

INVESTIGATION OF NONLINEAR WAVE PROPAGATION IN WATER

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In this paper the effects of nonlinear wave distortion have been described. This problem is characterized for the nonlinear medium. The experimental investigation results of nonlinear propagation have been presented. The wave distortion is connected with the harmonic generation. The measuring set-up used for the investigation of nonlinear wave distortion has been shown. It allows to measure wave distortion by determining harmonic amplitude and their numbers as a function of the distance from a wave source. As a primary wave source, the "sandwich" transducers have been used. The resonance frequencies of these transducers were 30 kHz and 81 kHz respectively. The recorded wave distortions are shown in diagrams and photos. Also the spectrum evolution as a function of distance from the wave source is shown. The investigation results are connected with the weak nonlinear wave distortion.

1. Finite amplitude wave propagation in nonlinear medium

The description of dynamical effects is carried out by means of equations of continuity, motion and state that characterize this phenomenon. The system of equations of continuity, motion, entropy and state makes it possible to obtain the nonlinear equation. This equation describes wave propagation with finite amplitude [2].

The assumption that the acoustic Mach number has small values in hydroacoustics is correct. This means that relative changes of density and pressure are small. Thanks to these facts, one can describe the nonlinear equation of acoustics as follows [3]:

$$\Delta p' - \frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} + \frac{b}{c_0^3 \rho_0} \frac{\partial}{\partial t} \Delta p' = -Q \quad (1)$$

where

$$Q = \frac{1}{c_0^4 \rho_0} \left(\frac{\partial p'}{\partial t} \right)^2 + \frac{\varepsilon - 1}{c_0^4 \rho_0} \frac{\partial^2 p'}{\partial t^2} + \frac{\rho_0}{2} \Delta v^2 + \rho_0 v \Delta v \quad (1a)$$

$p' = p - p_0$ – acoustic pressure, c_0 – speed of sound wave, ρ_0 – medium density at rest, b – attenuation factor, t – time, ε – nonlinearity parameter factor, v – vibration velocity.

The solution of Eq. (1) has not been found till now, but a few methods of its simplification are used. Sometimes the perturbation method is used. It gives good results in the case when the nonlinear effects are relatively small. This method takes into account the spare assumptions: the wave is a plane wave and the attenuation factor is equal to zero.

The other form of the known method which allows to solve Eq. (1) is connected with the quasi-optical assumption. It is assumed that the wave distortion is very small on the path equal to the wave length and the form of energy flux can change not only in the direction of wave propagation but in the transverse one, too. The transverse changes are larger than the longitudinal ones because of the diffraction effect. By fixing a coordinate system in the zero phase of the wave that propagates with c_0 speed, one can transform Eq. (1) to the following form [7]:

$$\frac{\partial}{\partial \tau} \left[\frac{\partial p'}{\partial z} - \frac{\varepsilon}{c_0^3 \rho_0} p' \frac{\partial p'}{\partial \tau} - \frac{b}{2 c_0^3 \rho_0} \frac{\partial^2 p'}{\partial \tau^2} \right] = \frac{c_0}{2} \left(\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} \right) \quad (2)$$

where: $\tau = t - z/c_0$ - time in the Lagrange description, z - wave propagation direction, x, y - axis orthogonal to z . This equation is called the Chochlov-Zabolotska-Kuznetsov equation. The above presented equation has no solution. When the right side of the equation equals zero, then one obtains the Burgers equation [7]:

$$\frac{\partial p'}{\partial z} - \frac{\varepsilon}{\rho_0 c_0^3} p' \frac{\partial p'}{\partial \tau} - \frac{b}{2 \rho_0 c_0^3} \frac{\partial^2 p'}{\partial \tau^2} = 0. \quad (3)$$

It is a nonlinear equation that can be solved. The solution of Eq. (3) is very useful in analyzing the nonlinear effects occurring on the energy flux axis. Unfortunately, it does not take into account the diffraction phenomenon because of one dimension of Burger's equation. Its solution describes the plane waves. Another advantage of this solution is the lack of any limits of acoustic Reynolds number values determined by [5]

$$Re_a = \frac{\rho_0 c_0 \omega}{b \omega} \quad (4)$$

where $\omega = 2\pi f$, f - frequency.

Burgers' equation can be solved for the two main cases, that is for a small Reynolds number and for a large one. The first case is strictly connected with the large dissipative effects and the second one when the nonlinear effects are dominating.

