

ACOUSTICAL PROPERTIES OF POROUS LAYER – UNDEFORMABLE HALFSpace SYSTEM AT NORMAL INCIDENCE OF HARMONIC WAVE

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The acoustical properties of a system composed of a porous layer and an undeformable solid halfspace, immersed in a barotropic fluid, are analyzed for the case of normal incidence of a harmonic wave. The explicit forms of expressions of the wave absorption coefficients were obtained for different particular configurations of the system. This allowed us to discuss the dependence of the absorption coefficient on the dissipative properties of a fluid and on the parameters characterizing the pore structure of a porous layer, in a wide range of frequencies of the incident wave. It was shown that the dissipative properties of the fluid do not considerably change the value of resonance frequencies. However, these properties as well as the parameters of the skeleton pore structure strongly influence the coefficient of wave absorption.

1. Introduction

The determination of acoustical properties of systems composed of porous elements is of great importance in many technical problems occurring, for example, in aircraft and machinery noise control or in architectural acoustics. In these systems the porous material in the form of layers, plates or halfspaces (ground), immersed in a fluid, strongly interacts with waves propagating in the fluid in a wide range of frequencies.

The complexity of a theoretical investigation of the properties of such systems is connected with the variety of transfer ways of acoustic energy. In the general case of a deformable skeleton of porous material the acoustic waves are transmitted both by the skeleton and by the fluid filling its pores, and also by vibrational movement of particular elements of the system.

In the majority of papers devoted to the interaction of waves with porous material (e.g. [1], [7], [11], [13], [16]) and to the investigation of its properties (e.g. [2], [14],

[15]), the authors exploit the analogy between the propagation of plane waves in the absorbant media and the propagation of electric disturbances in the loss lines. They characterize the acoustical properties of a porous medium by two quantities: the propagation constant of a wave and the wave impedance. They also formulate the boundary conditions at the surface of a porous medium by means of the surface impedance. Some authors (e.g. [1], [16]) use at the same time the existing equations of the dynamics of porous media to determine the relations between the quantities and the parameters characterizing a porous medium filled with a fluid. Such a characteristic, although sufficient for the media which can be modelled as a modified fluid (the skeleton being undeformable), needs to be extended by other quantities in the case of a deformable skeleton [16].

The other approach, rarely presented in the papers on this subject, consists in solving the boundary problem formulated strictly within mechanical notions.

In spite of a great variety of papers concerning the interaction of acoustic waves with porous materials and with their systems in the literature, there is a lack of theoretical papers devoted to the analysis of the influence of the parameters characterizing the properties of porous media and the geometry of the system on its acoustical properties.

The main purpose of this paper is to analyze the acoustical properties of a system consisting of a rigid immovable porous layer and undeformable solid halfspace, immersed in a fluid, at the normal incidence of a harmonic wave.

The starting point for the description of the dynamics of a fluid in pores of a rigid skeleton are the equations of the two-parameter theory of deformable porous media filled with a fluid ([5], [6], [8–10]), in which the skeleton pore structure is characterized by two parameters: volume porosity and structural permeability parameter. These parameters are explicitly present in the continuity and motion equations of the fluid as well as in the boundary conditions representing the continuity of the fluid mass flux and its effective pressure at both surfaces of a porous layer.

Solving the boundary problem resulted in obtaining the explicit forms of the absorption, reflection and transmission coefficients of waves for different configurations of the system. This enabled us to discuss the dependence of these coefficients on the dissipative properties of the fluid filling the pores of the layer, on the pore structure parameters and the geometry of the system in a wide frequency range of the incident wave.

2. Interaction of a plane acoustic wave with the porous layer-undeformable halfspace system

Formulation of the problem.

We analyze the acoustical properties of the system consisting of a rigid immovable porous layer of thickness b immersed in a fluid at distance d from the undeformable solid halfspace. We consider the case when a plane harmonic wave of frequency f ($\omega = 2\pi f$) and of amplitude A_1 , propagating in a fluid, falls normally at the surface of the porous layer (Fig. 1.). We assume that the fluid is barotropic, i.e., the effective pres-

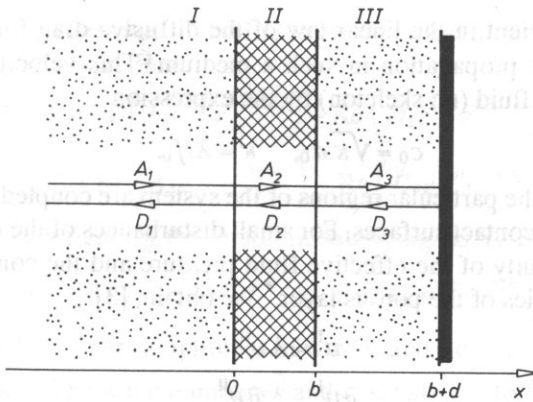


FIG. 1. Scheme of wave interaction with a system composed of a porous layer and undeformable solid halfspace.

sure p^f is in one to one relation with its effective mass density ρ^f ($p^f = p^f(\rho^f)$), and that the viscosity of the fluid does not influence its macroscopic state of stress (the deviators of the stress tensors in the bulk fluid and in the fluid filling porous medium are omitted) but it is taken into account in the interface interaction force with the porous skeleton.

