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RIGOROUS *P*-MATRIX FOR UNIFORM NSPUDT

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A rigorous scattering matrix in mixed representation (*P*-Matrix) is presented for natural single-phase unidirectional transducer (NSPUDT) of surface acoustic wave (SAW). It results from spectral theory of propagation of SAW in periodic system of elastic metal strips on arbitrary anisotropic piezoelectrics.

1. Introduction

Uniform (unapodized) interdigital transducers find many applications in SAW devices. Typically, one or more such IDTs are included in SAW filters, delay lines or SAW resonators. It is then evident that the calculation of frequency response of uniform IDT is frequently required when designing SAW devices.

An useful $\sin x/x$ formula for IDT frequency response has been derived without taking account of SAW reflection from transducer electrodes (neither $\Delta v/v$ nor mechanical) and without admitting peculiar property of piezoelectric anisotropic substrate that may lead to a transducer natural unidirectionality [1]. The above effects, as well as element factor [2] may cause considerable distortion of the above-mentioned simple description of the transducer property.

Below, a corresponding rigorous formula is presented for IDT frequency response that results from spectral theory of propagation and generation of SAW in periodic metal strips on general anisotropic piezoelectric substrate, including possible effect of natural unidirectionality. The theory is a generalization of that presented in [3] to a case of Rayleigh wave.

2. Description of uniform IDT

We consider uniform IDT having N (even number) equally spaced metal strips (periods Λ, strip width $\Lambda/2$, strip thickness $H << \Lambda$, and IDT aperture width W). The strips are perfectly conducting and their mechanical properties are described by mass density ρ and Lamé constants μ and λ.

The mechanical interaction of strips with a plane harmonic wave

$$\exp(j\omega t - jrx)$$

of particle displacement on the piezoelectric substrate surface supporting strips is characterized by a diagonal matrix with diagonal elements g_i

$$-h\begin{bmatrix} \rho\omega^2 - 4\mu\dfrac{\lambda+\mu}{2\mu+\lambda}r(r-K) & 0 & 0 \\ 0 & \rho\omega^2 & 0 \\ 0 & 0 & \rho\omega^2 - \mu r(r-K) \end{bmatrix} \tag{1}$$

resulting from perturbation theory, where $K = 2\pi/\Lambda$ ($K > r > 0$ assumed here), ω – angular frequency, r-wave-number, and $h = H/\pi$ for strips with rectangular cross-section, otherwise h can have comples value.

What concerns electric interaction, we assume that the strips are ideal, thin conductors shielding electric field on the substrate surface. Electric potential of strips can be either $-U/2$ or $U/2$, as the every second strip is connected to either lower, or upper IDT bus. Due to the transducer symmetry, the current I flowing to the IDT upper bus is exactly opposite to that flowing to the lower one. In this paper we assume for simplicity, that the element factor is the same for each transducer fingers, even for the edge ones.

3. Characterization of piezoelectric substrate

Here, we entirely neglect bulk waves, and assume that the transducer works at frequency close to its fundamental frequency. That results in $K - r \approx r$ in (1) and enables us to apply approximated relations between complex amplitudes of electric potential φ, particle displacement vector components u_n, stress components t_{yn} ($n = 1, 2, 3$ corresponding to x, y, z) and electric charge d at the substrate surface $y = 0$ in form

$$u_n = z_n\frac{\sqrt{\varepsilon_e}}{\kappa}\left(-\varphi + \frac{1+\kappa^2}{r\varepsilon_e}d\right) \tag{2}$$

$$(r-k_v)r\varphi - (r-k_0)\frac{d}{\varepsilon_e} = r\frac{\kappa}{\sqrt{\varepsilon_e}}\sum_n z_n^* t_{yn}$$

where ε_e – effective surface permittivity, k_v, k_0 – SAW wave-number for free or metallized substrate surface, and

$$\kappa^2 = k_0/k_v - 1 = \Delta u/v$$

$$z_n = -\frac{\kappa}{\sqrt{\varepsilon_e}}\frac{U_n^{(v)}}{\Phi^{(v)}} = \kappa k_v\sqrt{\varepsilon_e}\frac{U_n^{(0)}}{D^{(0)}} \tag{3}$$

where capital letters denote normalized SAW amplitudes for free (index v), or metallized (index 0) substrate surface [4].

These relations result from approximation of corresponding Green's function in spectral representation, taken at $r \approx k_v$. Equation (2) holds for weak piezoelectrics when $\Delta v/v$ is small, and generally when Eq. (3) are not contradicting, particularly if

$$\varepsilon_e = -\frac{U_n^{(v)}}{k_v \Phi^{(v)}} \frac{D^{(0)}}{U_n^{(0)}} \text{ and } \kappa^2 \approx -\frac{1}{4}\frac{\omega}{k_v} \Phi^{(v)} D^{(0)*}$$

for every n. Otherwise, for our purpose we may apply Eqs. (2) instead of so-called "equivalent isotropic" substrate description. If necessary, we can suitably change parameters characterizing SAW modes in (3) to take account of average loading of the substrate surface by the strips [4].

