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SOUND SOURCES OF HIGH DIRECTIVITY

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The reversibility of Hankel transform suggests the possibility of constructing such a sound source which radiates only within a certain cone. Both the approximate and accurate theories of that source are given. It is proved in the paper that the accurate source is better than the aaproximate one. The source is called the source of high directivity.

Introduction

In paper [7] was suggested the theoretical possibility of constructing a sound source radiating only within a certain cone. Such a property is exhibited by a baffled piston with a special distribution of the velocity amplitude, given by the Bessel function $J_1(n\frac{r}{a})$ (n will be explained later, "a" – radius of the piston, r – cylindrical coordinate) divided by the argument. The distribution must be extended theoretically to infinity. Such a source was called the source of high directivity. In the present paper we consider the case of the real distribution (only on the piston itself) and we prove that its directivity is better than the theoretical one.

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1. Theoretical basis

The possibility of realizing the directivity pattern mentioned above is the conclusion of reversibility of the Hankel transform [1]. That transform of the velocity distribution $u(r)$ is the main part of the R-directivity index [7]

$$R = \frac{2\pi}{Q}\int_0^a u(r)J_0(kr\sin\gamma)\,r\,dr, \tag{1}$$

where Q is the output of the source, γ – the angle between the z – axis perpendicular to the plane of the piston and the given direction, k – the wave number.
The Hankel transform of zero order is defined by the formula [1]

$$H_0(\rho) = \int_0^\infty u(r) J_0(\rho r) r dr, \tag{2}$$

where $J_0(\rho r)$ denotes the Bessel function of zero order. If the amplitude of velocity on the piston is 0 for $r \geq a$ and $u(r)$ for $r < a$, then

$$H_0(\rho) = \int_0^a u(r) J_0(\rho r) r dr \tag{3}$$

and the directivity index is

$$R = \frac{2\pi}{Q} H_0(k \sin\gamma). \tag{4}$$

It is well known that for the constant amplitude u_0 we have

$$u(r) = \begin{cases} u_0 & r < a, \\ 0 & r \geq a, \end{cases} \tag{5}$$

and the directivity index has the form

$$R = 2\frac{J_1(ka\sin\gamma)}{ka\sin\gamma}, \tag{6}$$

where $J_1(ka\sin\gamma)$ denotes the Bessel function of the order one. One may expect the velocity amplitude distribution of the form

$$u(r) = 2u_0 \frac{J_1(n\frac{r}{a})}{n\frac{r}{a}}, \tag{7}$$

to have the directivity index

$$R = \begin{cases} \text{const} & \gamma \leq \gamma_{\lim}, \\ 0 & \gamma > \gamma_{\lim}, \end{cases} \tag{8}$$

where $\gamma_{\lim}$ denotes the so-called limiting angle – the half of the cone angle, in which the sound is radiated. Of course, the constant in the formula (8) will be normalized to unity. The directivity (8) can be realized only if the distribution (7) is extended on the entire plane of the baffle. We choose the gauge factor n to diminsh the influence of the area $r > a$ under the integral. Such a method is called the approximate one. In the accurate method we choose the velocity distribution as follows:

$$u(r) = \begin{cases} 2u_0 \dfrac{J_1(n\frac{r}{a})}{n\frac{r}{a}} & r < a \\ 0 & r \geq a. \end{cases} \tag{9}$$

In the present paper both methods were applied and the results compared. One may expect that the distribution (9) will not give the sharp break-off of the directivity pattern for $\gamma = \gamma_{\lim}$.