At the above assumptions the propagation of disturbances with small amplitude in halfspace $x < 0$ (region I) and in the layer of the fluid (region III) is described by a linear wave equation for the barotropic fluid

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{a_0^2} \frac{\partial^2 v}{\partial t^2} \quad (2.1)$$

where v is the velocity of fluid particles and

$$a_0 = \left(\frac{dp^f}{d\rho^f} \right)^{\frac{1}{2}} \bigg|_{\rho^f = \rho_0^f}$$

is the velocity of wave propagation in the bulk fluid, whereas ρ_0^f stands for its mass density in the undisturbed state of medium.

The description of fluid motion in the pores of an undeformable porous layer (region II) is based on the two-parameter theory of deformable porous medium filled with a fluid ([5], [6], [8–10]) in which the skeleton pore structure is characterized by two macroparameters: the volume porosity f_v and structural permeability parameter λ ($\lambda \leq f_v$). The problem of fluid motion in pores of an undeformable skeleton is then the particular case of this theory, and the equation describing the propagation of waves with small amplitude takes the form [3]

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{c_0^2} \left(\frac{\partial^2 v}{\partial t^2} + \bar{k} \frac{\partial v}{\partial t} \right), \quad (2.2)$$

$$\bar{k} = \bar{k}' / (\lambda \rho_0^f)$$

where \bar{k}' is the coefficient in the linear law of the diffusive drag force while c_0 is the velocity of wave front propagation in such a medium. This velocity is related to the velocity a_0 in the bulk fluid (no skeleton) by the expression

$$c_0 = \sqrt{\kappa} a_0, \quad \kappa = \lambda/f_v. \quad (2.3)$$

Acoustic fields in the particular regions of the system are coupled via the compatibility conditions at their contact surfaces. For small disturbances of the medium these conditions are: the continuity of the effective fluid pressure and the continuity of its mass fluxes at both boundaries of the porous layer.* We obtain, [3]

$$v^I = \lambda v^{II}, \quad (2.4)$$

$$\frac{\partial v^I}{\partial x} = \kappa \frac{\partial v^{II}}{\partial x} \quad (2.5)$$

for $x = 0$, and

$$\lambda v^{II} = v^{III}, \quad (2.6)$$

$$\kappa \frac{\partial v^{II}}{\partial x} = \frac{\partial v^{III}}{\partial x} \quad (2.7)$$

for $x = b$, where v^α ($\alpha = I, II, III$) is the resultant velocity field in the region α of the system.

An additional limitation for the velocity field v^{III} in the layer of the bulk fluid is the boundary condition at the surface $x = b + d$. Due to the underformability of halfspace $x > b + d$, we have

$$v^{III} = 0. \quad (2.8)$$

Equation (2.1) and (2.2) together with the conditions (2.4)–(2.8) describe the dynamic behaviour of the fluid in the system shown in Fig. 1. It is evident that apart from the parameters b , d and \bar{k} which characterize the geometry of the system and the dissipative properties of the fluid in its viscous interaction with the pores of the layer, the real influence on the acoustical properties of the system is exerted by the pore structure parameters of the porous layer. These parameters are explicitly present both in the motion equation (2.2) and compatibility conditions (2.4)–(2.7).

Solution of the problem

The resultant acoustic field in each region of the system consists of two waves propagating in opposite directions (Fig. 1). These waves are the superposition of all elementary waves with proper directions resulting from the subsequent reflection and transmission of the incident wave at the boundaries of the particular regions.

* Continuity conditions of this kind are analogical to the conditions imposed at places of a rapid change of the cross-section in the analysis of wave propagation in wave guides of stepwise-changing cross-sections [13].

The acoustic fields in regions I and III, being the solution of equation (2.1), may be represented by the functions

$$v^I = \operatorname{Re} \left(A_1 e^{i\omega t} e^{-2\pi i \hat{k} x} \right) - \operatorname{Re} \left(D_1 e^{i\omega t} e^{2\pi i \hat{k} x} \right). \quad (2.9)$$

$$v^{III} = \operatorname{Re} \left(A_3 e^{i\omega t} e^{-2\pi i \hat{k} x} \right) - \operatorname{Re} \left(D_3 e^{i\omega t} e^{2\pi i \hat{k} x} \right). \quad (2.10)$$

respectively, and such a field for the region II, satisfying Eq. (2.2), takes the form

$$v^{II} = \operatorname{Re} \left(A_2 e^{i\omega t} e^{-2\pi i k' x} \right) - \operatorname{Re} \left(D_2 e^{i\omega t} e^{2\pi i k' x} \right). \quad (2.11)$$