The effect of nonlinear distortion in water is similar to the phenomenon which takes place during shock wave propagation in air. In air the finite amplitude wave distortion takes place due to the difference of phase velocity. In the compression area the phase velocity is higher than in the rest area as well as in the expansion one. Finally this phenomenon causes a wave distortion from sine form to a saw-tooth one. The wave distortion is strictly connected with harmonic generation whose numbers and amplitudes are a function of the distortion degree.

The effect of wave shape change can be observed on the basis of Riemman's solution of an equation that describes the plane finite amplitude wave in an idealized fluid.

The Lagrange description is used. The particle velocity v can be described in the form [4]:

$$v(z, \tau) = v_0 \sin \omega \left(\tau - \frac{z \varepsilon v}{c_0^2} \right) \quad (5)$$

where v_0 – source vibration velocity.

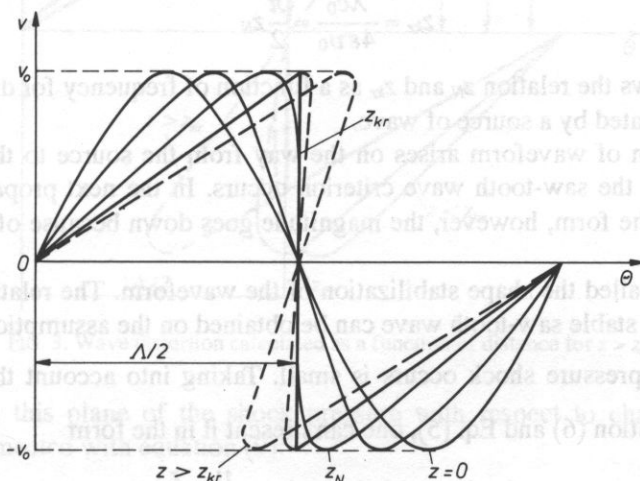


FIG. 1. Wave distortion calculated as a function of distances for $z \leq z_{kr}$.

According to Eq. (5), the waveform changes the shape (see Fig. 1) step by step up to the point where it reaches the range $z = z_N$. The value z_N is a boundary value of a coordinate z for which the functional described by the relation (5) is single-valued function. This value can be determined by means of the following formula [4]:

$$z_N = \frac{c_0^2}{\omega \varepsilon v_0} = \frac{c_0 \lambda}{2 \pi \varepsilon v_0} \quad (6)$$

At the range from the source $z > z_N$, the solution of Eq. (5) is not a single-valued function. This range is sometimes called the distance of loss of solution continuity or simply the discontinuity distance. The loss of a continuity of waveform description takes place only in mathematical formalism because it takes into account the wave propagation in the idealized fluid. In the case of sound propagation in the real medium, that is water, with the increase of distortion the loss of wave energy increases, too. As a result of these interactions, the harmonic wave becomes saw-tooth shaped at some distance from the source. This distance is called the critical distance. This distance can be determined on the basis of the following approach. The finite amplitude wave speed can be taken apart as the sum of two components of speed. The first one is a sound speed in the rest medium and the second component is connected with the rise of the total speed as a result of the medium nonlinearity [5]:

$$c = c_0 + \varepsilon v(z, \tau). \quad (7)$$

To create the saw-tooth wave, the top of the wave must overcome the way $\lambda/4$ (λ – wavelength) with the speed εv_0 at the same time as the distance $z = z_{kr}$ is overcome with c_0 speed. Assuming that these times are equal, one can obtain the following relation [5]:

$$z_{kr} = \frac{\lambda c_0}{4 \varepsilon v_0} = \frac{\pi}{2} z_N \quad (8)$$

Figure 2 shows the relation z_N and z_{kr} as a function of frequency for different values of pressure generated by a source of wave.

The distortion of waveform arises on the way from the source to the critical distance (z_{kr}) where the saw-tooth wave criterion occurs. In the next propagation region the wave keeps the form, however, the magnitude goes down because of the nonlinear damping (Fig. 3).

