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ULTRASONIC WAVES IN SOME BIOLOGICAL SUSPENSIONS AND EMULSIONS

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The frequency dependence is established for the propagation velocity and attenuation coefficient of ultrasonic waves in dilute suspensions and emulsions and digital computations are performed for the aqueous emulsion of sunflower oil. The results show that the measurements of the propagation velocity of ultrasonic waves enable us to estimate the volume fraction of the suspended particles of both dilute and highly concentrated suspensions and emulsions.

Keywords: suspensions; emulsions; ultrasonic waves; propagation velocity; attenuation coefficient.

1. Introduction

In many areas of research such as cloud physics, underwater acoustics, medicine and in engineering application such as rocket propulsion, lubrication and so on are of interest the effective dynamic properties of some types of suspensions and emulsions. These properties are related to the acoustic wave velocities in the materials under study and their structure. Therefore some properties and structure parameters of suspensions and emulsions can be estimated on the basis of ultrasonic measurements.

In this paper, the two-component media are described using Truesdell's concept of replacing the noncontinuous components by fictitious continuous constituents [1]. The basic phenomenon responsible for attenuation and dispersion is, in the approach presented, relaxation of the phases (components) due to the velocity difference between them. In other words, attenuation and dispersion are caused by the inability of the phases to follow each other in the changes of the mechanical state, the changes being induced by the ultrasonic waves. The frequency dependence of the wave velocity and attenuation is evaluated by using a secular equation which, in turn, is obtained from hydrodynamic considerations.

2. Equations of continuity and balance of linear momentum

As previously mentioned we consider a two-component medium with a Newtonian fluid as one of the components (phases). Throughout the paper this fluid is called the f – phase and all the quantities concerning this phase are denoted by abbreviations with the subscript or superscript f. The other phase is called the s – phase and is taken to have the form of an elastic skeleton with a statistical distribution of interconnected pores or a set of particles with arbitrary shape and size. The particles are assumed to be made up of a solid material or of another Newtonian fluid which is immiscible in the first one (f – phase) and chemically non-reacting with that. Throughout the paper every abbreviation with the subscript or superscript s denotes a quantity referred to the s – phase.

If $\rho_{(\alpha)}$ denotes the density of the phase component occupying the set of disjoint domains V_α of the Lebesgue measure $m[V_\alpha]$ and b_α is its volume fraction, then ρ_α defined as

$$\rho_\alpha = b_\alpha \rho_{(\alpha)}, \; b_\alpha = \frac{m[V_\alpha]}{m[V]}, \; \alpha = s, f \tag{2.1}$$

represents the mass of the α-th component per unit volume, and the formula

$$\rho = \sum_\alpha \rho_\alpha \tag{2.2}$$

defines the density of the medium occupying the domain V of the Lebesgue measure (volume) $m[V]$. The formulae (2.1) can be regarded as the definitions of the density ρ_α of the α-th fictitious constituent and its volume fraction with the constituent considered to be present in every point of the domain V.

Such a concept of treating an n – phase medium ($n = 2, 3, 4...$) as a mixture of n fictitious continuous constituents was proposed by TRUESDELL [1] and is employed throughout the paper. Equations (2.1) define the volume fraction of the α-th constituent and density to be set functions $b_\alpha(V)$ and $\rho_\alpha(V)$, respectively. However, for making the mathematical analysis methods suitable, all the scalar, vector and tensor fields considered in this paper are required to be point functions of the vector $\mathbf{r} = (x_1, x_2, x_3)$ of the position in the heterogeneous medium under consideration. The Radon-Nikodym theorem enables a wide class of set functions, $F(V)$, to be converted into point functions, as it was explained in the paper [2] by converting $b_\alpha(V)$ in the form given by the formula (2.1) into $b_\alpha(\mathbf{r})$. Use was made of the fact that $m[V_\alpha] = 0$ whenever $m[V] = 0$, where one can conclude that the additive set function $m[V_\alpha]$ is absolutely continuous with respect to Lebesgue measure. Hence, on the strength of the Radon-Nikodym theorem, there exists a point function $b_\alpha(\mathbf{r})$ such that [2]

$$m[V_\alpha] = b_\alpha(V) m[V] = \int_V b_\alpha(\mathbf{r}) \, d^3\mathbf{r}, \; \mathbf{r} \in V \tag{2.3}$$

or, equivalently

$$b_\alpha(\mathbf{r}) = \lim_{m[V] \to 0} \frac{m\,[V_\alpha]}{m\,[V]} \tag{2.4}$$