where A_α, D_α ($\alpha = 1, 2, 3$) are amplitudes of waves, and $\operatorname{Re}(\cdot)$ stands for the real part of a complex expression. The wave numbers \hat{k} and k' satisfy the following relations:

$$\hat{k} = f/a_0, \quad k'^2 = k^2(1 - ik_0/k) \quad (2.12)$$

where

$$k = f/c_0, \quad k_0 = \bar{k}/(2\pi c_0). \quad (2.13)$$

The expressions (2.9)–(2.11) involve five unknown amplitudes of waves. To determine them, we have five boundary conditions (2.4)–(2.8). Introducing Eqs. (2.9)–(2.11) into the proper boundary conditions (2.4)–(2.8), we obtain the following algebraic system of equations

$$\begin{aligned} A_1 - D_1 &= \lambda(A_2 - D_2), \\ A_1 + D_1 &= \sqrt{\kappa}K(A_2 + D_2), \\ \lambda(A_2 e^{-2\pi i \eta K} - D_2 e^{2\pi i \eta K}) &= A_3 e^{-2\pi i \sqrt{\kappa} \eta} - D_3 e^{2\pi i \sqrt{\kappa} \eta}, \\ \sqrt{\kappa}K(A_2 e^{-2\pi i \eta K} + D_2 e^{2\pi i \eta K}) &= A_3 e^{-2\pi i \sqrt{\kappa} \eta} + D_3 e^{2\pi i \sqrt{\kappa} \eta}, \\ D_3 &= A_3 e^{-4\pi i \sqrt{\kappa} \eta(1+\varepsilon)} \end{aligned} \quad (2.14)$$

where

$$K = k'/k, \quad \eta = bk, \quad \varepsilon = d/b. \quad (2.15)$$

The above equations allow us to determine the ratios of wave amplitudes propagating in the system to the amplitude of the incident wave as the explicit functions of the quantities characterizing the pore structure of a porous layer, the dissipative properties of the fluid and the macroscopic geometry of the system, for various frequencies of the incident wave. In particular, they allow us to determine the absorption coefficient α defined as the ratio of energy absorbed by the system to the energy of the incident wave. This coefficient takes the form

$$\alpha = 1 - |D_1/A_1|^2 = f_1(\eta, \eta_0, \varepsilon, f, \kappa) \quad (2.16)$$

where $|\cdot|$ stands for the absolute value of a complex number, and

$$\eta_0 = bk_0$$

is the dimensionless parameter characterizing the dissipative properties of the system.

The explicit form of the function f_1 is given in Appendix A.

From the practical point of view, the absorption coefficient α is the most important quantity characterizing the global properties of the system under consideration. Further, we analyze the influence of internal parameters of the system: η_0 , ε , f_v , κ on the absorption coefficient α for various frequencies of the incident wave and different configurations of the system.

3. Influence of internal parameters of the system on wave absorption coefficient. Special configuration of the system

3.1. Absorption properties of halfspace of a porous medium immersed in a fluid

The simplest case of a configuration of the system depicted in Fig. 1 is the halfspace of a porous medium immersed in a fluid (Fig. 2). This case is obtained by increasing the thickness b of a porous layer to infinity.

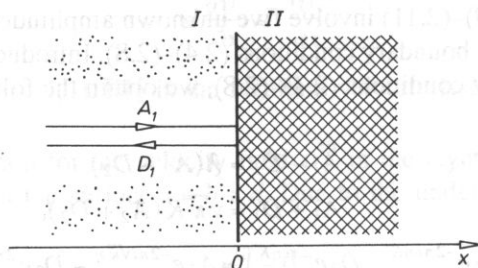


FIG. 2. Scheme of wave interaction with a halfspace of a porous material.

The absorption coefficient α_∞ of a such a system is given by the expression (2.16) for $b \rightarrow \infty$ ($k_0 \neq 0$) and takes the form

$$\alpha_\infty = \frac{4\sqrt{\kappa}f_v P}{(\sqrt{\kappa}f_v + P)^2 + Q^2} \quad (3.1)$$

where

$$P = \text{Re}(K) = \sqrt{\left(1 + \sqrt{1 + (k_0/k)^2}\right)/2}, \quad (3.2)$$

$$Q = \text{Im}(K) = -\sqrt{\left(-1 + \sqrt{1 + (k_0/k)^2}\right)/2}.$$

The dependence of the coefficient α_∞ on the dimensionless wave frequency $2\pi f/\bar{k}$ for two different pore structures of a porous medium is depicted in Fig. 3. This figure shows that the absorption coefficient α_∞ for low frequencies of the wave is small and increases when the frequency increases, approaching asymptotically the value