2. Approximate method

We choose the distribution of the velocity as in the formula (7). It is evident that the best option is to accept for $r = a$ $u(a) = 0$; therefore we must have

$$J_1(n) = 0. \tag{10}$$

The gauge factor n must be equal to the zeros of the function $J_1(n)$ denoted as α_{1m}

$$n = \alpha_{1m}, \quad m = 1, 2, \ldots \tag{11}$$

where $\alpha_{10} = 0$, $\alpha_{11} = 3.8317$
The formula (7) now takes the form

$$u(r) = 2u_0 \frac{J_1(\alpha_{1m}\frac{r}{a})}{\alpha_{1m}\frac{r}{a}} \tag{12}$$

For $r \to 0$ we have [7]:

$$\lim_{r \to 0} u_0 \frac{J_1(\alpha_{1m}\frac{r}{a})}{\alpha_{1m}\frac{r}{a}} = \frac{1}{2} \tag{13}$$

and

$$u(0) = u_0. \tag{14}$$

That explains the presence of the factor 2 in the formula (12). Owing to the extension of the distribution (12) to infinity, the Hankel transform of $u(r)$ is

$$H_0(k\sin\gamma) = 2u_0 \frac{a}{\alpha_{1m}} \int_0^\infty J_1(\alpha_{1m}\frac{r}{a}) J_0(kr\sin\gamma)dr. \tag{15}$$

The integral in the formula (15) is given in the tables of integrals [1; p. 681]. It has the value

$$\int_0^\infty J_1(\alpha_{1m}\frac{r}{a})J_0(kr\sin\gamma)\,dr = \begin{cases} \dfrac{a}{\alpha_{1m}} & \text{for } k\sin\gamma < \dfrac{\alpha_{1m}}{a} \\ \dfrac{a}{2\alpha_{1m}} & \text{for } k\sin\gamma = \dfrac{\alpha_{1m}}{a} \\ 0 & \text{for } k\sin\gamma > \dfrac{\alpha_{1m}}{a} \end{cases} \tag{16}$$

From the formula (15) we calculate the directivity index (4) in the form

$$R = \begin{cases} \dfrac{4\pi u_0}{Q\left(\dfrac{\alpha_{1m}}{a}\right)^2} & \text{for } k\sin\gamma < \dfrac{\alpha_{1m}}{a} \\ \dfrac{4\pi u_0}{Q\left(\dfrac{\alpha_{1m}}{a}\right)^2} & \text{for } k\sin\gamma = \dfrac{\alpha_{1m}}{a} \\ 0 & \text{for } k\sin\gamma > \dfrac{\alpha_{1m}}{a} \end{cases} \tag{17}$$

The output of the source for the velocity distribution, determined by (7), is

$$Q = 4\pi u_0 \frac{a}{\alpha_{1m}} \int_0^\infty J_1(\alpha_{1m}\frac{r}{a})\,dr, \tag{18}$$

and solving the elementary integral we get

$$Q = 4\pi u_0 \left(\frac{a}{\alpha_{1m}}\right)^2. \tag{19}$$

Substituting (19) into (17) we get

$$R = \begin{cases} 1 & \text{for } k\sin\gamma \dfrac{\alpha_{1m}}{a} \\ \dfrac{1}{2} & \text{for } k\sin\gamma = \dfrac{\alpha_{1m}}{a}. \\ 0 & \text{for } k\sin\gamma > \dfrac{\alpha_{1m}}{a} \end{cases} \tag{20}$$

We see that in our problem does exist a limiting angle

$$\gamma_{\text{lim}} = \sin^{-1}\frac{\alpha_{1m}}{ka}, \tag{21}$$

above which we have no sound field i.e. no radiation of the sound energy. In the physical sense only $\gamma_{\text{lim}} < \pi/2$ or $\sin\gamma_{\text{lim}} < 1$ is acceptable. For that reason only for the case $ka > \alpha_{1m}$ we have the required source.

The condition $ka > \alpha_{1m}$ can be expressed by the corresponding wavelength λ

$$\lambda < \frac{2\pi a}{\alpha_{1m}}. \tag{22}$$

If we take

$$ka = \alpha_{1m}, \tag{23}$$

then:

$$\lambda_{\lim} = \frac{2\pi a}{\alpha_{1m}}. \tag{24}$$

In the physical sense $\gamma_{\lim}$ takes its maximum value

$$\gamma_{\lim}^{\max} = \frac{\pi}{2}. \tag{25}$$

Figure 1 represents the values $\gamma_{\lim}$ versus ka for α_{11}, α_{12} and α_{13}.