Now consider the ith, $i = 1, 2, 3$, component of the displacement on the α-th component during the unit time. Let $Q_i^\alpha(V)$ and $q_i^\alpha(V)$ denote the volumes sweeped out due to this movement by the α-th component contained in the whole volume of the domain V and in its unit volume element, respectively. Therefore we can write

$$Q_i^\alpha(V) = q_i^\alpha(V)\, m[V],\ i = 1, 2, 3. \tag{2.5}$$

In such a way we define the volume flux $q_i^\alpha(V)$ to be a set function. It can be converted to a point function, $q_i^\alpha(\mathbf{r})$, $\mathbf{r} \in V$, by employing the fact that $Q_i^\alpha(V) = 0$ whevener $m[V] = 0$. Hence we conclude that the additive set function $Q_i^\alpha(V)$ is absolutely continuous with respect to Lebesgue measure. Thus, by the Radon-Nikodym theorem, there exists a function $q_i^\alpha(r)$ such that

$$Q_i^\alpha(V) = q_i^\alpha(V)\, m[V] = \int_V q_i^\alpha(\mathbf{r})\, d^3\mathbf{r},\ \mathbf{r} \in V \tag{2.6}$$

Equivalently, one may write

$$q_i^\alpha(\mathbf{r}) = \lim_{m[V] \to 0} \frac{Q_i^\alpha(V)}{m[V]}. \tag{2.7}$$

On defining the volume flux as a point function $q_i^\alpha(r)$, we can write

$$q_i^\alpha(\mathbf{r}) = s_0\, v_i^\alpha(\mathbf{r});\quad v_i^\alpha(\mathbf{r}) = b_\alpha(\mathbf{r})\, v_i^{(\alpha)}(\mathbf{r}),\quad i = 1, 2, 3. \tag{2.8}$$

s_0 denotes the unit surface. Equations (2.6) and (2.8) or, equivalently (2.7) and (2.8) define the velocities $v_i^{(\alpha)}(\mathbf{r})$ and $v_i^\alpha(\mathbf{r})$ of the α-th component and fictitious continuous constituent, respectively, as point functions.

After defining the point functions $b_\alpha(\mathbf{r})$ and, consequently, $\rho_\alpha(\mathbf{r})$, and $v^\alpha(\mathbf{r})$, the equations of continuity and balance of linear momentum can be derived in the usual way (see, e.g. [2]) for the two-component medium under consideration. In this way we arrive at the following continuity equations:

$$\frac{\partial \rho_\alpha}{\partial t} + \operatorname{div}(\rho_{(\alpha)} \mathbf{v}_\alpha) = 0,\quad \alpha = s, f \tag{2.9}$$

which express the laws of mass conservation on the α-th continuous constituent, $\alpha = s, f$. If we add Eqs. (2.9) to each other and next if we add and subtract the expression $\operatorname{div}(\rho_f \mathbf{v}^s)$ to and from the resulting equation then we arrive at the following equation, after some manipulation:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho\, \mathbf{v}^{(s)} - \rho_f \mathbf{u}) = 0 \tag{2.10}$$

where the so-called diffusion velocity, $\mathbf{u}$, is defined as follows $\mathbf{u} = \mathbf{v}^{(s)} - \mathbf{v}^{(f)}$.

To obtain Eqs. (2.9) and, consequently, Eq. (2.10) by following [2], a volume $V(t)$ has been considered which is allowed to change with increasing time, t, due to progressing flow so that the mass, m, of the mixture contained remains constant. Then the changes in the momentum of the mass m can be equated to the sum of the forces acting on the medium occupying the volume $V(t)$. These forces can be divided into forces $\mathbf{F}_S$ acting on the surface $S(t)$ of $V(t)$, and body forces $\mathbf{F}_V$ which act upon the heterogeneous medium occupying every infinitesimal element $dV(t)$ of the volume $V(t)$. Moreover, the phases s and f act on each other but the forces of the phase interaction cancel out each other during summation over $\alpha = s, f$. Therefore, if we neglect the body forces the exterior forces due to gravity, etc. and represent the forces acting on the surface $dS(t)$ of the volume $dV(t)$ by the so-called effective stress tensor, σ, then, following [2, 3] we arrive at the equations

$$\rho_f \frac{\partial \mathbf{v}^{(f)}}{\partial t} + \rho_s \frac{\partial \mathbf{v}^{(s)}}{\partial t} + \rho_f (\mathbf{u}^{(f)} \cdot \operatorname{grad}) \mathbf{v}^{(f)} + \rho_s (\mathbf{v}^{(s)} \cdot \operatorname{grad}) \mathbf{v}^{(s)} = \operatorname{div} \sigma \tag{2.11}$$

σ denotes the stress acting in the medium as a whole.

In the remainder of this paper, we confine ourselves to flows in which the momentum transferred from the continuum f to the continuum s depends linearly on the velocities $\mathbf{v}^{(f)}$, $\mathbf{u}$, and the pressure p_f in the continuum f. Accordingly, we can write

$$\rho_s \frac{d\mathbf{v}^{(s)}}{dt} = \hat{P}\mathbf{u} + \hat{Q}\mathbf{v}^{(f)} + \hat{S}p_f \tag{2.12}$$

$\hat{P}$, $\hat{Q}$ and $\hat{S}$ denote operators which should be determined for every particular model of the medium and flow under study.