$$\alpha_\infty^0 = 4\sqrt{\kappa}f_v/(\sqrt{\kappa}f_v + 1)^2, \quad (3.3)$$

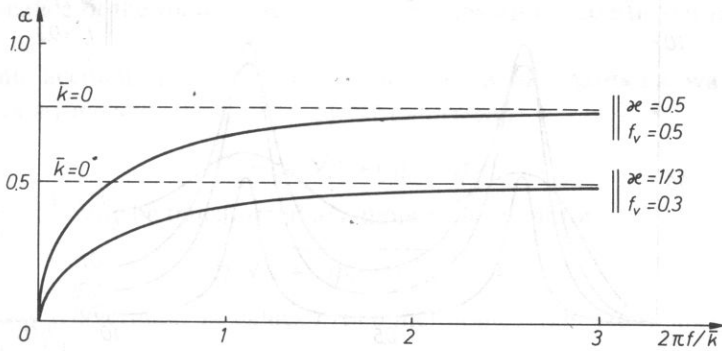


FIG. 3. Dependence of the absorption coefficient α on the dimensionless frequency $2\pi f/\bar{k}$ (for an acoustic plaster filled with air [16]; $2\pi f/\bar{k} = 1$ corresponds to $f = 400$ Hz).

which is entirely defined by the pore structure parameters. The quantity α_∞^0 is at the same time the absorption coefficient of the porous halfspace for the case when the fluid is inviscid ($\bar{k} = 0$). This means that in the range of higher frequencies the pore structure is the main factor determining the value of the coefficient of wave absorption by the porous halfspace, whereas in the range of low frequencies the predominant influence is exerted by the diffusive drag force characterized by the parameter \bar{k} .

3.2. Absorption properties of a porous layer with an impervious back surface

Let us now analyze the absorptive properties of the considered system in the case when the porous layer lies on the surface of an underformable solid material ($\varepsilon = 0$, Fig. 4). For such a configuration of the system, the absorption coefficient α given by the expression (2.16) takes the form

$$\alpha = f_i(\eta, \eta_0, \varepsilon, f_v, \kappa)|_{\varepsilon=0} \quad (3.4)$$

The dependence of the coefficient α on the dimensionless wave frequency η is shown in Fig. 5.

An important element in understanding the character of the course of the curves in Fig. 5 are the notions of (anti) resonance (resonance and/or antiresonance) frequencies of

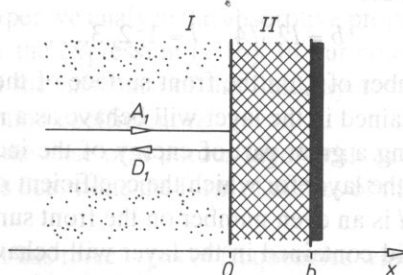


FIG. 4. Scheme of wave interaction with a porous layer with an impervious back surface.

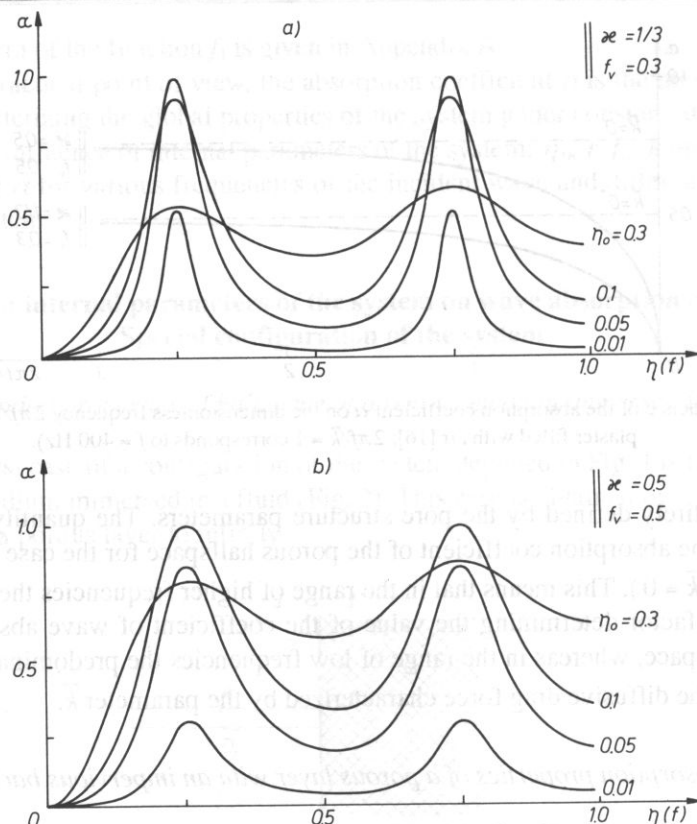


FIG. 5. Dependence of the absorption coefficient α on the dimensionless frequency η for two different pore structures of a porous layer and various values of the coefficient η_0 (for air and $b = .02$ m; a) $\eta = 1$ corresponds to $f \approx 9.8$ kHz; b) $\eta = 1$ corresponds to $f \approx 12$ kHz).

the fluid filling the porous layer. These frequencies determine the position of extremal values of the coefficient α . Taking into account the fact that both waves propagating in the layer form a standing wave, the node of which is placed on the contact surface with the undeformable halfspace, the (anti) resonance of the fluid in the layer occurs when the multiple of one fourth of the wave length λ_0 , propagating in the layer, is equal to the thickness of the layer, i.e., for

$$b = l\lambda_0/4 \quad l = 1, 2, 3, \dots \quad (3.5)$$

Then, for an odd number of l , on the front surface of the layer, the loop of a wave appears and the fluid contained in the layer will behave as a material of great flexibility, intercepting and dissipating a great part of energy of the incident wave. These are the resonance frequencies of the layer for which the coefficient α takes maximal values.

