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DESIGN PRINCIPLES OF TRANSDUCERS WITH MATCHING LAYERS BASED ON ADMITTANCE MEASUREMENTS

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Basing on modified Mason's equivalent circuit of an acoustic probe the authors carried out the computation of admittances and pulses reflected from the ideal reflector immersed in water.

The relation between the admittance of a PZT transducer with acoustic layers (matching the acoustic impedance of the transducer to the acoustic impedance of the human body) and the thickness of the layer is given.

It is shown the optimal layer thickness should be determined as a quarter wave-length $\lambda_e/4$ (calculated for the electric resonance frequency). In such a case the admittance curve is symmetrical and reaches the minimum, and the reflected pulse is shortest. Small thickness changes of the layer causes more distinct changes of the admittance than of the reflected pulses.

The influence of the acoustic impedance of layers and electromechanical coupling coefficient k_t on the admittance curves is discussed.

The admittances of the transducer measured for different thicknesses of the acoustic matching layer are shown. The results are in good agreement with computations.

1. Introduction

The sensitivity of the ultrasonic diagnostic equipment depends on the ultrasonic probe construction. In order to increase the sensitivity of the probe, layers matching the acoustic impedance of the transducer to the human body are applied. As yet, the method of preparing the layers was very time consuming. To determine the optimal thickness of the layer the pulse reflected from a reflector immersed in water was observed. After each polishing of the layer surface the probe was connected to the measurement device, its position was corrected to obtain the maximum pulse amplitude. The polishing was finished when the reflected pulse ceased to increase.

Now we have devised a new method of determination of the optimal thickness of the layer. During polishing of the layer admittance of the probe immersed in water is

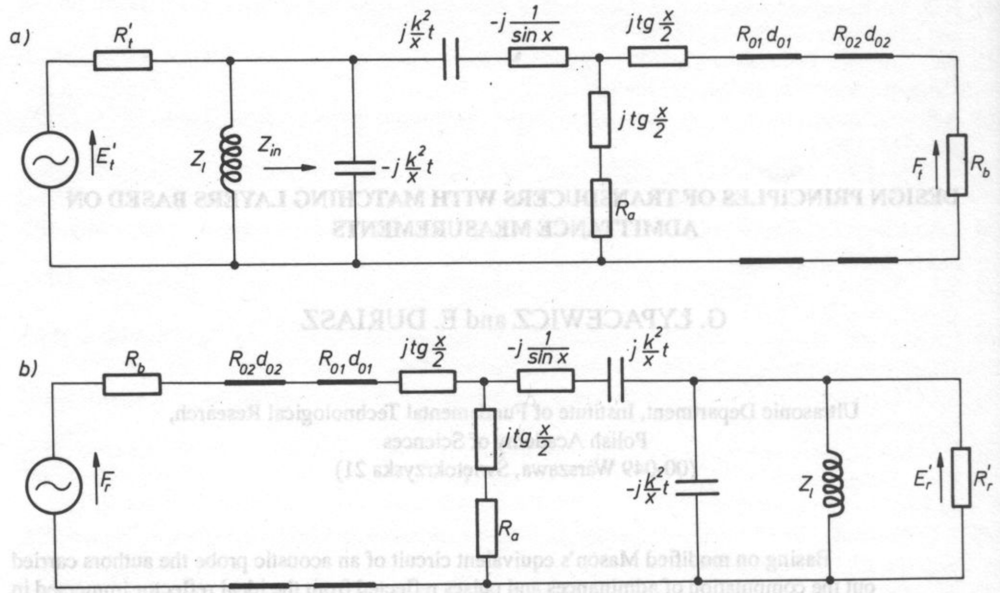

FIG. 1. Equivalent circuit of ultrasonic transducer with acoustic matching layers and parallel compensating inductance, a – transmitting, b – receiving.

measured automatically. Calculation are carried out, modified Mason's model described in [6] being assumed (Fig. 1).

All quantities describing the probe and the input transmitting-receiving system have been transferred to the mechanical side of the electromechanical transformer, and the corresponding relative values of parameters have been introduced:

– frequency $x = \pi f/f_m$;

– acoustic impedance of the transducer $R_p/R_p = 1$; where $R_p = A\,\rho_p c_p$;

– acoustic impedance of back load $R_a = A\,\rho_a c_a/R_p$,

– acoustic impedance of investigated medium $R_b = A\,\rho_b c_b/R_p$;

– acoustic impedance of matching layers

$$R_{01} = A\,\rho_{01} c_{01}/R_p,\ R_{02} = A\,\rho_{02} c_{02}/R_p;$$

– thickness of matching layers – d_{01}, d_{02}, where $d_{01} = \lambda_m/n$, $d_{02} = \lambda_m/p$;

– impedance of parallel inductance $Z_l = k_t^2 x/\pi m^2$;

– voltage of transmitter $E'_t = E_t N$;

– voltage of pulse reflected from an ideal reflector $E'_r = E_r N$;

– resistance of transmitter $R'_t = R_t N^2/R_p$;

– resistance of receiver $R'_r = R_r N^2/R_p$; where f_m – frequency of transducer of the mechanical resonance, A – surface of transducer, $\rho_p c_p$, $\rho_a c_b$, $\rho_{01} c_{01}$, $\rho_{02} c_{02}$ – acoustic impedance of the transducer, back load, the investigated medium and the matching layers respectively; λ_m, λ_e – wavelength for the mechanical f_m and electric f_e transducer resonance frequency; n, p are parameters of layer thickness (for instance for $n = 4$ thickness of layer $d_{01} = \lambda_m/4$, for $n = 4 f_e/f_m$ the thickness $d_{01} = \lambda_e/4$), m is the parameter describing parallel inductance (for $m = 1$ inductance compensates the clamped capacity C_0 for frequency f_m, while for the electric resonance $m = f_e/f_m$); R_t, R_r – input resistances of the transmitter and receiver, $N^2 = 2 k_t^2 f_m C_0 R_p$ – turns ratio of the electromechanical transformer, C_0 – clamped capacitance of the transducer, k_t – electromechanical coupling coefficient for the thickness vibration, E_t – voltage of the incident wave, E_r – voltage of the reflected wave.