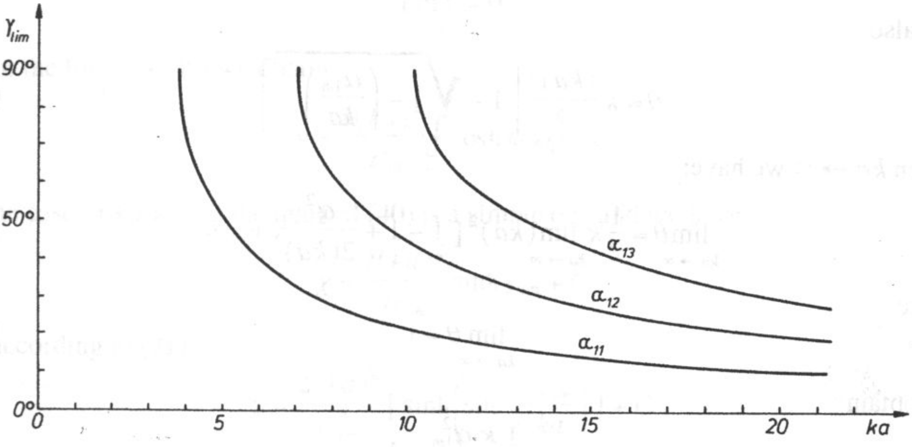

FIG. 1. Values of $\gamma_{\lim}$ versus ka for α_{11}, α_{12} and α_{13}.

Further reduction of ka leads us to a undirectional source. Coming now to the calculation of the specific impedance of our idealised source, we denote by θ its real part and by χ its imaginary part. To find θ we will modify the method applied in [6] for the constant velocity amplitude. For that purpose we introduce in the formula given in [6] a normalizing coefficient κ and we get

$$\theta = \kappa \frac{(ka)^2}{2} \int_0^{\pi/2} R^2(ka \sin\gamma) \sin\gamma d\gamma. \tag{26}$$

It should be remembered that formula (26) for $\kappa = 1$ was obtained for the constant amplitude by equating the acoustic power emitted by the source (expressed by θ) to the power obtained in the farfield by integrating the square of the directivity index. When the amplitude is not constant we must remember that the first quantity is proportional to the mean value of squared velocity amplitude, but the second one is based on the output and is proportional to the square of the mean value. When we calculate the accurate values we will demonstrate the method of calculating κ. In the present case we can find it simply by equating the $\lim_{ka \to \infty} \theta$ to unity. From (20) and (26) we have

$$\theta = \kappa \frac{(ka)^2}{2} \int_0^{\gamma_{\lim}} \sin\gamma d\gamma, \tag{27}$$

from which we directly obtain

$$\theta = \kappa \frac{(ka)^2}{2} (1 - \cos\gamma_{\lim}) \tag{28}$$

Substituting (21) for the value of $\sin\gamma_{\lim}$, we may write

$$\cos\gamma_{\lim} = \sqrt{1 - \sin^2\gamma_{\lim}} = \sqrt{1 - \left(\frac{\alpha_{1m}}{ka}\right)^2} \tag{29}$$

and also

$$\theta = \kappa \frac{(ka)^2}{2} \left[1 - \sqrt{1 - \left(\frac{\alpha_{1m}}{ka}\right)^2} \right]. \tag{30}$$

When $ka \to \infty$ we have:

$$\lim_{ka \to \infty} \theta = \frac{1}{2} \kappa \lim_{ka \to \infty} (ka)^2 [\, 1 - 1 + \frac{\alpha_{1m}^2}{2(ka)^2} + \ldots]. \tag{31}$$