In turn, in the case when l is an even number on the front surface of the layer, a node of wave appears and the fluid contained in the layer will behave as a material with small flexibility, reflecting a great part of the energy of the incident wave. In this case we deal

with antiresonance of the fluid in the layer and the absorption coefficient α takes minimal values.

Taking into account the fact that the phase velocity of a harmonic wave in a fluid filling pores of a rigid skeleton is given by the expression

$$V_\varphi = c_0 / \sqrt{1 + (\eta_0/2\eta)^2}, \quad (3.6)$$

the condition (3.5) may be transformed to a more convenient form:

$$\eta_l \sqrt{1 + (\eta_0/2\eta_l)^2} = l/4 \quad (3.7)$$

which allows us to determine the values η_l of dimensionless (anti) resonance frequencies of the fluid in the layer.

As Fig. 5 shows, the parameter η_0 characterizing the dissipative properties of the fluid in a porous layer does not influence significantly the position of extremal values of the coefficient α . This means that in order to determine the (anti) resonance frequencies and the values of the coefficient α corresponding to them, the approximated form of the condition (3.7) and of the expression (3.4) may be used.

For $\eta_0/\eta \gg 1$, from Eqs. (3.7) and (3.4) we have

$$\eta_l = l/4, \quad (3.8)$$

$$\alpha = \frac{4\sqrt{\kappa} f_v \text{th}(\pi\eta_0)}{(1 + \sqrt{\kappa} f_v \text{th}(\pi\eta_0))^2 - \sin^2(2\pi\eta)(1 - \kappa f_v^2)/\text{ch}^2(\pi\eta_0)}. \quad (3.9)$$

From Fig. 5 and the expression (3.9) it is evident that both parameters η_0 and κ strongly change the form of curves of the coefficient. Moreover, the pore structure parameter κ influences the position of extremal values of the coefficient α . This parameter is explicitly present in the expression defining the dimensionless frequency η and therefore its influence, however not evident in Fig. 5, appears as a change of the scale on the axis of frequency.

The condition (3.8) together with the expression (3.9) are convenient for the calculation of the parameter κ of the pore structure and of the coefficient k' from the experimental data for the absorption coefficient α .

3.3. The absorption properties of the porous layer – solid halfspace system

In this section of the paper we analyze the absorptive properties of the system shown in Fig. 1 and described by the expression (2.16). In our considerations we put special stress on the discussion of the influence of the fluid layer separating both parts of the system on the wave absorption coefficient α .

Similarly as it was in the case of a porous layer with an impervious back surface, (anti)resonance frequencies determine the form of curves of the absorption coefficient of a system of two layers.

The system shown in Fig. 1 has three types of frequencies which determine the positions of extremal values of the coefficient α . The first type of these frequencies is connected

with (anti)resonance of the fluid filling porous layer and it appears when the condition (3.7) or (3.8) is satisfied. The second type of (anti) resonance frequencies results from anti resonance of the fluid contained between the porous layer and the undeformable halfspace. They are given by the condition

$$d = m\lambda_0/4 \quad m = 1, 2, 3, \dots,$$

which, due to constant phase velocity in a bulk fluid, equal to a_0 , may be expressed in the form

$$\eta_m = \frac{m}{4} \frac{1}{\sqrt{\kappa \varepsilon}}. \quad (3.10)$$

The third type of (anti) resonance frequencies of the system results from (anti) resonance of the fluid contained in both layers as a whole. In this case the condition for the (anti) resonance frequencies is obtained by requiring the time of transition of the wave through both layers to be equal to the multiple of one fourth of the wave period.

When the phase velocities of waves in each layer is considered, this condition takes the form

$$\eta_n (\sqrt{1 + (\eta_0/2\eta_n)^2} + \sqrt{\kappa \varepsilon}) = n/4 \quad n = 1, 2, 3, \dots \quad (3.11)$$

or

$$\eta_n = \frac{n}{4} \frac{1}{1 + \sqrt{\kappa \varepsilon}} \quad (3.12)$$

for $\eta/\eta_0 \gg 1$.

The conditions (3.7), (3.9) and (3.11) or their approximated forms allow one to evaluate the influence of particular types of (anti) resonances (parameters of the system) on the form of the curves of the coefficient α . These conditions determine the exact position of extremal values of the coefficient α only in the case when the parameters of the system are so chosen that all three conditions are fulfilled at the same time, i.e., when all three (anti) resonances are present. Then the value of the number n determines the type of extremum. For even n there appears a minimum of the coefficient α , and for odd n , there appears its maximum. In the remaining cases the positions of extrema are determined by neighbouring (anti) resonance frequencies of different types.

