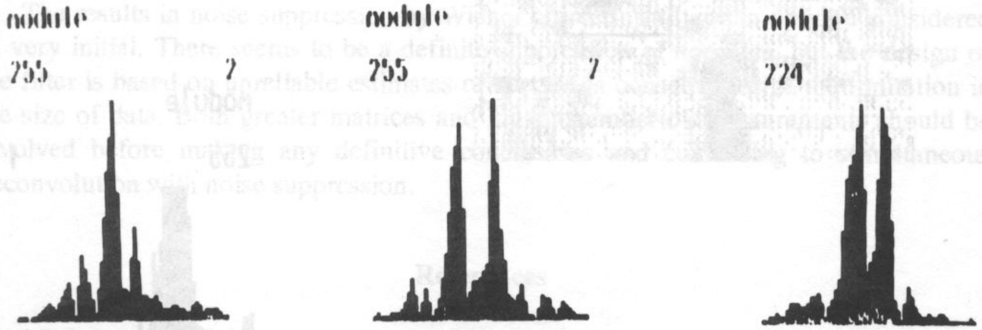

FIG. 13. Restorations of images from Fig. 11 by the modified inverse filter.

Fig. 15 (by theoretical filter) and Fig. 16 (by modified filter). As the original image matrices filled only upper left quarter of the whole matrix, so should do even the restored images which is not exactly the case; certain level of spurious signals (different from speckles) fills the whole image area. Anyway it seems that the original speckles have been removed though it is difficult to determine the extent of improvement in signal to noise ratio. Also, the distortion of useful structures is definitely present.

A comment should be inserted on the attempts to linearize the problem by homomorphic transform, inverse to the average nonlinearity of the imaging process as derived experimentally in [9, 10]. The rectifying transform strongly emphasizes the high level components of the image thus suppressing the noise components including speckles. The unwanted textures are practically invisible in the transformed images so that no further improvement could have been found in the images filtered by a filter based on the average spectra of the rectified images.

7. Conclusions

The form (7) of the Wiener filter seems to be a good tool for experimenting with restoration of ultrasonic tomograms because it enables to separate the problem of pure deconvolution and the connected problem of identification of transfer function of the imaging system from the other problem of suppressing the noise and designing the corresponding filter by determining the statistical properties of average image and average noise. It also removes the necessity to determine the average statistics of the unavailable undistorted image as needed for the common form (4).

The results for the deconvolution problem are rather promising: in spite of the nonadditive character of the responses the resolution can be significantly improved even in

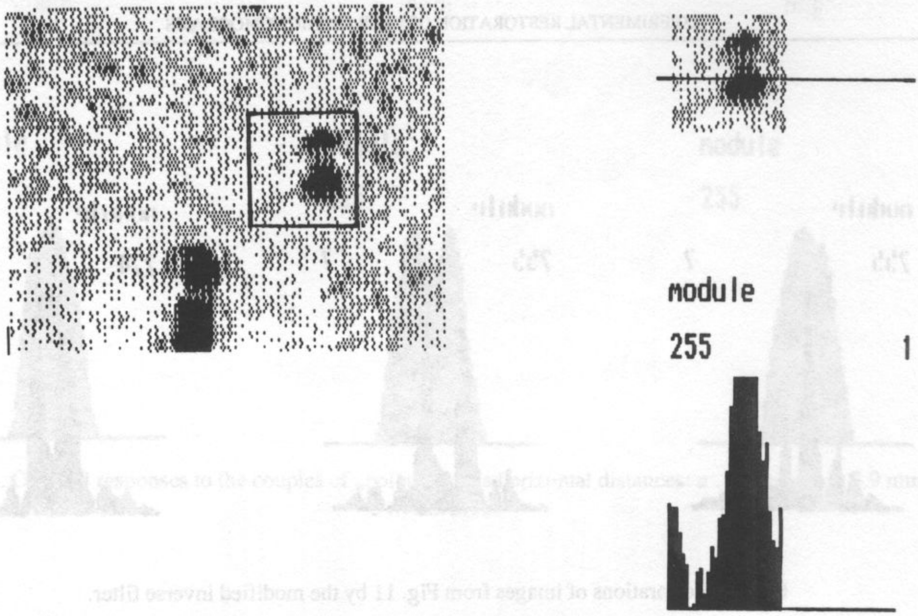

FIG. 14. Part of original image data with marked area to be restored.

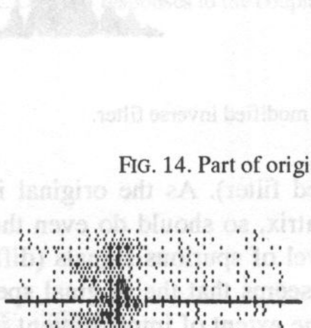

FIG. 15. Noisy image area from Fig. 14 processed by theoretical correcting filter.

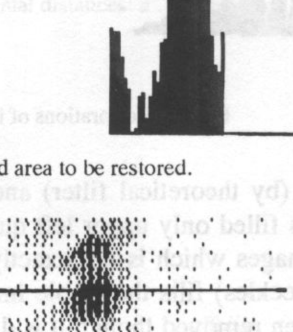

FIG. 16. Noisy image area from Fig. 14 processed by modified correcting filter.

cases of responses seriously damaged by wave interference. An important condition contributing to the positive results is probably working with the non-format-converted data. Any success of the deconvolution is of course dependent on the possibilities of noise suppression, either by filtering or by other means, as averaging.

The results in noise suppression by Wiener correcting filter can only be considered as very initial. There seems to be a definite suppression of speckles, but the design of the filter is based on unreliable estimates of spectra in consequence of the limitation in the size of data. Both greater matrices and higher number of measurements should be involved before making any definitive conclusions and continuing to simultaneous deconvolution with noise suppression.

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