In our work we assumed that transducer exhibits no mechanical and dielectric losses, the voltage exciting the transducer is the Dirac pulse – $E_t = \delta(t)$, $R_a = 0$, the ceramics is of PZT type ($\rho_p c_p = 34 \cdot 10^6$ kgm^2/s, $k_t = 0.4 – 0.7$ the investigated medium is water ($\rho_b c_b = 1.5 \cdot 10^6$ kgm^2/s), $R_t = 0.01\, Z_{in}(f_e)$, $R_r = 10\, Z_{in} f_e$ where Z_{in} – electrical impedance of the transducer.

2. Relation between layer thickness and admittance

The admittance of the transducer is distinctly dependent on the thickness of the matching layer. In Fig. 2 the admittance calculated for the PZT transducer ($\rho_p c_p = 34 \cdot 10^6$ kgm^2/s, $k_t = 0.5$) with matching layer of a different thickness is shown. The acoustic impedance of the layer was determined according to DE SILETS'S formulae [7, 8].

As we can see, the value of admittance reaches its minimum when the thickness of the layer is equal to $(2k + 1)\lambda_e/4$ (Fig. 2a, g) (where $k = 0, 1, 2, ...k$), whereas for the layer thickness equal to $2k\lambda_e/4$ the admittance is close to that of the transducer without any layers, loaded by water (Fig. 2c, l).

It should be noticed that minimal admittance is obtained when the layer thickness is calculated not for mechanical, but the for electric resonance frequency $d_{01} = \lambda_e/4$ (Fig. 2g, h) [6].

It is obvious that the amplitude and shape of the reflected pulse depends on the matching layer. In Fig. 3 the reflected pulses calculated for the same layer thickness as in Fig. 2 are shown. As it could be expected, the optimal pulse is obtained for $d_{01} = \lambda_e/4$ (Fig. 3g).

Small changes (5%) of layer thickness do not noticeably influence the reflected pulse (Fig. 4), whereas we can see a distinct differences of admittance for the same changes of layer thickness (Fig. 5a, b, c). The acoustic impedance of matching layer was determined by the authors on the basis of various assumption. In this paper the impedance calculated according to CHEBYSHEFF [2], SUOQUET [9] and DE SILETS [7, 8] are taken into account.

For optimal thickness equal to $\lambda_e/4$, in all cases the corresponding curves approach symmetric form (Fig. 5b, e, h). It means that measurement of the admittance is a more sensitive method of the layer thickness than measurement of the reflected pulse.

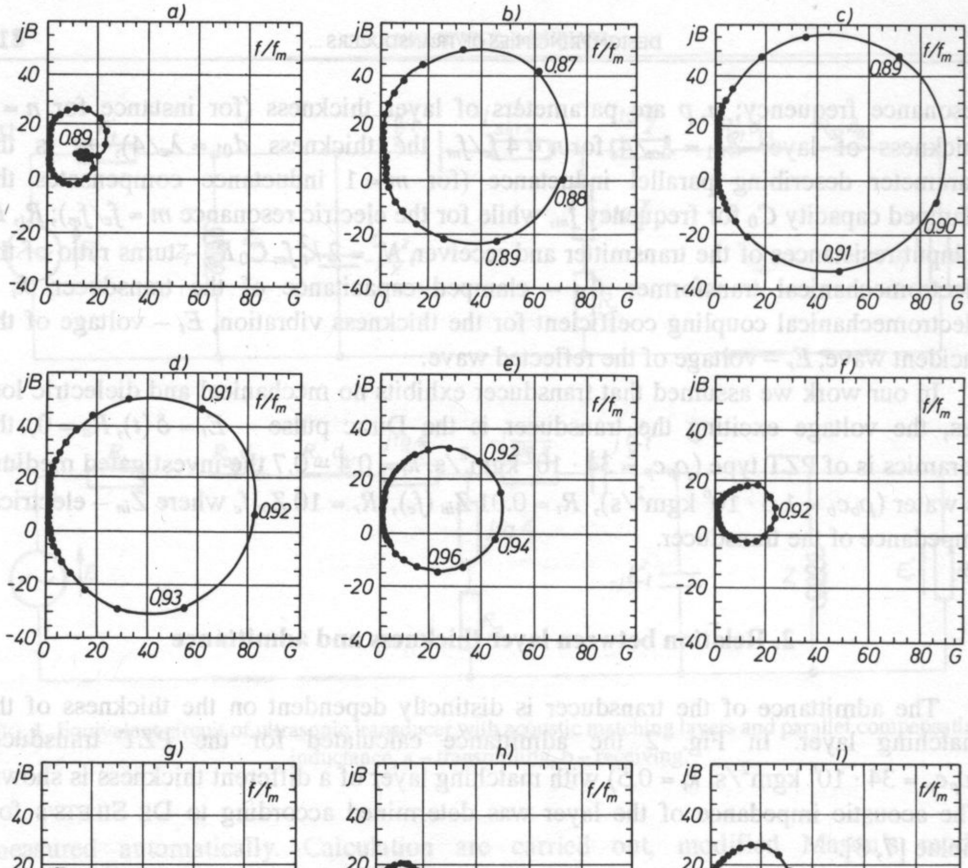

FIG. 2. Relative admittance calculated for PZT transducer ($k_t = 0.5$, $R_b = 0.044$) with acoustic matching layer calculated according to De Silet's formulae ($R_{01} = 0.122$) for different layer thicknesses.

a – $n = 1.2$ ($d_{01} = \frac{3}{4}\lambda_e$)	b – $n = 1.5$	c – $n = 1.78$ ($d_{01} = \lambda_e/2$)
d – $n = 2$ ($d_{01} = \lambda_m/2$)	e – $n = 2.5$	f – $n = 3$
g – $n = 3.55$ ($d_{01} = \lambda_e/4$)	h – $n = 4$ ($d_{01} = \lambda_m/4$)	i – $n = 5$
j – $n = 7.5$	k – $n = 10$	l – without layer.