The conditions (3.8), (3.10) and (3.12) provide that for small values of the parameter ε , the (anti) resonance frequencies of the fluid in a porous layer and in both layers as a whole are close to one another, and the (anti) resonance of the layer of a bulk fluid occurs at higher frequencies of the wave. This means that for low frequencies and small values of ε , the position of extrema of the coefficient α is determined by the (anti) resonance frequencies of the fluid in a porous layer and both layers as a whole.

Since the (anti) resonance frequencies given by the formula (3.12) decrease when the parameter ε increases, the extrema of the coefficient α displace in the direction of lower frequencies (Fig. 6a). From Fig. 6b it is seen that a further increase of the parameter ε does not change significantly the absorption coefficient α for low frequencies but it causes the appearance of extrema connected with (anti) resonances of the layer of a bulk fluid.

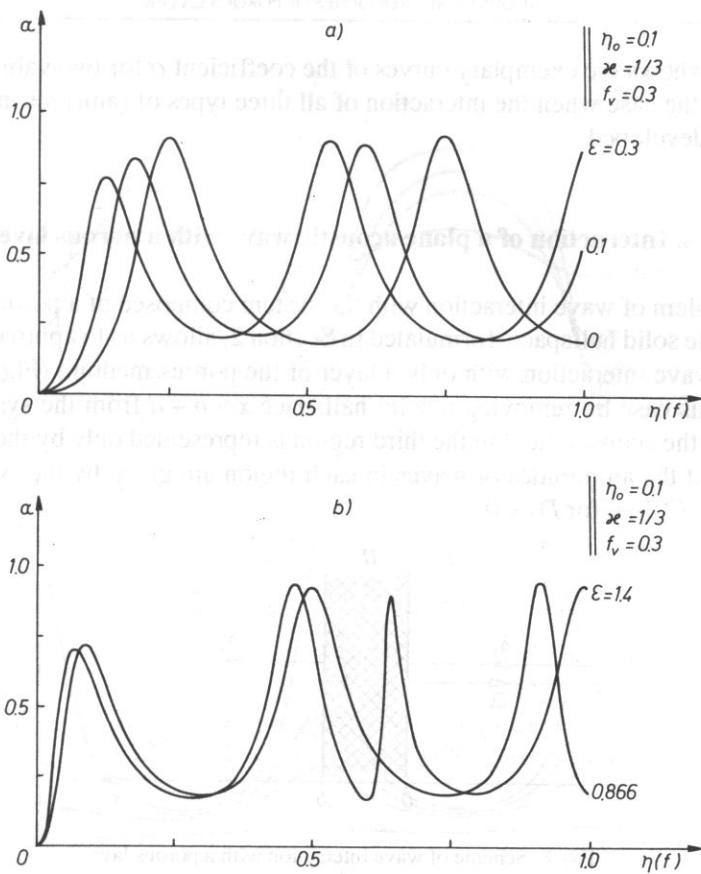


FIG. 6. Dependence of the absorption coefficient α on the dimensionless frequency η for two different pore structures of a porous layer and various values of the coefficient ϵ (for air and $b = .02\text{m}$; a) $\eta = 1$ corresponds to $f \approx 9.8$ kHz; b) $\eta = 1$ corresponds to $f \approx 12$ kHz).

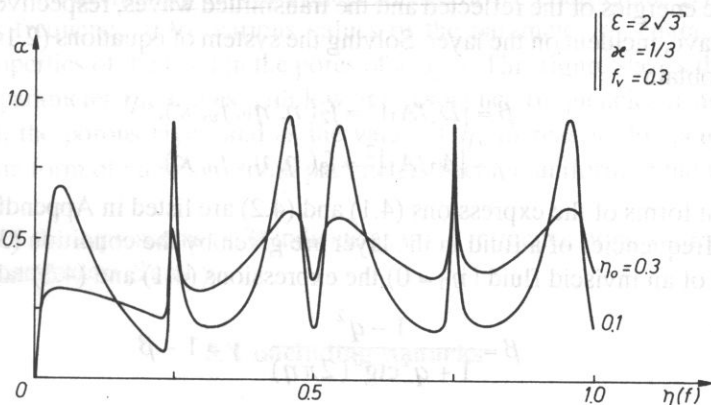


FIG. 7. Dependence of the absorption coefficient α on the dimensionless frequency η (for air and $b = .02$ m $\eta = 1$ corresponds to $f = 9.8$ kHz).

Figure 7 shows the exemplary curves of the coefficient α for two values of the parameter η_0 , in the case when the interaction of all three types of (anti) resonance frequencies is fully developed.