To define the pressure p in the medium as a whole, consider the mean internal compression δF_C exerted by the surrounding medium on the surface δS of a small volume δV. If δV can be treated as a good approximation of an infinitesimal volume element dV, then p is to be thought as the ratio of δF_C to δS and its value is supposed to be equal approximately to the mean value between the pressures inside the components occupying the volume δV, i.e.,

$$p = b_f p_{(f)} + b_s p_{(s)} := p_f + p_s \tag{2.13}$$

where $p_{(\alpha)}$, $\alpha = s, f$, stands for the pressure inside the α-th component.

In a similar way, if the s – component is also a Newtonian fluid, we define the dynamic viscosity η, and the second viscosity ζ of the medium as

$$\eta = b_f \eta_{(f)} + b_s \eta_{(s)}, \quad \zeta = b_f \zeta_{(f)} + b_s \zeta_{(s)}. \tag{2.14}$$

$\eta_{(\alpha)}$ and $\zeta_{(\alpha)}$ denote the dynamic and the second viscosity of the α-th component, respectively.

3. Linearized acoustic equations of the solid-fluid heterogeneous media

The scalar, vector and tensor quantities which are involved in the system of Eqs. (2.9) – (2.12) as unknown functions, $F(\mathbf{r}, t)$ are assumed to be of the form

$$F(\mathbf{r}, t) = \overline{F} + \Delta F(\mathbf{r}, t), \ \overline{F} = const. \tag{3.1}$$

$\Delta F(\mathbf{r}, t)$ denotes the local and instant fluctuation in a quantity $F(\mathbf{r}, t)$ about its equilibrium value which is denoted by the overbar. It is assumed that

$$\left| \frac{\Delta F(\mathbf{r}, t)}{\overline{F}} \right| << 1.$$

Henceforth the fluctuations $\Delta F(\mathbf{r}, t)$ only in the form of acoustic ultrasonic disturbances are of interest to us, simple harmonic time and position dependence being assumed, i.e.,

$$\Delta F(\mathbf{r}, t) = \Delta F_0 \exp\left[-i(\omega t - \mathbf{k} \cdot \mathbf{r})\right], \ \ \mathbf{k} = \left(\frac{\omega}{c} + i\mu\right)\mathbf{e}_k, \ \ i = (-1)^{1/2}, \ \mathbf{e}_k = 1. \tag{3.3}$$

The expressions (3.3) describe an attenuated plane wave propagating in the direction $\mathbf{e}_k$ and with the velocity c through a medium with the attenuation coefficient μ. The abbreviation ΔF_0 stands for the amplitude of the fluctuation $\Delta F(\mathbf{r}, t)$. Therefore the acoustic disturbances under consideration are assumed to be the periodic fluctuations (3.3) in the density $\Delta\rho_\alpha(\mathbf{r}, t)$, pressure $\Delta p_\alpha(\mathbf{r}, t)$, and the velocity $\mathbf{v}^\alpha(\mathbf{r}, t)$ about the respective equilibrium values

$$\overline{\rho}_\alpha(\mathbf{r}, t) = \overline{\rho}_\alpha = \text{const}, \ \ \overline{p}_\alpha(\mathbf{r}, t) = \overline{p}_\alpha = \text{const},$$

$$\overline{\mathbf{v}^{(\alpha)}}(\mathbf{r}, t) = 0, \ \ p_\alpha = b_\alpha p_\alpha, \ \ \alpha = s, f. \tag{3.4}$$

The acoustic disturbances are assumed to be adiabatic. Among others, this assumption implies the validity of the following equation:

$$\frac{dp_{(f)}(\mathbf{r}, t)}{dt} = c_{(f)}^2 \frac{d\rho_{(f)}(\mathbf{r}, t)}{dt} \Rightarrow c_{(f)}^2 = \frac{\Delta p_{(f)}}{\Delta \rho_{(f)}}. \tag{3.5}$$

$c_{(f)}$ denotes the velocity of the propagation of the wave given by the formulae (3.3) through the fluid f. If the following inequalities are assumed to be valid:

$$\left| \frac{d(b_f q_{(f)})}{dt} \right| >> q_{(f)} \frac{db_f}{dt}; \ \ q_f = b_f q_{(f)}, \ \ q = \rho, p \tag{3.6}$$

then

$$\frac{dq_{(f)}}{dt} = \frac{d(q_f/b_f)}{dt} \approx \frac{1}{b_f}\frac{dq_f}{dt}. \tag{3.7}$$

On substituting the relations (3.7) into Eqs. (3.5), we obtain

$$\frac{dp_f(\mathbf{r}, t)}{dt} = c_{(f)}^2 \frac{d\rho_f(\mathbf{r}, t)}{dt} \Rightarrow c_{(f)}^2 = \frac{\Delta p_{(f)}}{\Delta \rho_{(f)}} = \frac{\Delta p_f}{\Delta \rho_f} = c_f^2. \tag{3.8}$$

Thus, if the inequalities (3.6) are fulfilled, the velocities of the adiabatic propagation of the wave considered through both the component (fluid) f and f-th continuous constituent are equal to each other. Further in this paper, we will be interested only in this case.