Since

$$\lim_{ka \to \infty} \theta = 1 \tag{32}$$

we obtain

$$\frac{1}{2} \frac{\kappa\, \alpha_{1m}^2}{2} = 1 \tag{33}$$

and the normalizing coefficient:

$$\kappa = \frac{4}{\alpha_{1m}^2} \tag{34}$$

Substituting (34) into (30) we get:

$$\theta = \frac{2(ka)^2}{\alpha_{1m}^2} \left[1 - \sqrt{1 - \left(\frac{\alpha_{1m}}{ka}\right)^2} \right]. \tag{35}$$

The formula (35) is valid only when $ka \geq \alpha_{1m}$.
When $ka \to \alpha_{1m}$, we have:

$$\lim_{ka \to \alpha_{1m}} \theta = 2. \tag{36}$$

As it was explained before, the case $ka < \alpha_{1m}$ does not present any interest. But, to keep continuity of our reasoning, we can admit that for $ka < \alpha_{1m}$, $\gamma_{\lim}$ remains constant and equal to $\pi/2$ in the formula (28), and

$$\theta = \frac{2(ka)^2}{\alpha_{1m}^2}. \tag{37}$$

Of course, when $ka \to 0$, the value of θ tends to zero. To obtain the imaginary part of the specific impedance we use the method given by W. RDZANEK in [4], and substitute in the formula (26), representing the real part, $\cosh\psi$ for $\sin\gamma$ and integrating with respect to ψ from 0 to ∞. We have therefore, due to (34):

$$\chi = \frac{2(ka)^2}{\alpha_{1m}^2} \int_0^\infty R^2(ka\cosh\psi)\cosh\psi\, d\psi. \tag{38}$$

In the considered case $R(\cosh\psi)$ is equal to 1 for γ ranging from 0 to $\gamma_{\lim}$, and 0 for $\gamma > \gamma_{\lim}$. It is well known [4] that the application of integral transform (38) gives us the result to within the accuracy of a constant. That value must be found from the condition

$$\lim_{ka\to\infty} \chi = 0. \tag{39}$$

From the formula (38) we obtain

$$\chi = \frac{2(ka)^2}{\alpha_{1m}^2} \int_0^{\gamma_{\lim}} \cosh\psi\, d\psi + C. \tag{40}$$

In the case of $ka > \alpha_{1m}$, the integral (40) is a simple one and we have

$$\chi = \frac{2(ka)^2}{\alpha_{1m}^2} \sinh\gamma_{\lim} + C. \tag{41}$$

or, according to (21)

$$\chi = \frac{2(ka)^2}{\alpha_{1m}^2} \left[\sinh\left(\sin^{-1}\frac{\alpha_{1m}}{ka}\right)\right] + C. \tag{42}$$

Since integration in (40) is performed with respect to ψ, C may be a function of ka, and should be calculated from the condition (39)

When $ka \to \infty$ we have

$$\sinh\left(\sin^{-1}\frac{\alpha_{1m}}{ka}\right) \approx \sinh\frac{\alpha_{1m}}{ka} \approx \frac{\alpha_{1m}}{ka} + \ldots \tag{43}$$

and

$$\lim_{ka\to\infty}\chi = 2\lim_{ka\to\infty}\frac{(ka)^2}{\alpha_{1m}^2}\frac{\alpha_{1m}}{ka} + C = 0. \tag{44}$$

The only possibility of fulfilling the above condition is to take

$$C = -\frac{2ka}{\alpha_{1m}}, \tag{45}$$

and finally we obtain

$$\chi = \frac{2(ka)^2}{\alpha_{1m}^2}\sinh\left(\sin^{-1}\frac{\alpha_{1m}}{ka}\right) - \frac{2ka}{\alpha_{1m}}. \tag{46}$$