4. Interaction of a plane acoustic wave with a porous layer

The problem of wave interaction with the system composed of a porous layer and an undeformable solid halfspace, formulated in Section 2, allows us to approach directly the problem of wave interaction with only a layer of the porous medium (Fig. 8). Formally, we obtain this case by removing a solid halfspace $x > b + d$ from the system shown in Fig. 1. Then the acoustic field in the third region is represented only by the wave leaving the layer, and the amplitudes of waves in each region are given by the system of equations (2.14)₁–(2.14)₄ for $D_3 = 0$.

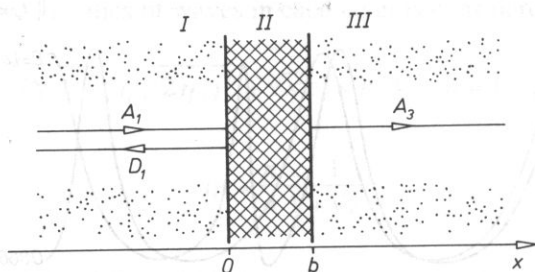


FIG. 8. Scheme of wave interaction with a porous layer.

The acoustical properties of a porous layer are characterized by two quantities: the reflection coefficient β and the transmission coefficient γ . These quantities are defined as ratios of the energies of the reflected and the transmitted waves, respectively, to the energy of the wave, incident on the layer. Solving the system of equations (2.14)₁–(2.14)₄, for $D_3 = 0$ we obtain

$$\beta = |D_1/A_1|^2 = f_2(\eta, \eta_0, f_v, \kappa), \quad (4.1)$$

$$\gamma = |A_3/A_1|^2 = f_3(\eta, \eta_0, f_v, \kappa). \quad (4.2)$$

The explicit forms of the expressions (4.1) and (4.2) are listed in Appendix B. The (anti) resonance frequencies of a fluid in the layer are given by the condition (3.8) or (3.9).

In the case of an inviscid fluid ($\eta_0 = 0$) the expressions (4.1) and (4.2) take the forms

$$\beta = \frac{1 - q^2}{1 + q^2 \operatorname{ctg}^2(2\pi\eta)}, \quad \gamma = 1 - \beta \quad (4.3)$$

where

$$q^2 = \frac{4\kappa f_v^2}{(\kappa f_v^2 + 1)^2}.$$

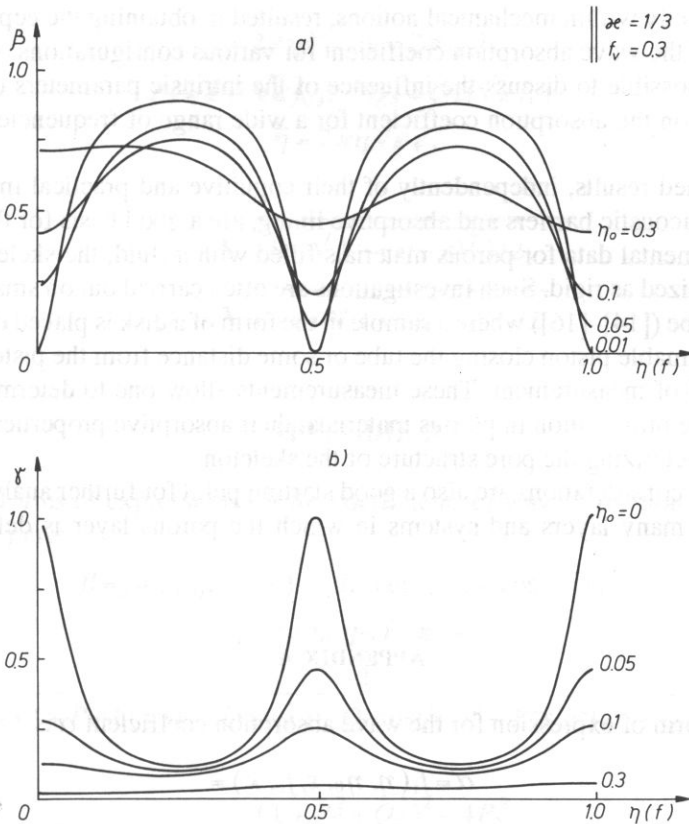


FIG. 9. Dependence of the reflection coefficient β a) and the transmission coefficient γ b) on the dimensionless frequency η (for air and $b = .02$ m $\eta = 1$ correspond to $f \approx 9.8$ kHz).

Figure 9 shows the exemplary curves of dependence of the coefficients β and γ on the dimensionless frequency η for various values of the parameter η_0 characterizing the dissipative properties of the fluid in the pores of a layer. This figure shows that for small values of the parameter η_0 , waves with low and resonance frequencies penetrate intensively through the porous layer, and as the value of η_0 increases, this penetration decreases, and the form of curves for both parameters become uniform in the whole range of frequencies.

The form of the expressions (4.3) indicate a strong influence of pore structure on the values of the parameters β and γ .