Now the unknown functions $\rho_\alpha, p_\alpha, \mathbf{v}^{(\alpha)}, \alpha = s, f$ involved in Eqs. (2.9)–(2.12) are taken to be of the form given by the formulae (3.1). After such a substitution, only terms up to those linear in the acoustic disturbances $\Delta F(\mathbf{r}, t)$ are kept in these equations. This acoustic linearization of the flow equations yields, after some manipulation,

$$\frac{\partial(\Delta\rho_\alpha)}{\partial t} + \overline{\rho}_\alpha \operatorname{div} \mathbf{v}^{(\alpha)} = 0 \tag{3.9}$$

$$\frac{\partial(\Delta\rho)}{\partial t} + \overline{\rho} \operatorname{div} \mathbf{v}^{(s)} - \overline{\rho}_f \operatorname{div} \mathbf{u} = 0 \tag{3.10}$$

$$\overline{\rho}\frac{\partial \mathbf{v}^{(s)}}{\partial t} - \overline{\rho}_f \frac{\partial \mathbf{u}}{\partial t} = \operatorname{div}\sigma^L \tag{3.11}$$

$$\overline{\rho}_s \frac{\partial(\mathbf{u}^{(f)} + \mathbf{u})}{\partial t} - \hat{Q}\mathbf{u}^f - \mathbf{F}^s = 0. \tag{3.12}$$

$\mathbf{F}^s$ is the density of the viscous drag force experienced by the continuum s when it executes oscillations. $\hat{Q}\mathbf{v}^{(f)}$ is the external force produced by the sound field.

In this paper Eqs. (3.10)–(3.12) and one equation arbitrarily chosen from Eqs. (3.9) are treated as the system of equations describing the propagation of the acoustic disturbances through the media considered. For instance, if Eq. (3.9) for $\alpha = s$ is chosen, a system of 8 equations (3.9)–(3.12) is obtained with 10 unknown functions: $\Delta\rho_s, \Delta\rho_f, p_s, p_f, v_i^{(s)}, u_i, i = 1, 2, 3$. To equate the number of acoustic equations with that of the acoustic disturbances (unknown functions), we add to Eqs. (3.10)–(3.12) the relations expressing the assumption that both phases are disturbed adiabatically. Then the unknown functions can be sought for in the wave form given by the formulae (3.3). On substituting such forms of the disturbances into the set of acoustic equations, the set of algebraic linear and homogeneous equations is obtained for the disturbance amplitudes. The condition of the existence of the non-trivial solution to the system of linear homogeneous equations is obtained for the disturbance amplitudes. The condition of the existence of the non-trivial solution to the system of linear homogeneous equations leads to a secular (determinant) equation which enables the dispersion laws to be found for the medium considered. These dispersion laws express the frequency dependence of the propagation velocity and attenuation coefficient of acoustic waves which propagate through the two-component medium under study.

It should perhaps be stressed that it will be possible to establish the dispersion laws in the above way for every particular type of the two-component media under consideration if the respective forms are found for the effective stress tensor σ and operators $\hat{P}$ and $\hat{Q}$. In the next section such considerations are presented for dilute suspensions and emulsions.

4. Propagation velocity and attenuation of ultrasonic waves in dilute suspensions and emulsions

Basic assumptions

In this section dilute suspensions and emulsions are considered and use is made of the above described hydrodynamic approach to derive the dispersion laws for the media under study.

An emulsion is thought of as being a mixture of two chemically non-reacting and immiscible viscous fluids. One fluid called the f – phase is volumetrically dominant and the other called the s – phase is uniformly dispersed in the form of a large number of spherical particles with a radius R. The viscosities of both fluids are assumed to be independent of the disturbance frequency ω. The limit when the ratio $\eta_{(f)}/\eta_{(s)}$ is negligibly small as compared with a quantity of order unity, i.e.,

$$\frac{\eta_{(f)}}{\eta_{(s)}} << 1 \tag{4.1}$$

corresponds to the case when the considered mixture is a suspension of rigid spheres (s – phase) in a Newtonian fluid (f – phase).