When $ka = \alpha_{1m}$ we must replace $\frac{ka}{\alpha_{1m}}$ by 1 and we obtain

$$\chi = 2\sinh\frac{\pi}{2} - 2 = 2.5986. \tag{47}$$

When $ka < \alpha_{1m}$, the value of $\gamma_{\lim}$ in the formula (41) remains equal to $\pi/2$. In that case we must choose another value of the constant (from the condition of continuity for $\alpha_{1m} = ka$ and the positive value of χ_2 i.e.

$$\kappa = \frac{2(ka)^2}{\alpha_{1m}^2}(\sinh\frac{\pi}{2} - 1) \tag{48}$$

or, substituting the value of $\sinh\pi/2$,

$$\sinh\pi/2 = 2.2993,$$

we get

$$\chi = 2.5986\frac{(ka)^2}{\alpha_{1m}^2} \tag{49}$$

We see that for $ka \to 0$ we have

$$\lim_{ka\to 0}\chi = 0$$

Of course, the case of $ka < \alpha_{1m}$ is of theoretical interest only, since it has no application in practice.

3. The accurate method

We assume the distribution function of the velocity amplitude as

$$u(r) = \begin{cases} 2u_0\dfrac{J_1(\alpha_{1m}\frac{r}{a})}{\alpha_{1m}\frac{r}{a}} & \text{for} \quad 0 < r < a \\ 0 & \text{for} \quad r \geq a \end{cases} \tag{50}$$

The directivity index (1) takes then the form:

$$R = u_0\frac{4\pi a}{Q\alpha_{1m}}\int_0^{a} J_1(\alpha_{1m}\frac{r}{a})J_0(kr\sin\gamma)\,dr. \tag{51}$$

In order to calculate the output of the source we must replace in (18) the upper limit ∞ by a.

$$Q = 4\pi u_0 \frac{a}{\alpha_{1m}} \int_0^{\alpha} J_1\left(\alpha_{1m}\frac{r}{a}\right) dr. \tag{52}$$

The integral is an elementary one and we get

$$Q = 4\pi u_0 \left(\frac{a}{\alpha_{1m}}\right)^2 [1 - J_0(\alpha_{1m})]. \tag{53}$$

Substituting (53) into (51) we obtain the directivity index in the form

$$R = \frac{\alpha_{1m}}{a[1 - J_0(\alpha_{1m})]} \int_0^{\alpha} J_1\left(\alpha_{1m}\frac{r}{a}\right) J_0(kr\sin\gamma)\, dr. \tag{54}$$

Evidently, for $\gamma = 0$ we get $R = 1$. In the formula (54) we introduce a new variable:

$$x = \frac{r}{a} \tag{55}$$

and we get:

$$R = \frac{\alpha_{1m}}{1 - J_0(\alpha_{1m})} \int_0^1 J_1(\alpha_{1m}x) J_0(kax\sin\gamma)\, dx, \tag{56}$$

Figures 2, 3, 4 represent the directivity index versus the angle γ for α_{11}, α_{12} and α_{13}. The continuous lines represent the approximate case the dashed line the accurate solutions (56). Of course, the value $\gamma_{\lim}$ has not the same meaning as before. Nevertheless, it is easy to calculate the directivity pattern for $\gamma = \gamma_{\lim}$ (21) because the above integral takes then a form given in the tables of integrals. In that case we obtain ($ka\sin\gamma = \alpha_{1m}$)

$$R_{\lim} = \frac{\alpha_{1m}}{1 - J_0(\alpha_{1m})} \int_0^1 J_1(\alpha_{1m}x) J_0(\alpha_{1m}x)\, dx. \tag{57}$$

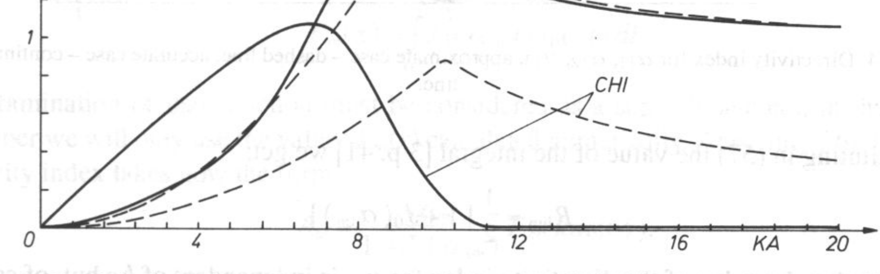

FIG. 2. Directivity index for α_{11}, α_{12}, α_{13}: approximate case – dashed line, accurate case – continuous line.