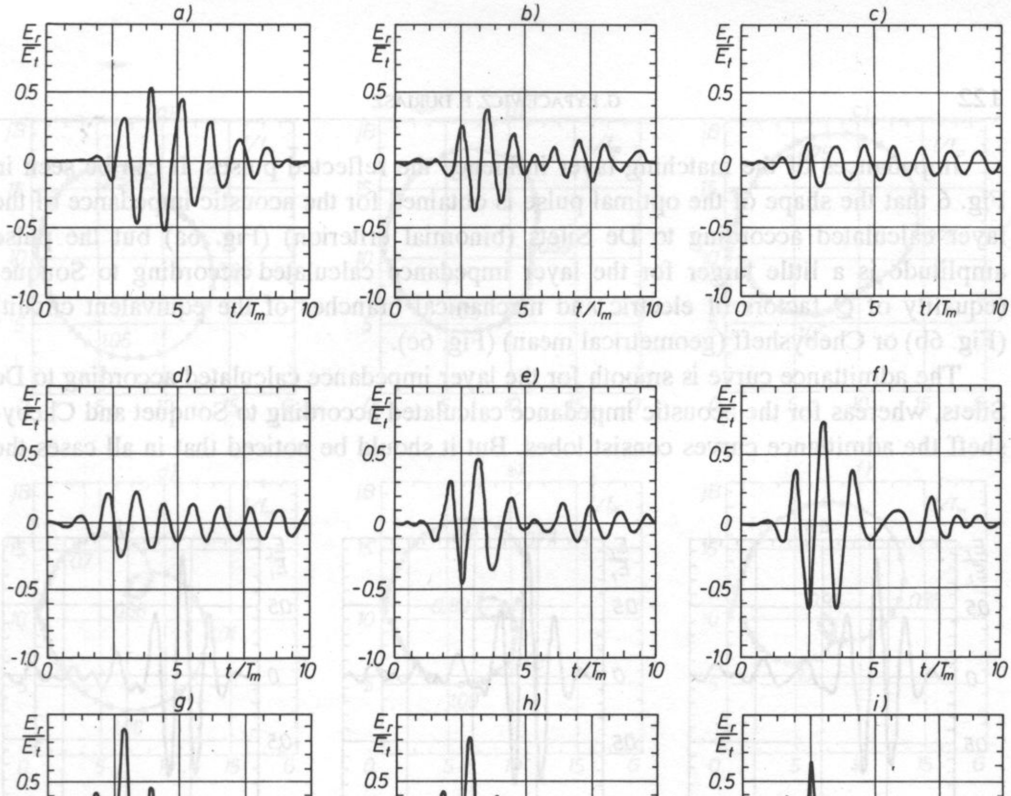

FIG. 3. Pulses reflected from an ideal reflector received by PZT transducer ($k_t = 0.5$, $R_b = 0.044$) with acoustic matching layer calculated according to De Silet's formulae ($R_{01} = 0.122$) and with parallel inductance compensating capacitance C_0 at electric resonanse frequency f_e ($m = 0.89$) for different layer thicknesses.

a – $n = 1.2$ ($d_{01} = \frac{3}{4}\lambda_e$) b – $n = 1.5$ c – $n = 1.78$ ($d_{01} = \lambda_e/2$)
d – $n = 2$ ($d_{01} = \lambda_m/2$) e – $n = 2.5$ f – $n = 3$
g – $n = 3.55$ ($d_{01} = \lambda_e/4$) h – $n = 4$ ($d_{01} = \lambda_m/4$) i – $n = 5$
j – $n = 7.5$ k – $n = 10$ l – without laye.r

Impedances of the matching layer influence the reflected pulses. It can be seen in Fig. 6 that the shape of the optimal pulse is obtained for the acoustic impedance of the layer calculated according to De Silets (binomial criterion) (Fig. 6a) but the pulse amplitude is a little larger for the layer impedance calculated according to Souquet (equality of Q factors of electric and mechanical branches of the equivalent circuit) (Fig. 6b) or Chebysheff (geometrical mean) (Fig. 6c).

The admittance curve is smooth for the layer impedance calculated according to De Silets, whereas for the acoustic impedance calculated according to Souquet and Chebysheff the admittance curves consist lobes. But it should be noticed that in all cases the

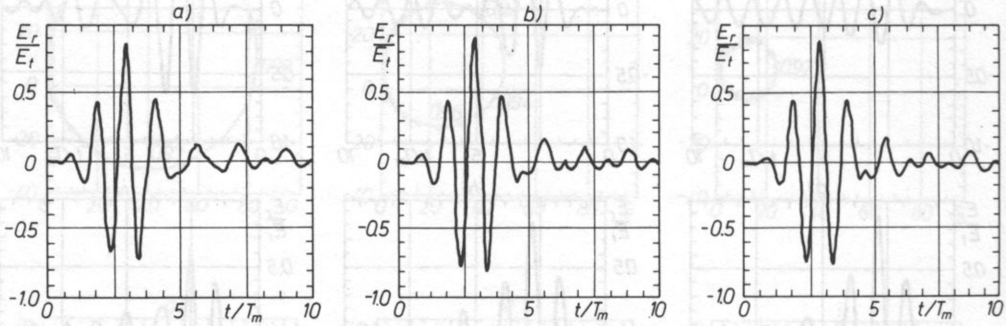

FIG. 4. Pulses reflected from an ideal reflector received by PZT transducer ($k_t = 0.5, R_b = 0.044$) with acoustic matching layer calculated according to De Silet's formulae ($R_{01} = 0.122$) and with parallel inductance compensating capacitance C_0 at electric resonance frequency f_e ($m = 0.89$) for different layer thicknesses: a $-\lambda_e/4 + 5\%$, b $- \lambda_e/4$ ($n = 3.55$), c $- \lambda_e/4 - 5\%$.

curved approaches symmetric formes for the layer thickness equal to $\lambda_e/4$. It means that the optimal thickness of the layer is independent of the acoustic impedance of the layer.