The assumption that the considered suspensions and emulsions are dilute, i.e.,

$$b_s << 1 \tag{4.2}$$

allows us to neglect the forces with which the particles act on each other and take the divergence of the effective stress tensor to be

$$\operatorname{div}\sigma^L = \operatorname{grad}\left[-\Delta p + \eta_f \operatorname{grad}\mathbf{v}^{(f)} + \left(\frac{\eta_f}{3} + \zeta_f\right)\operatorname{div}\mathbf{v}^{(f)}\right]. \tag{4.3}$$

According to the formula (2.13), the pressure disturbance Δp, in the emulsion as a whole is thought of as being given by the formula

$$\Delta p = b_f \Delta p_{(f)} + b_s \Delta p_{(s)} := \Delta p_f + \Delta p_s \tag{4.4}$$

where Δp_α and $\Delta \rho_\alpha$ are related with each other by Eqs. (3.8), i.e.,

$$c_\alpha^2 = \frac{\Delta p_\alpha}{\Delta \rho_\alpha}, \quad \alpha = s, f \tag{4.5}$$

Calculation of $\mathbf{F}^s$

When an ultrasonic wave meets a particle, freely suspended in a viscous, compressible, non-heat-conducting fluid, both the wave and the particle are affected by the frequency-dependent force of the viscous interaction between the fluid and particle. The wave induces changes in the mixture and motion of the particle which, however, is unable to follow closely the changes in the environment. Therefore, the basic phenomenon responsible for attenuation and dispersion is relaxation of the dispersed phase due to the difference between the velocity of the suspended particles and the suspending fluid.

To calculate $\mathbf{F}^s$, it is necessary to consider the interaction $\mathbf{F}^s$ between a single particle oscillating with the velocity $\mathbf{v}^{(s)}$ and the fluid surrounding the particle closely and occupying an influence domain of the volume of order much less than $(2\pi \frac{c_f}{\omega})^3$. The fluid occupying the influence domain oscillates as a whole with the velocity $\mathbf{v}^{(f)}$ where $\mathbf{v}^{(f)}$ is here to be thought of as the velocity of the fluid f at the centre of the particle if it were absent. Before going on to calculate the interaction, we replace the finite influence region by an infinite one and assume that the interaction in the last fictitious case differs negligibly from that in the former real case. In such a model, the infinitely extended fluid oscillates as a whole with the velocity $\mathbf{v}^{(f)}$ and exert a surface force, $\mathbf{F}^s$, on the particle which oscillates with the velocity $\mathbf{v}^{(s)}$ In order to calculate the force $\mathbf{F}^s$, it is necessary to find the velocity and pressure fields $\mathbf{v}^{\text{ex}}(\mathbf{r}, t)$ and $p_{\text{ex}}(\mathbf{r}, t)$ in the fluid f at the boundary S of the particle (sphere).

Solving the auxiliary problem of determining $\mathbf{v}^{\text{ex}}$ and p_{ex}, both the fluids f and s are treated to be viscous and incompressible. Then the unknown quantities are involved in the following set of equations:

$$\frac{\partial \mathbf{v}^{(m)}}{\partial t} + (\mathbf{v}^{(m)} \cdot \operatorname{grad})\, \mathbf{v}^{(m)} = \frac{1}{\rho_{(m)}} \nabla(- p_{(m)} + \eta_{(m)} \operatorname{grad} \mathbf{v}^{(m)}) \tag{4.6}$$

$$\operatorname{div} \mathbf{v}^{(m)} = 0, \; m = \text{in}, \text{ex} \tag{4.7}$$

Throughout the considerations concerning the calculation of the force $\mathbf{F}^s$, every abbreviation with sub- or superscript "in" and "ex" denotes a quantity referred to the particle or surrounding fluid, respectively. Equations (4.6) and (4.7) are to be solved with the following conditions at the boundary S of the particle (sphere):

1) the normal components of velocity both inside and outside the particle vanish;
2) the tangential components of velocity and stress are continuous.

Moreover, every suspended particle is assumed to maintain its spherical shape due to surface tension. Therefore, calculating $\mathbf{F}^{(s)}$, we take into account the motions of the particle (sphere) as a whole, all shearing and frictional effects, but neglect the expansion and deformation effects.

Although the applied method of calculation of $\mathbf{F}^{(s)}$ follows to a wide extent AHUJA'S method [4], it is more general and the utility of their results is released from some limitations. Ahuja omitted the term with the time derivative in the Navier-Stokes equation describing the viscous and incompressible flow of the fluid inside an oscillating particle. In our calculation this term is not omitted but only the nonlinear (with respect to velocity) terms in Eqs. (4.6) are omitted. Introducing the respective dimensionless variables into Eqs. (4.6), one can verify that the nonlinear terms may be neglected if

$$\mathrm{St} = \frac{a}{R} << 1,\ \mathrm{Ru}_{(m)} = \frac{\overline{p}_{(m)}}{\rho_{(m)} a^2 \omega^2} << 1,\ \mathrm{Re}_{(m)} = \frac{\mathrm{Ra}\omega\overline{\rho}_{(m)}}{\eta_{(m)}} << 1. \tag{4.8}$$