This area is called the shape stabilization of the waveform. The relation describing the motion of the stable saw-tooth wave can be obtained on the assumption that the distance where the pressure shock occurs is small. Taking into account that $\tau = t - \frac{z}{c_0}$, $k = \omega/c_0$, the relation (6) and Eq. (5), one can present it in the form

$$v(z, t) = v_0 \sin(\omega t - kz + \frac{v}{v_0} \cdot \frac{z}{z_N}). \quad (9)$$

The plane where the pressure shock occurs moves in the space with phase speed c_0 and during time t it overcomes the way from a transmitter equal to $z = c_0 t$. The position of this plane is described by means of the phase equation $\omega t - kz = 0$. The condition of

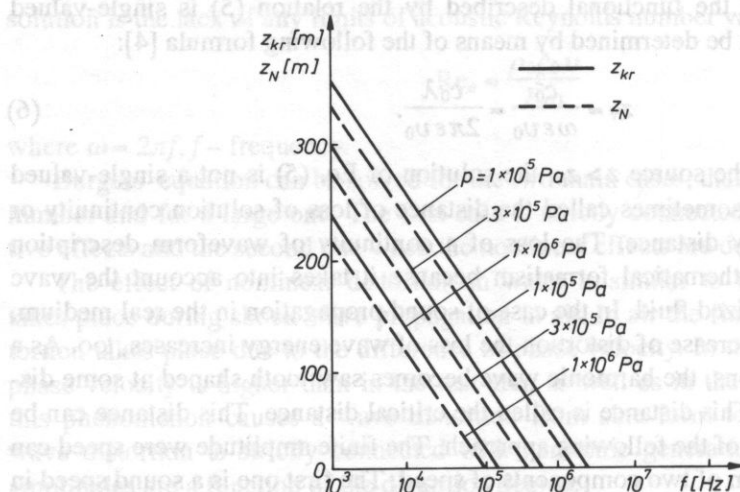


FIG. 2. The dependence of the distances z_N and z_{kr} on the frequency calculated for different values of pressure generated by the wave source.

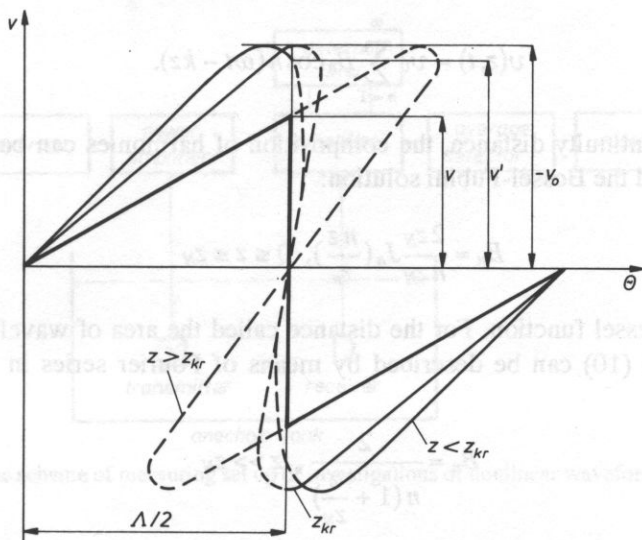


FIG. 3. Wave distortion calculated as a function of distance for $z > z_{kr}$

occurrence on this plane of the shock pressure with respect to changes of particle velocity is connected with equation (9):

$$v = v_0 \sin\left(\frac{v}{v_0} \cdot \frac{z}{z_N}\right) \quad (10)$$

where $z > z_{kr}$ and v is the magnitude of the saw-tooth wave. In the distance of $z/z_N \gg 1$, the expression $\sin\left(\frac{v}{v_0} \cdot \frac{z}{z_N}\right)$ can be represented by a Taylors' series expansion close to point π , allowing the formula (10) to show in the simpler form

$$v = v_0 \frac{\pi}{1 + \frac{z}{z_N}} \quad (11)$$

that describing the decrease of the wave magnitude.

At the large distance from the transmitter, as a result of dissipation of wave energy, the nonlinear distortion falls down and the wave-front changes. The shape of the wave goes to the harmonic wave.

2. Finite amplitude plane wave spectrum

The change of the shape of the finite amplitude harmonic wave during propagation in nonlinear medium is connected at the same time with a change of wave spectrum.