Various types of PZT ceramics have different electromechanical coupling coefficients k_t. For increasing k_t the band-width is larger [1] and the pulse becames shorter (Fig. 7) but amplitudes of the pulses are almost the same (the same electrical matching of the probe to the transmitter and receiver has been assumed).

The influence of the coupling coefficient k_t on the shape of the admittance curve is rather small, although it should be noticed that for greater k_t the imaginary part of admittance is smaller [5].

3. The second matching layer

Application of two matching layers increases the reflected the pulse (cf. Fig. 9a and c) and widens and flattens characteristics of the modulus and phase of the transducer impedances (cf. Fig. 9b and d).

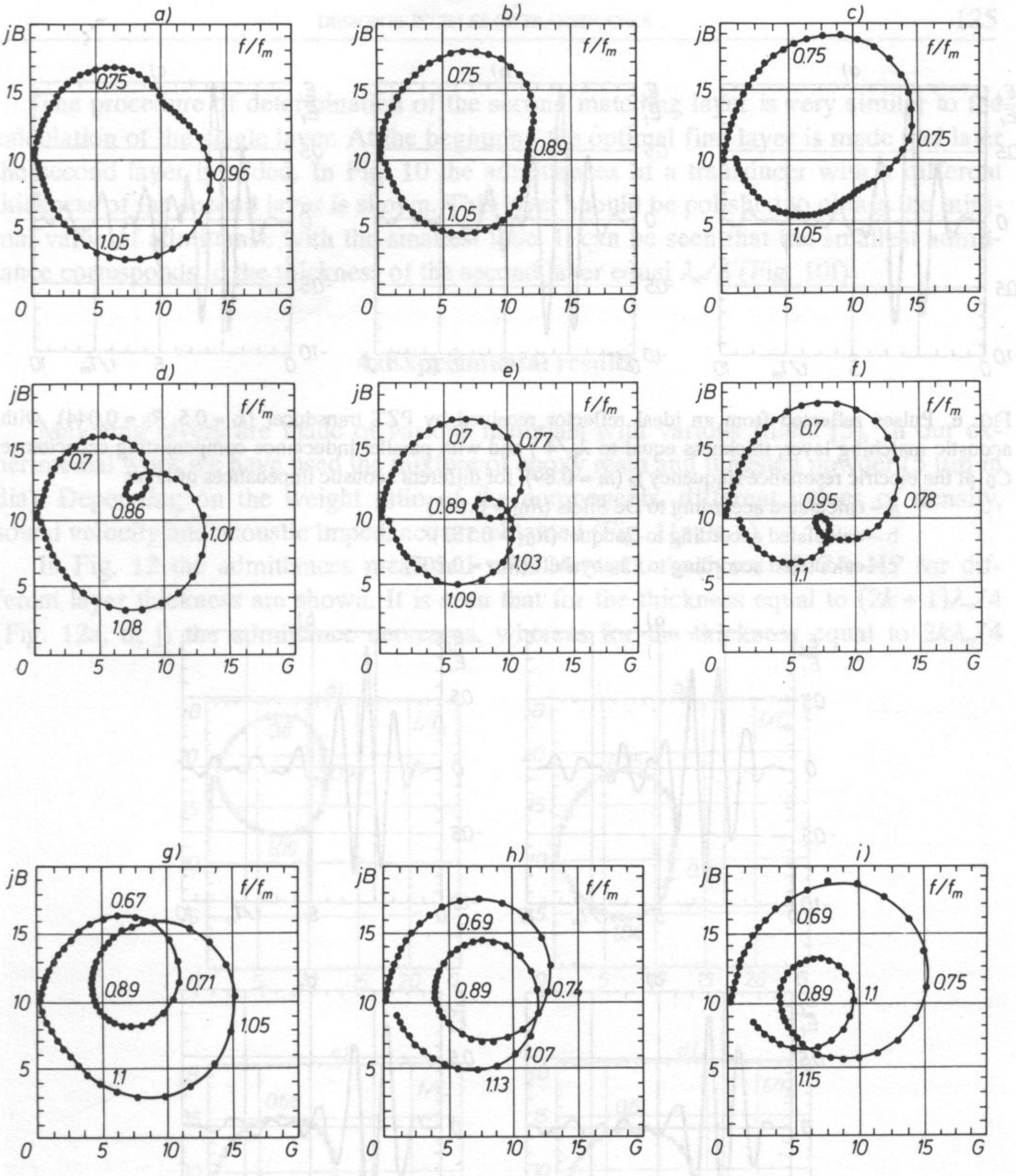

FIG. 5. Relative admittance calculated for PZT transducer ($k_t = 0.5, R_b = 0.044$) with acoustic matching layer calculated according to:

De Silet's formulae ($R_{01} = 0.122$) – a, b, c
Souquet's formulae ($R_{01} = 0.153$) – d, e, f
Chebysheff's formulae ($R_{01} = 0.207$) – g, h, i
for different layer thicknesses:

a, d, g – $\lambda_e/4 + 5\%$
b, e, h – $\lambda_e/4$ ($n = 3{,}55$)
c, f, i $\lambda_e/4 - 5\%$.