a denotes the oscillation amplitude. The abbreviations St, $\mathrm{Ru}_{(m)}$, and $\mathrm{Re}_{(m)}$, m = in, ex, denotes the Strouhall, Ruark and Reynolds number, respectively. On finding the expressions for $\mathbf{v}^{ex}$ and p_{ex} from Eqs. (4.6) and (4.7) with their boundary conditions and conditions (4.8), we substitute the right hand-sides of $\mathbf{v}^{ex}$ and p^{ex} into the following well-known formula:

$$\mathbf{F}^{(s)} = \int_S (-p_{ex}\cos\theta + \sigma_{rr}^{ex}\cos\theta - \sigma_r^{ex}\sin\theta)\, ds \tag{4.9}$$

where the integration is to be performed over the surface of the spherical particle. The integrand of Eq. (4.9) is presented in the polar spherical coordinates with the polar axis parallel to $\mathbf{u}$. After evaluating the integral (4.9), we get the drag force as

$$\mathbf{F}^{(s)} = -[4\pi(i\kappa + \kappa^2 R)\eta_{(f)} C_0 - \frac{2}{3} i\omega\overline{\rho}_{(f)} R^3]\mathbf{u} \tag{4.10}$$

$$\kappa = (1+i)\left[\frac{\omega\overline{\rho}_{(f)}}{2\eta_{(f)}}\right]^{1/2} \tag{4.11}$$

$$C_0 = \frac{3}{2}\,\frac{3\eta_{(s)} + 2\eta_{(f)} + 2\eta_{(s)}\Psi}{\frac{3}{R} i\kappa(\eta_{(f)} + \eta_{(s)}) + \eta_{(s)}\kappa^2 + \frac{2}{R} i\kappa\eta_{(s)}\Psi} \tag{4.12}$$

$$\Psi = \sum_{n=1}^{\infty} n(n+1)H_n \,/\, [1 + \sum_{n=1}^{\infty}(n+1)H_n] \tag{4.13}$$

$$H_n = (-i)^n \left[\frac{R^2\omega\overline{\rho}_{(s)}}{\eta_{(s)}}\right]^n \Big/ \prod_{k=1}^{n}(2k+2)(2k+5) \tag{4.14}$$

Now the density $\mathbf{F}^s$ of the viscous drag force experienced by the continuum s when it executes oscillations may be evaluated from the formula

$$F^s = \frac{3}{4} b_s \pi^{-1} R^{-3} F^{(s)}. \tag{4.15}$$

The last expression which is required to be found before going on to derive the secular equation is the operator $\hat{Q}$ which has been introduced into Eqs. (2.12) and (3.12), Making use of [5, Eq. (7)], we get the operator $\hat{Q}$ as

$$Q = \overline{\rho}_s B \frac{\partial}{\partial t},\ B = \frac{b_s\overline{\rho}_f}{b_f\overline{\rho}_s} = \frac{\overline{\rho}_{(f)}}{\overline{\rho}_{(s)}}. \tag{4.16}$$

Secular equation

Upon substituting the expressions (3.8), (4.3), (4.15) and (4.16) into Eqs. (3.11) and (3.12), we obtain from Eqs. (3.9)–(3.11), after some manipulation

$$\frac{\partial(\Delta\rho)}{\partial t} + \bar{\rho}\,\mathrm{div}\,\mathbf{v}^{(f)} + \bar{\rho}_s\mathrm{div}\,\mathbf{u} = 0 \tag{4.17}$$

$$\frac{\partial(\Delta\rho_f)}{\partial t} + \bar{\rho}_f\mathrm{div}\,\mathbf{v}^{(f)} = 0 \tag{4.18}$$

$$\bar{\rho}\frac{\partial \mathbf{v}^{(f)}}{\partial t} + \bar{\rho}_s\frac{\partial \mathbf{u}}{\partial t} + c_f^2\,\mathrm{grad}\,(\Delta\rho_f) - \eta_f\Delta\mathbf{v}^{(f)} - (\zeta_f + \frac{\eta_f}{3})\,\mathrm{grad\,div}\,\mathbf{v}^{(f)} = 0 \tag{4.19}$$

$$[\frac{\partial}{\partial t}(1+\frac{1}{2}B) + D]\mathbf{u} + (1-B)\frac{\partial \mathbf{v}^{(f)}}{\partial t} = 0; \tag{4.20}$$

$$D = 3B(\bar{\rho}_f)^{-1}R^{-2}(i\kappa R^{-1} + \kappa^2)\eta_f C_0. \tag{4.21}$$

Now let us suppose that the unit vector $\mathbf{e}_k$ in the formulae (3.3) is parallel to the unit vector $\mathbf{e}_1$ along the direction of the reference axis Ox_1. Then the substitution of the formulae (3.3) into Eqs. (4.17)–(4.20) leads us to the following secular equation:

$$\begin{vmatrix} -i\omega & 0 & ik\bar{\rho} & ik\bar{\rho}_s \\ 0 & -i\omega & ik\bar{\rho}_f & 0 \\ 0 & ic_f^2k & [-i\omega\bar{\rho} + k^2(\frac{4}{3}\eta_f + \zeta_f)] & -i\omega\bar{\rho}_s \\ 0 & 0 & -Z & 1 \end{vmatrix} \tag{4.22}$$

where the successive column of the determinant correspond to the amplitudes of the disturbances $\Delta\rho$, $\Delta\rho_f$, $\mathbf{v}^{(f)}$ and $\mathbf{u}$, respectively.