The evolution of the wave spectrum can be shown using the Fourier series of the solution of Burgers' equation for large Reynolds numbers [1]:

$$v(z, t) = v_0 \sum_{n=1}^{\infty} B_n \cos n(\omega t - kz). \quad (12)$$

For the discontinuity distance, the composition of harmonics can be shown in the form that is called the Bessel-Fubini solution:

$$B_n = \frac{2z_N}{n z_N} J_n\left(\frac{nz}{z_N}\right), \quad 0 \leq z \leq z_N \quad (13)$$

where $J_n(\cdot)$ – Bessel function. For the distance called the area of waveform stabilization, the formula (10) can be described by means of Fourier series in the following form:

$$B_n = \frac{2}{n(1 + \frac{z}{z_N})}, \quad z \gg z_N \quad (14)$$

Distortion of the harmonic wave is connected with the phenomenon of pumping of the primary wave energy to harmonics. The nonlinear distortion can be characterized by a change of the wave spectrum. The spectrum of a radiated wave at the transmitter ($z = 0$) has only spike whose value equals v_0 . In the area $0 < z \leq z_{kr}$, the number of spectrum spikes related to higher harmonics increases and their amplitudes increase too, but the amplitude of primary wave decreases. In the distance of the stabilizing waveform, the loss of energy is due to nonlinear attenuation. The value of the differential attenuation coefficient is not a function of frequency. With regard to this fact, the number of harmonics is constant but the values of their amplitudes decrease. At a large distance from the transmitter $z \gg z_{kr}$, the spectral spikes related to higher harmonics slowly decay. Taking into account the above data one can notice that for investigation of large intensity waveform distortion the method of measuring harmonics amplitudes is very useful.

3. The Measuring system

The observation of nonlinear distortion of waveform during its propagation in water was carried out by measuring the harmonic amplitudes. The measure were carried out by means of a set up that is shown in Fig. 4.

Piezoelectric transducers in "sandwich" form are used as transmitters with the frequency 30 kHz and 81 kHz, respectively. In both cases the wave intensity was limited by cavitation phenomena [6].

The measurements of harmonic amplitudes were carried out on the axis of the transmitters' beams. The receiving signal from the receiving transducers was amplified by means of a measuring amplifier with a set of band pass filters allowing to measure and register the value of harmonic amplitudes.

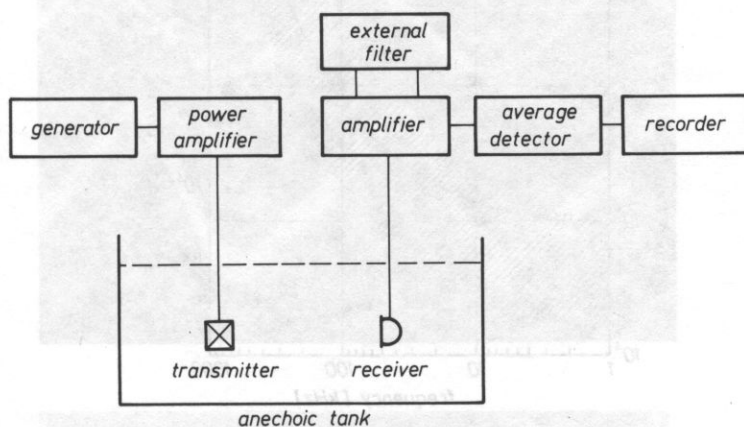


FIG. 4. The scheme of measuring set up of investigations of nonlinear waveform distortions.

4. Investigation results

Our investigations consisted in observing primary wave distortion and the creation of harmonics. In Fig. 6 the change of primary waveform shape (frequency 30 kHz and value of Reynolds number $Re_a = 8.5$) in water at the distance of $12 R_0$ is shown. R_0 is the Rayleigh length $R_0 = S/\lambda$, S – area of transmitter. Figure 7 shows the primary wave shape, the first harmonic and the second one at the distance of $6 R_0$ from the transmitter. The primary wave distortion ($f = 81$ kHz, $Re_a = 14.2$) is shown in Fig. 8. During investigation, time histories are recorded by means of a digital oscilloscope. One can notice the distortion of waveform shape and the first harmonic. On the second line of the oscilloscope view, the reference signal that is put into the transmitter is shown.

The distortion of wave shapes shown in the figures and photos are not too large. However, one can notice the differences between the length of the increase and decrease slopes.

The study carried out allows to observe the changes of wave spectrums during propagation in water. The wave's spectrum of the primary wave with frequency equal to 81 kHz ($Re_a = 14.2$) at the range $15 R_0$ from the transmitter (Fig. 8) is shown in Fig. 9.

The measuring results of the harmonic components of the 30 kHz primary wave ($Re_a = 8.5$) as a function of the range from the transmitter is presented in Fig. 10. The correction factor connected with the spherical spreading of the waveform outside the Rayleighs area ($z > R_0$) is taken into account.

The registered nonlinear distortion are relatively small as a result of the limitation of the wave intensity due to the occurrence of a cavitation in both cases. The increase of the cavitation threshold for the same frequency can be obtained following the increase of the static pressures or by increasing the localization of the depth of the transmitting transducer. A similar result can be achieved by using cleaned and degased water.