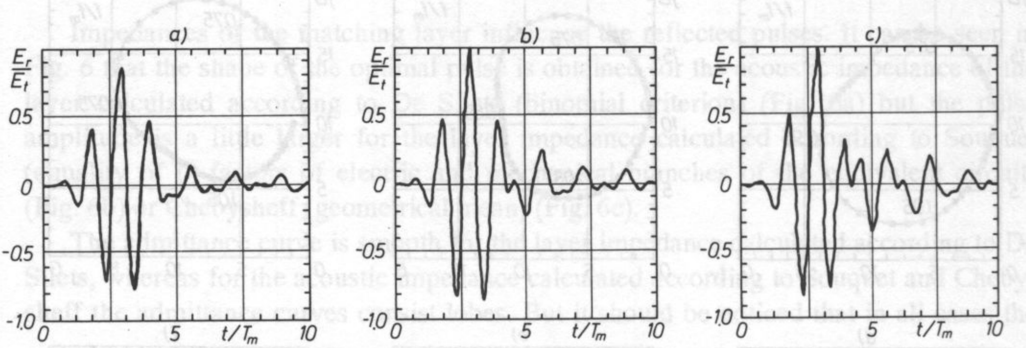

FIG. 6. Pulses reflected from an ideal reflector received by PZT transducer ($k_t = 0.5, R_b = 0.044$) with acoustic matching layer, thickness equal to $\lambda_e/4$) and with parallel inductance compensating capacitance C_0 at the electric resonance frequency f_e ($m = 0.89$) for different acoustic impedances of layer

a – calculated according to De Silets ($R_{01} = 0.122$)
b – calculated according to Souquet ($R_{01} = 0.153$)
c – calculated according to Chebysheff ($R_{01} = 0.207$).

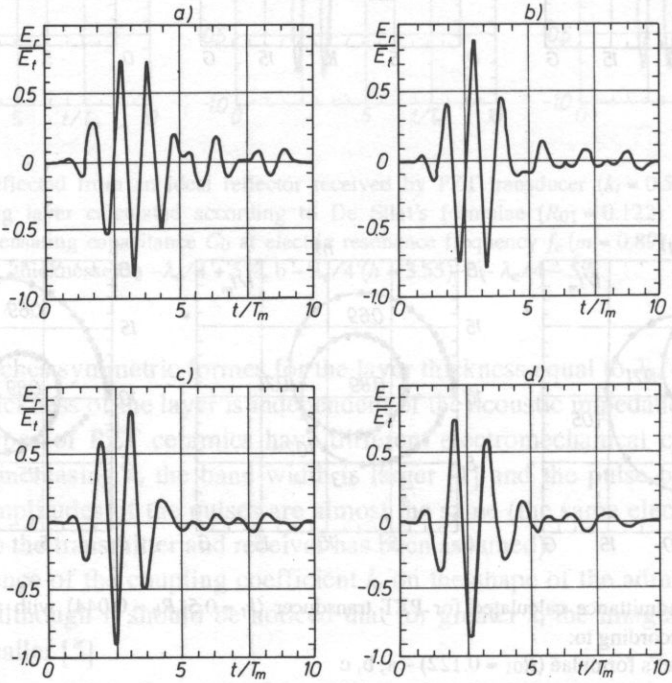

FIG. 7. Pulses reflected from an ideal reflector received by PZT transducer ($R_b = 0.044$) with different coupling coefficient k_t, with acoustic matching layer (impedance calculated according De Silets's formulae – $R_{01} = 0.122$, thickness equal to $\lambda_e/4$) and with parallel inductance compensating capacitance C_0 at electric resonance frequency f_e

a – $k_t = 0.4$ $n = 3.7$ $m = 0.93$ b– $k_t = 0.5$ $n = 3.55$ $m = 0.89$
c – $k_t = 0.6$ $n = 3.3$ $m = 0.82$ d – $k_t = 0.7$ $n = 3.0$ $m = 0.75$

The procedure of determination of the second matching layer is very similar to the calculation of the single layer. At the beginning the optimal first layer is made and later the second layer is added. In Fig. 10 the admittances of a transducer with a different thickness of the second layer is shown. This layer should be polished to obtain the minimal value of admittance with the smallest lobe. It can be seen that the smallest admittance corresponds to the thickness of the second layer equal $\lambda_e/4$ (Fig. 10f).

4. Experimental results

Matching layers are made of various materials with various fillers [5]. In our experimental work we have used the mixture of epoxy resin and tungsten powder (5 μm in dia). Depending on the weight ratio of the components, different values of density, sound velocity and acoustic impedance are obtained (Fig. 11a, b, c).

In Fig. 12 the admittances measured with Network Analyser 3577A–HP for different layer thickness are shown. It is seen that for the thickness equal to $(2k+1)\lambda_e/4$ (Fig. 12a, d, j) the admittance decreases, whereas for the thickness equal to $2k\lambda_e/4$

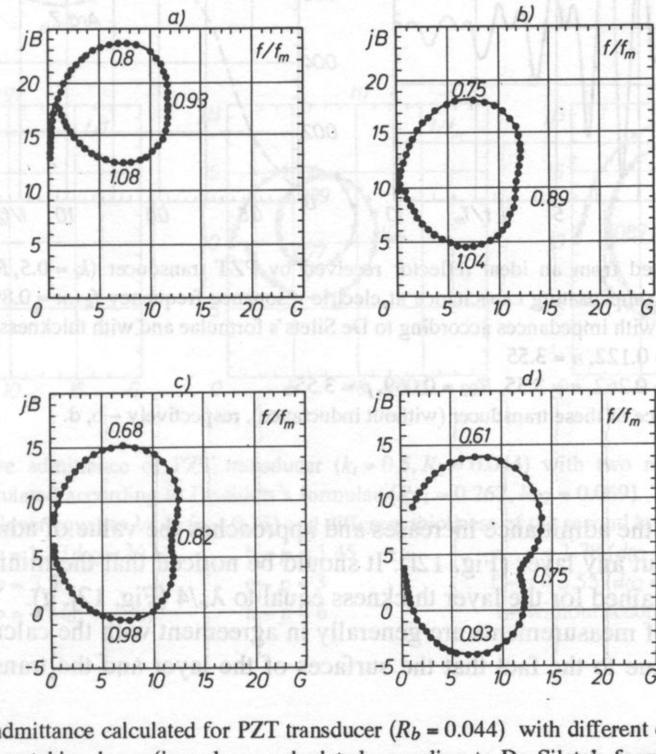

FIG. 8. Relative admittance calculated for PZT transducer ($R_b = 0.044$) with different coupling coefficient k_t, with acoustic matching layer (impedance calculated according to De Silets's formulae – $R_{01} = 0.122$, thickness equal to $\lambda_e/4$)