In accordance with Eq. (4.20), Z is defined as

$$Z = i\omega(1-B)[D - i\omega(1+\tfrac{1}{2}B)]^{-1}. \tag{4.23}$$

Equation (4.22) leads to the following equation

$$\left(\frac{1}{c^*} + i\mu^*\right)^2 = [\bar{\rho}_f + \bar{\rho}_s(Z+1)]c_f^2[\bar{\rho}_f c_f^2 - i\omega(\frac{4}{3}\eta_f + \zeta_f)]^{-1} \tag{4.24}$$

where the dimensionless propagation velocity c^* and attenuation coefficient μ^* of the two-component medium is defined as

$$c^* = \frac{c}{c_f},\ \mu^* = \frac{\mu c_f}{\omega}. \tag{4.25}$$

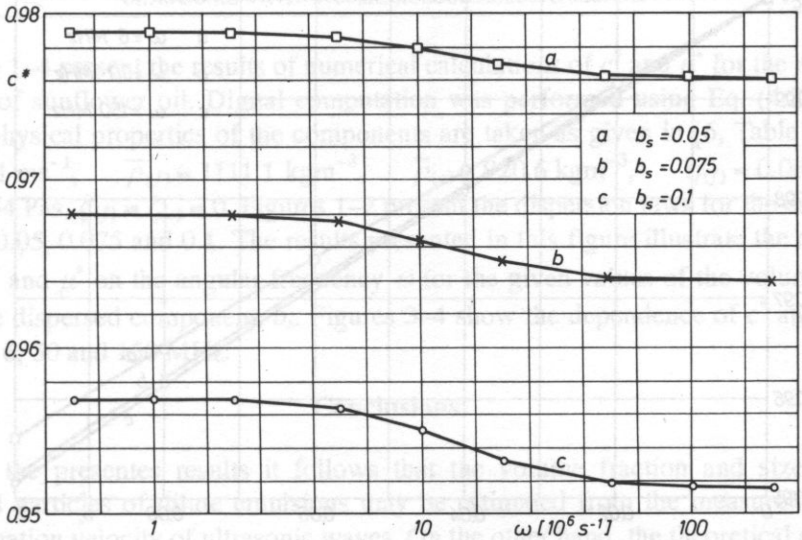

FIG. 1. The frequency dependence of the dimensionless propagation velocity c^* for $b_s = 0.05, 0.075, 0.1$ and $R = 2 \cdot 10^{-6}$ m.

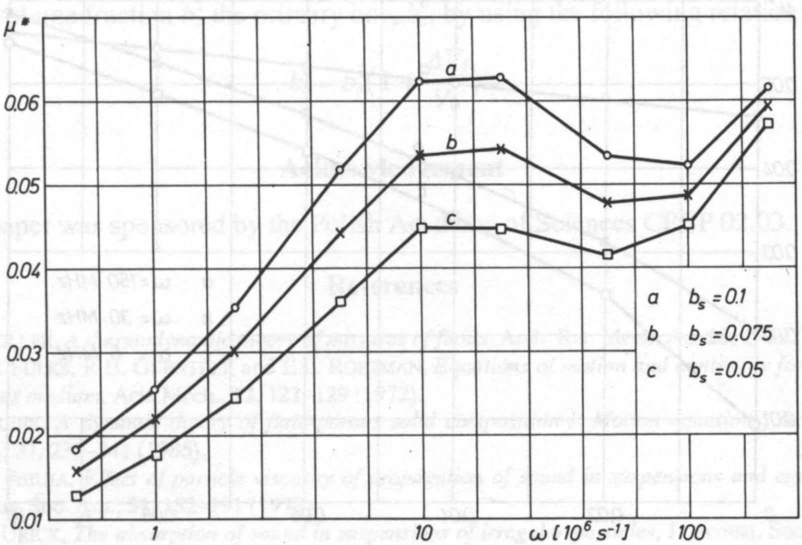

FIG. 2. The frequency dependence of the dimensionless attenuation coefficient μ^* for $b_s = 0.05, 0.75, 0.1$ and $R = 2 \cdot 10^{-6}$ m.