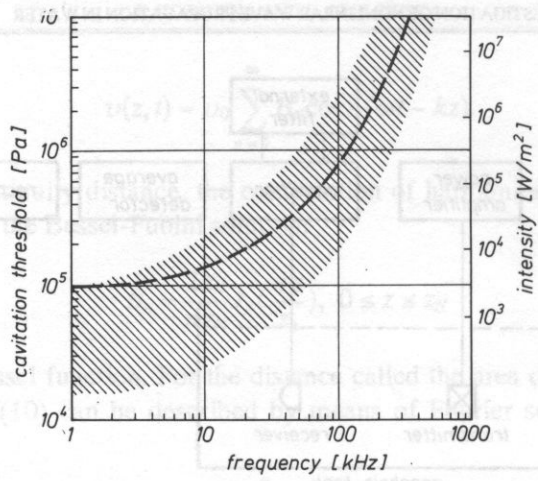


FIG. 5. The dependence of the cavitation threshold on the frequency [6].

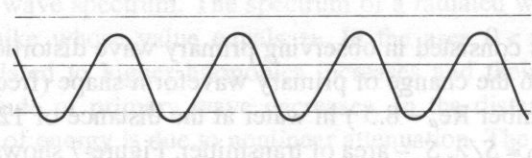


FIG. 6. The shape of the wave (frequency 30 kHz) measured at the distance of $12 R_0$ from the transmitter.

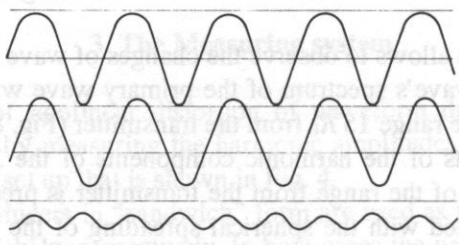
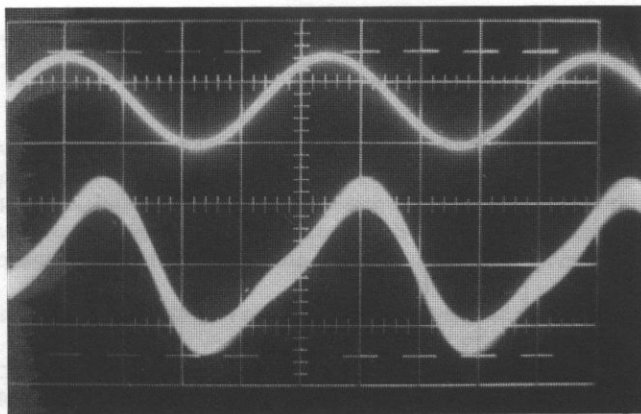
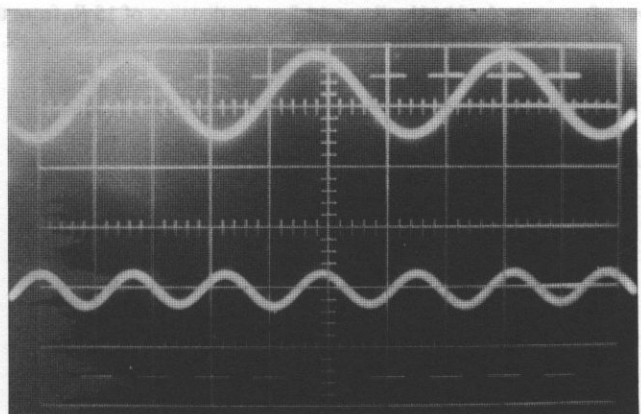


FIG. 7. The shape of the wave frequency 30 kHz and the first and the second harmonics measured at the distance of $6 R_0$ from the transmitter; $Re_a = 8.5$.

a)



b)



c)

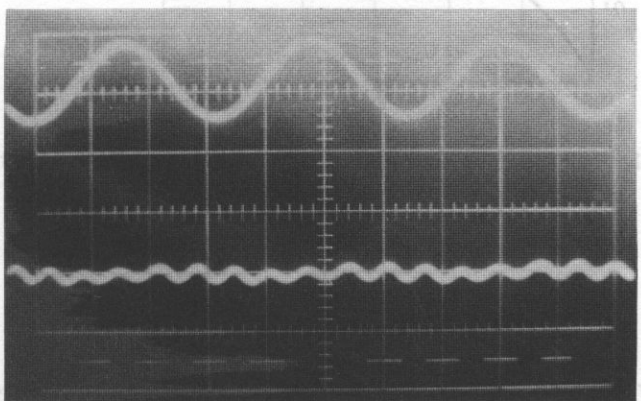


FIG. 8. The distortion of the wave shape (frequency 81 kHz) (a), the second harmonic (b) and the fifth one (c).