a – $k_t = 0.4$, $n = 3.7$ b – $k_t = 0.5$, $n = 3.55$

c – $k_t = 0.6$, $n = 3.3$ d – $k_t = 0.7$, $n = 3$.

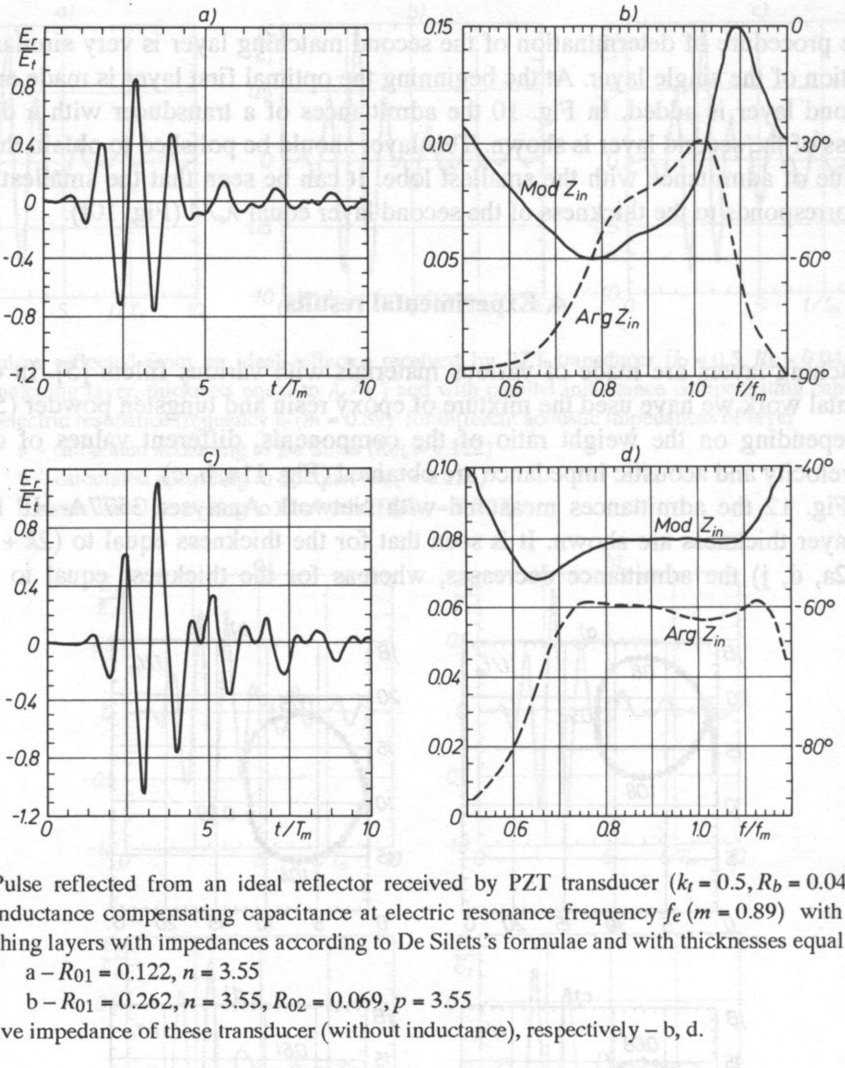

FIG. 9. Pulse reflected from an ideal reflector received by PZT transducer (k_t = 0.5, R_b = 0.044) with parallel inductance compensating capacitance at electric resonance frequency f_e (m = 0.89) with one and two matching layers with impedances according to De Silets's formulae and with thicknesses equal to $\lambda_e/4$

a – R_{01} = 0.122, n = 3.55

b – R_{01} = 0.262, n = 3.55, R_{02} = 0.069, p = 3.55

and relative impedance of these transducer (without inductance), respectively – b, d.

(Fig. 12b, g) the admittance increases and approaches the value of admittance of the transducer without any layer (Fig. 12l). It should be noticed that the minimum value of admittance is obtained for the layer thickness equal to $\lambda_e/4$ (Fig. 12j, g).

The results of measurements are generally in agreement with the calculations, some differences are due to the fact that the surfaces of the layer and the transducer are not ideally parallel.