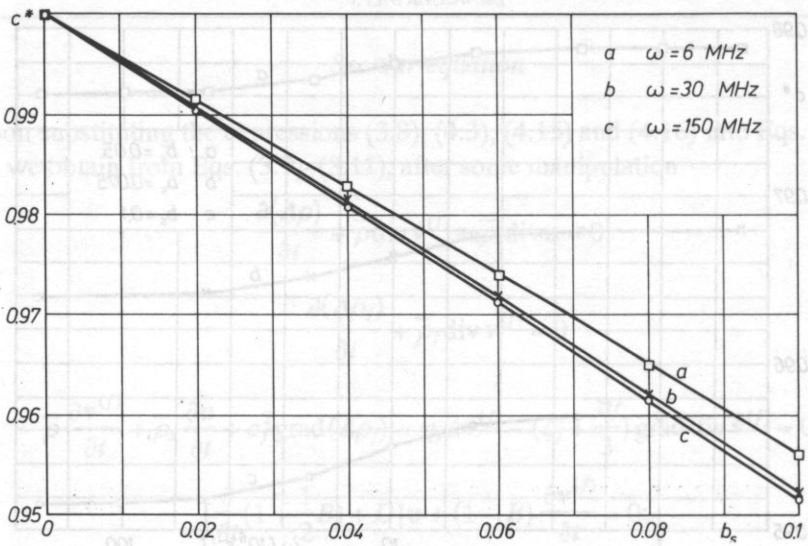

FIG. 3. The values of the dimensionless propagation velocity c^* as the function of the volume fraction b_s for $\omega = 6$, 30, 150 MHz and $R = 2 \cdot 10^{-6}$m.

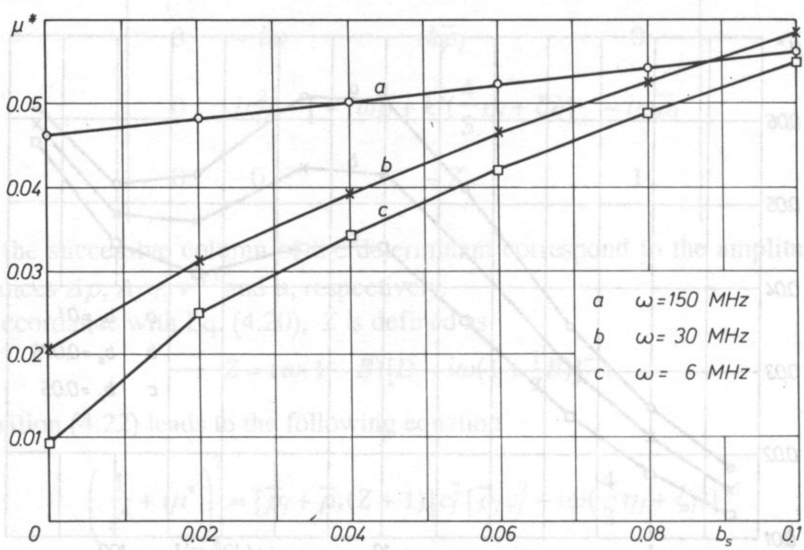

FIG. 4. The values of the dimensionless attenuation coefficient μ^* as the function of the volume fraction b_s for $\omega = 6$, 30, 150 MHz and $R = 2 \cdot 10^{-6}$m.

Figure 1–4 present the results of numerical calculations of c^* and μ^* for the aqueous emulsion of sunflower oil. Digital computation was performed using Eq. (4.24). The required physical properties of the components are taken as given in [6, Table 1], i.e., $c_f = 1604.1\ \mathrm{ms^{-1}}$, $\overline{\rho}_{(f)} = 1111.1\ \mathrm{kgm^{-3}}$, $\overline{\rho}_{(s)} = 920.6\ \mathrm{kgm^{-3}}$, $\eta_{(f)} = 0.0677$ Pas, $\eta_{(s)} = 0.054$ Pas, $\zeta_{(f)} = \zeta_{(s)} = 0$. Figures 1–2 present the dispersion laws for the emulsion with $b_s = 0.05, 0.075$ and 0.1. The results presented in this figure illustrate the dependence of c^* and μ^* on the angular frequency ω for the given values of the volume fraction of the dispersed component, b_s. Figures 3–4 show the dependence of c^* and μ^* on b_s for $\omega = 6$, 30 and 150 MHz.

Conclusions

From the presentes results it follows that the volume fraction and size of the suspended particles of dilute emulsions may be estimated from the measurements of the propagation velocity of ultrasonic waves. On the other hand, the theoretical description of ultrasonic wave propagation in dilute emulsions is rather simple as compared with that for highly concentrated emulsions. It should perhaps be stressed that this method of the estimation of b_s and R could be used not only for dilute suspensions and emulsions which have been considered above, but also for highly concentrated two-phase media. It follows from the fact that every highly concentrated two-phase medium of the primary volume V^0 may be converted into a dilute one by introducing an amount ΔV_f of the suspending fluid. If the volume ΔV_f is known, then we may evaluate from the final volume fraction b_s^1 the primary one, b_s^0, by using the following relation:

$$b_s^0 = b_s^1\left(1 + \frac{\Delta V_f}{V_0}\right).$$

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