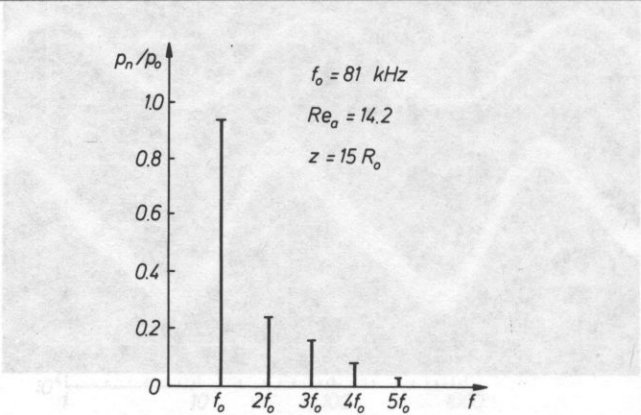


FIG. 9. Spectrum of wave frequency $f = 81 \text{ kHz}$, $Re_a = 14.2$ at the distance of $15 R_0$ from the transmitting transducer.

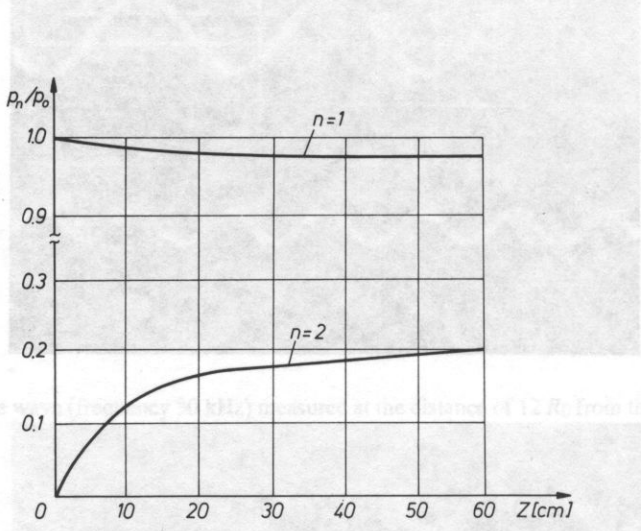


FIG. 10. The first and the second harmonic measurements as a function of the range from the primary wave transmitter $f = 30 \text{ kHz}$.

5. Conclusions

The results of the experimental investigation of the nonlinear wave propagation presented above allows to determine the wave distortion. The method based on measurements of harmonics amplitudes and their numbers as a function of the range from the wave source is effective in the case of a weak nonlinear interaction, too.

In particular, changes of the second harmonic amplitude can be used to determine the nonlinearity parameter B/A .

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Received on September 14, 1990

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The frequency dependence is established for the propagation velocity and attenuation coefficient of ultrasonic waves in dilute suspensions and emulsions and digital computations are performed for the aqueous emulsion of sunflower oil. The results show that the measurements of the propagation velocity of ultrasonic waves enable us to estimate the volume fraction of the suspended particles of both dilute and highly concentrated suspensions and emulsions.

Keywords: suspensions; emulsions; ultrasonic waves; propagation velocity; attenuation coefficient.

1. Introduction

In many areas of research such as cloud physics, underwater acoustics, medicine and in engineering application such as rocket propulsion, lubrication and so on are of interest the effective dynamic properties of some types of suspensions and emulsions. These properties are related to the acoustic wave velocities in the materials under study and their structure. Therefore some properties and structure parameters of suspensions and emulsions can be estimated on the basis of ultrasonic measurements.

In this paper, the two-component media are described using Truesdell's concept of replacing the noncontinuous components by fictitious continuous constituents [1]. The basic phenomenon responsible for attenuation and dispersion is, in the approach presented, relaxation of the phases (components) due to the velocity difference between them. In other words, attenuation and dispersion are caused by the inability of the phases to follow each other in the changes of the mechanical state, the changes being induced by the ultrasonic waves. The frequency dependence of the wave velocity and attenuation is evaluated by using a secular equation which, in turn, is obtained from hydrodynamic considerations.