5. Concluding remarks

In the paper we have considered the problem of wave interaction with a system composed of a porous layer and an undeformable halfspace immersed in a barotropic fluid, for the case of normal incidence of a harmonic wave. Solving the boundary problem,

formulated strictly within mechanical notions, resulted in obtaining the explicit forms of expressions of the wave absorption coefficient for various configurations of the system. This made it possible to discuss the influence of the intrinsic parameters of the system ($b, d, \kappa, f_v, \bar{k}$) on the absorption coefficient for a wide range of frequencies of the incident wave.

The obtained results, independently of their cognitive and practical importance for designing the acoustic barriers and absorptive lining, are a good basis for the interpretation of experimental data for porous materials filled with a fluid, the skeleton of which may be recognized as rigid. Such investigations are often carried out on small samples in a resonance tube ([14]–[16]) where a sample in the form of a disk is placed either directly on the undeformable piston closing the tube or some distance from the piston depending on the method of measurement. These measurements allow one to determine the parameters of wave propagation in porous materials, their absorptive properties, and the parameters characterizing the pore structure of the skeleton.

The above considerations are also a good starting point for further analysis extended to systems of many layers and systems in which the porous layer is deformable and movable.

APPENDIX A

The explicit form of expression for the wave absorption coefficient α is:

$$\alpha = f_1(\eta, \eta_0, \varepsilon, f_v, \kappa) =$$

$$= 1 - [(\operatorname{ch}(\bar{y}) + P_G \operatorname{sh}(\bar{y}))^2 - (\sin(\bar{x}) - Q_G \cos(\bar{x}))^2 + \\ + P_G^2 \sin^2(\bar{x}) + Q_G^2 \operatorname{ch}^2(\bar{y})] / [P_D^2 \sin^2(\bar{y}) + Q_D^2 \operatorname{ch}^2(\bar{y}) + \\ + (\operatorname{ch}(\bar{y}) + P_D \operatorname{sh}(\bar{y}))^2 - (\sin(\bar{x}) - Q_D \cos(\bar{x}))^2]$$

where

$$P_G = \frac{P_S + \sin(\bar{\eta})[P_S(P_S^2 + Q_S^2 - 1)\sin(\bar{\eta}) + Q_S(P_S^2 + Q_S^2 + 1)\cos(\bar{\eta})]}{P_S^2 + Q_S^2},$$

$$Q_G = \frac{-Q_S + \sin(\bar{\eta})[Q_S(P_S^2 + Q_S^2 + 1)\sin(\bar{\eta}) - P_S(P_S^2 + Q_S^2 - 1)\cos(\bar{\eta})]}{P_S^2 + Q_S^2},$$

$$P_D = -\frac{P_S + \sin(\bar{\eta})[P_S(P_S^2 + Q_S^2 - 1)\sin(\bar{\eta}) - Q_S(P_S^2 + Q_S^2 + 1)\cos(\bar{\eta})]}{P_S^2 + Q_S^2},$$

$$Q_D = \frac{Q_S - \sin(\bar{\eta})[Q_S(P_S^2 + Q_S^2 + 1)\sin(\bar{\eta}) + P_S(P_S^2 + Q_S^2 - 1)\cos(\bar{\eta})]}{P_S^2 + Q_S^2},$$

and

$$\bar{x} = 2\pi\eta P; \quad \bar{y} = 2\pi\eta Q;$$

$$P_S = P/(\sqrt{\kappa}f_v); \quad Q_S = Q/(\sqrt{\kappa}f_v);$$

$$\bar{\eta} = 2\pi\eta\sqrt{\kappa}\varepsilon,$$

whereas

$$P = \sqrt{(1 + \sqrt{1 + (\eta_0/\eta)^2})/2},$$

$$Q = -\sqrt{(-1 + \sqrt{1 + (\eta_0/\eta)^2})/2}.$$

APPENDIX B

The explicit forms of expressions for the coefficients of wave reflection β and of wave transmission γ are:

$$\beta = f_2(\eta, \eta_0, f_v, \kappa) = \gamma K_0 (\text{ch}^2(\bar{y}) - \cos^2(\bar{x}))^2;$$

$$\gamma = f_3(\eta, \eta_0, f_v, \kappa) =$$

$$= \frac{1}{(\text{ch}(\bar{y}) + P_0 \text{sh}(\bar{y}))^2 - (\sin(\bar{x}) - Q_0 \cos(\bar{x}))^2 + P_0^2 \sin^2(\bar{x}) + Q_0^2 \text{ch}^2(\bar{y})}$$

where

$$K_0 = \frac{(1 + P_S^2 + Q_S^2)^2 - 4P_S^2}{4(P_S^2 + Q_S^2)},$$

$$P_0 = -\frac{P_S}{2} \frac{P_S^2 + Q_S^2 + 1}{P_S^2 + Q_S^2}; \quad Q_0 = -\frac{P_S}{2} \frac{P_S^2 + Q_S^2 - 1}{P_S^2 + Q_S^2},$$

and $\bar{x}, \bar{y}, P_S, Q_S$ are the quantities defined in Appendix A.

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