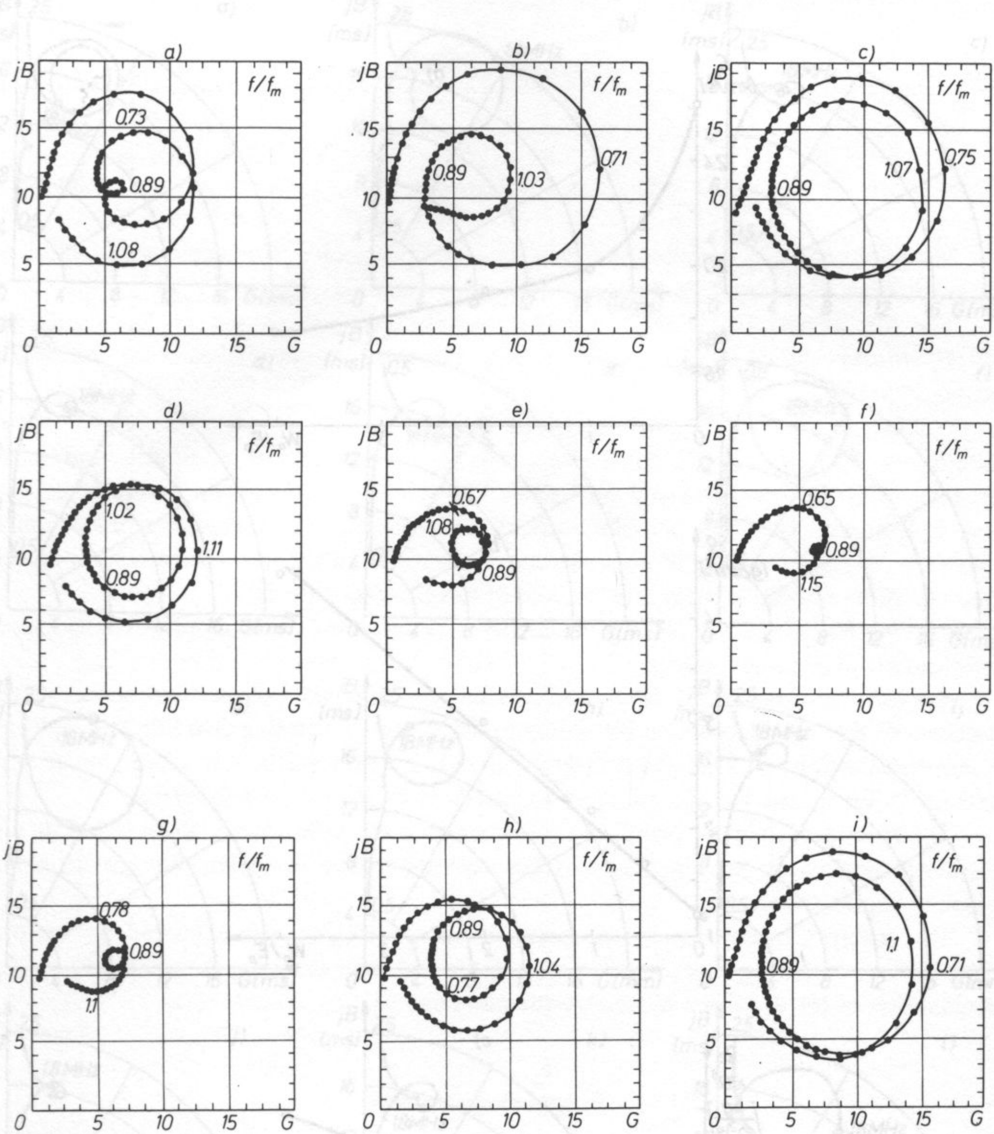

FIG. 10. Relative admittance of PZT transducer ($k_t = 0.5, R_b = 0.044$) with two matching layers with impedances calculated according to De Silets's formulae ($R_{01} = 0.262, R_{02} = 0.069$). Thickness of the first layer equal to $1/4\ \lambda_e$ ($n = 0.35$) and different thickness of the second layer.

a – $p = 1.2$ ($d_{02} = 3/4\lambda_e$) b – $p = 1{,}45$ c – $p = 1.78$ ($d_{02} = \lambda_e/2$)
d – $p = 2.5$ e – $p = 3$ f – $p = 3.55$ ($d_{02} = \lambda_e/4$)
g – $p = 4$ ($d_{02} = \lambda_e/4$) h – $p = 6$ i – without second layer

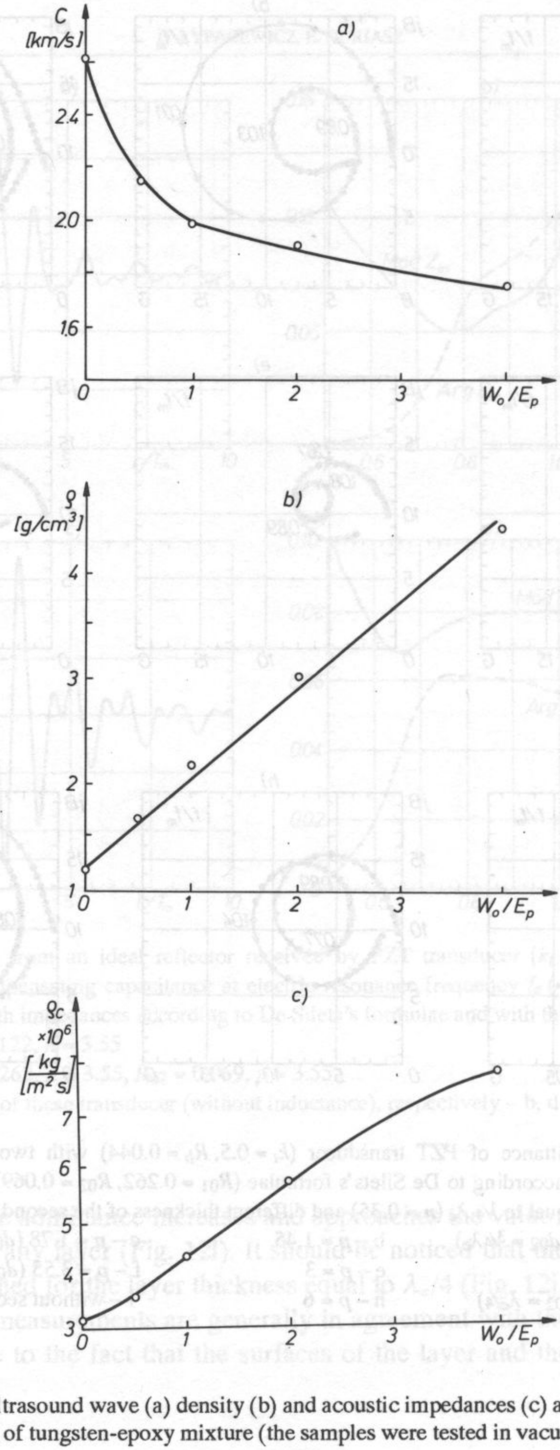

FIG. 11. Velocity of ultrasound wave (a) density (b) and acoustic impedances (c) as a function of weight ratio of tungsten-epoxy mixture (the samples were tested in vacuum).