4. *P*-Matrix for IDT

An useful representation for scattering matrix of IDT is P – Matrix which describes scattering property of IDT at acoustic ports, and admittance of IDT at its electric port, that is

$$[a_L^-, a_R^+, I]^T = [P]\,[a_L^+, a_R^-, U]^T \tag{4}$$

where a – SAW amplitude by definition related to SAW Poynting vector, $\Pi = \frac{1}{2}|a|^2$ (full energy carried by SAW is ΠW). Upper and lower indices mark waves on left (L) or right (R) side of IDT, and propagating in left (–) or right (+) direction. Phases of these complex amplitudes are related to lines which are $N\Lambda/2$ distant from transducer center. I is the transducer current and U is voltage applied to it.

The above-mentioned spectral theory yields

$$r_0 = \frac{K}{2} + Q\,,\ \chi = \sum_{i=1}^{3} g_i\,(z_i^*)^2\,,\ X = \frac{K}{2} - \frac{k_v + k_0}{2}$$

$$Q = \pm\sqrt{X^2 - |\chi|^2 - \kappa^2 K \frac{\sin \pi r_0/K}{\pi\,[P_{-r_0/K}(0)]^2}(X - \mathrm{Re}\{\chi\})}$$

$$\gamma^+ = \frac{X + Q - \chi^*}{X - Q - \chi}\,,\ \gamma^- = \frac{X + Q - \chi}{X - Q - \chi^*}$$

$$P_{11} = -\gamma^+ \frac{1 - e^{-jr_0 2N\Lambda}}{1 - \gamma^+\gamma^- e^{-jr_0 2N\Lambda}}$$

$$P_{12} = \frac{1 - \gamma^+\gamma^-}{1 - \gamma^+\gamma^- e^{-jr_0 2N\Lambda}}\, e^{-jr_0 2N\Lambda}$$

$$P_{13} = \kappa \sqrt{\frac{1}{2}\varepsilon_e \omega}\, \frac{\sin \pi r_0/K}{P_{-r_0/K}(0)} \, \frac{\sin N \dfrac{r_0 \Lambda - \pi}{2}}{\sin \dfrac{r_0 \Lambda - \pi}{2}} \, e^{j\pi \frac{N+1}{2}} e^{-jr_0 N\Lambda/2} \times$$

$$\times \frac{1 - \gamma^+ \gamma^- e^{-jr_0 N\Lambda} + \gamma^+ (1 - e^{-jr_0 N\Lambda})}{1 - \gamma^+ \gamma^- e^{-jr_0 2N\Lambda}} \tag{5}$$

$$P_{3n} = 2WP_{n3}\,,\; n = 1, 2$$

$$P_{33} = j\omega C_T + W\kappa^2 \varepsilon_e \omega \frac{\sin \pi r_0/K}{P_{-r_0/K}(0)} \frac{(1+\gamma^+)(1+\gamma^-)}{1 - \gamma^+ \gamma^-} \times$$

$$\left[j\frac{\sin N(r_0\Lambda - \pi) - N \sin (r_0 \Lambda - \pi)}{2 \sin^2 \dfrac{r_0 \Lambda - \pi}{2}} + \left(\frac{\sin N \dfrac{r_0 \Lambda - \pi}{2}}{\sin \dfrac{r_0 \Lambda - \pi}{2}} \right)^2 \times \right.$$

$$\left. \times \frac{1 - \left(\dfrac{1+\gamma^+}{1+\gamma^-}\gamma^- + \dfrac{1+\gamma^-}{1+\gamma^+}\gamma^+\right) e^{-jr_0 N\Lambda} + \gamma^+ \gamma^- e^{-jr_0 2N\Lambda}}{1 - \gamma^+ \gamma^- e^{-jr_0 2N\Lambda}} \right]. \tag{5}$$

C_T is static capacitance of the transducer and $r_0\Lambda - \pi = Q\Lambda$. The rule for sign of Q is that $\mathrm{Im}\{r_0\} < 0$ or that $r_0 \approx k_0$. To obtain P_{2n}, one should apply relation for P_{1n} with substitutions $\gamma^+ \Leftrightarrow \gamma^-$ and $\exp\left[j\pi\dfrac{(N+1)}{2}\right] \Rightarrow \exp\left[-j\pi\dfrac{(N+1)}{2}\right]$.

5. Comments

As it is seen in (5), there is an asymmetry in the transducer property as concern left, or right propagating SAW detection and generation by IDT, if only γ^+ differs from γ^-. One may then be interested in interpretation of these quantities. In theory they appear in solution for waves in periodic system of short-circuited strips, namely the wave propagating right is composed of forward-propagating wave and backward propagating wave as follows: $a^+(1 + \gamma^+ e^{jKx})e^{-jr_0 x}$, similarly the wave propagating left: $a^-(1 + \gamma^- e^{-jKx})e^{jr_0 x}$, r_0 is wave-number of SAW that can take complex value in stop-band. Thus we can interpret γ^+ and γ^- as reflection coefficients of SAW from half-infinite system of periodic strips, from the left, or the right bound of the system (see P_{11} in (5) above).

For typical piezoelectric substrates $\gamma^{+} = \gamma^{-}$ and Eqs. (5) describes a typical two-directional IDT, however accounting for mechanical property of the transducer fingers. For further discussion of the above relations and possible application in analysis of apodized transducers-see [3] and [5].

References

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