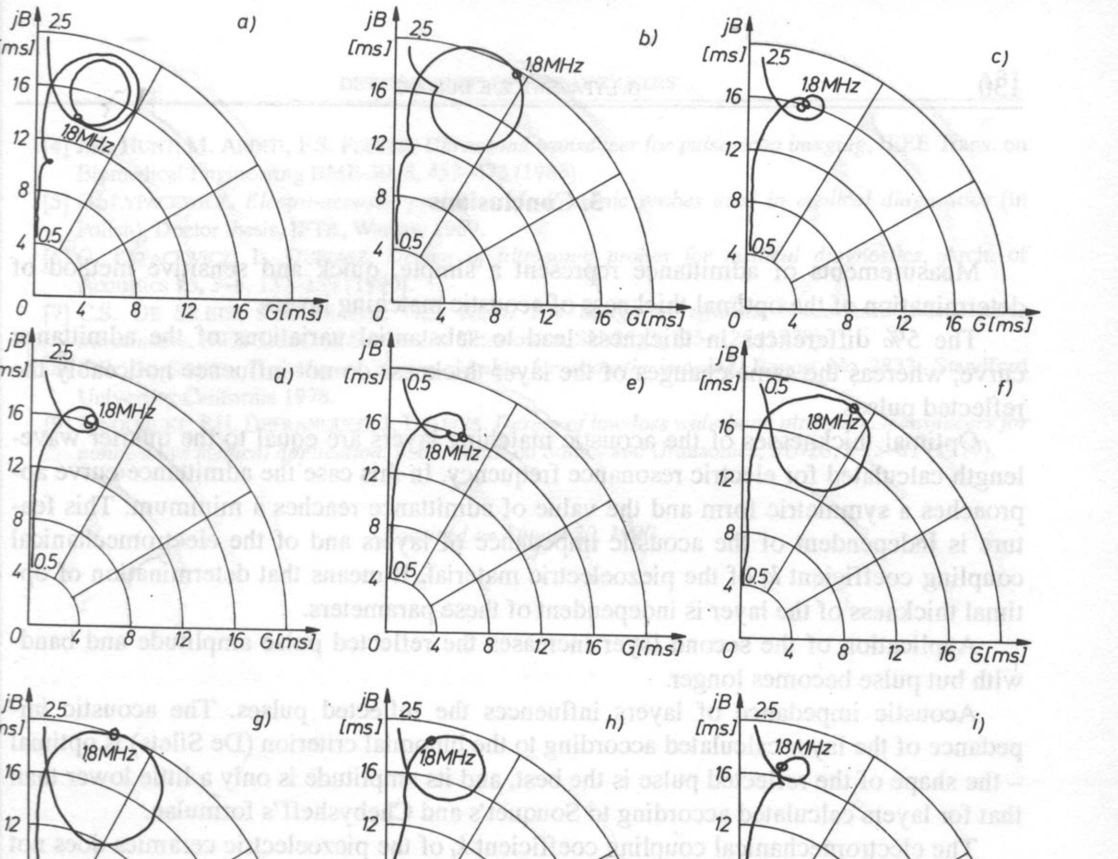

FIG. 12. Admittances of PZT transducer (CERAD PP-9) of 20 mm dia, with a matching layer made of tungsten-epoxy mixture (Wo: epoxy = 2.5), immersed in water measured for different thicnkesses of the layer

a – $d_{01} = 5/4\lambda_e$	b – $d_{01} = \lambda_e$	c – $d_{01} = 3/4\lambda_e + 5\%$
d – $d_{01} = 3/4\lambda_e$	e – $d_{01} = 3/4\lambda_e - 5\%$	f – $3/4\,\lambda_e > d_{01} > 1/2\,\lambda_e$
g – $d_{01} = 1/2\,\lambda_e$	h – $1/2\,\lambda_e > d_{01} > 1/4\,\lambda_e$	i – $d_{01} = 1/4\,\lambda_e + 5\%$

5. Conclusions

Measurements of admittance represent a simple, quick and sensitive method of determination of the optimal thickness of acoustic matching layers.

The 5% differences in thickness lead to substantial variations of the admittance curve, whereas the same changes of the layer thickness do not influence noticeably the reflected pulses.

Optimal thicknesses of the acoustic matching layers are equal to the quarter wavelength calculated for electric resonance frequency. In this case the admittance curve approaches a symmetric form and the value of admittance reaches a minimum. This feature is independent of the acoustic impedance of layers and of the electromechanical coupling coefficient k_t of the piezoelectric material. It means that determination of optimal thickness of the layer is independent of these parameters.

Application of the second layer increases the reflected pulse amplitude and bandwith but pulse becomes longer.

Acoustic impedance of layers influences the reflected pulses. The acoustic impedance of the layer calculated according to the binomial criterion (De Silets) is optimal – the shape of the reflected pulse is the best, and its amplitude is only a little lower than that for layers calculated according to Souquet's and Chebysheff's formulae.

The electromechanical coupling coefficient k_t of the piezoelectric ceramics does not influence noticeably the amplitude of the reflected pulses; however, for greater k_t the pulse becomes shorter.

The results of admittance measurements in transducers characterized by various thickness of matching layer are in good agreement with the calculations based on the assumption of the modified Mason's equivalent circuit.

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