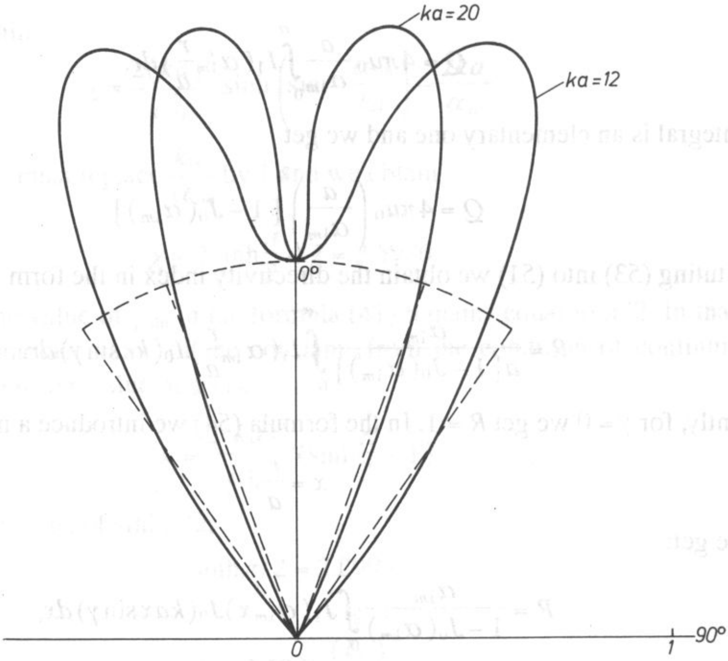

FIG. 3. Directivity index for α_{11}, α_{12}, α_{13}: approximate case – dashed line, accurate case – continuous line.

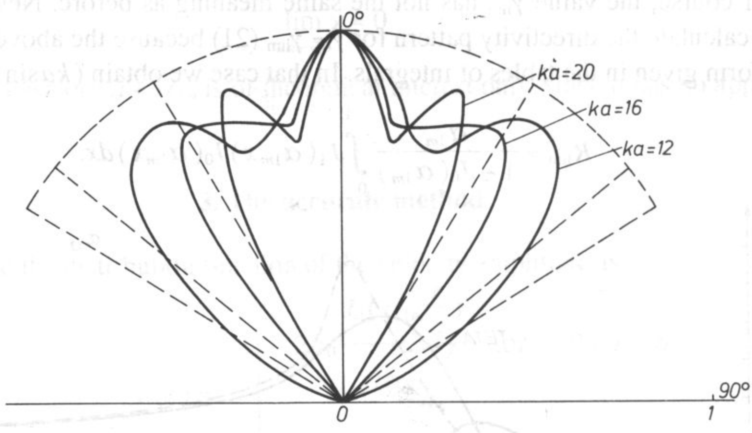

FIG. 4. Directivity index for α_{11}, α_{12}, α_{13}: approximate case – dashed line, accurate case – continuous line.

Substituting in (57) the value of the integral [3 p. 41] we get:

$$R_{\lim} = \frac{1}{2}\,[\,1 + J_0(\,\alpha_{1m}\,)\,]. \tag{58}$$

We see that the value of the directivity index for $\gamma_{\lim}$ is independent of ka but, of course, it is obtained for a fixed $\gamma_{\lim}$ which depends on ka. For example, we have for

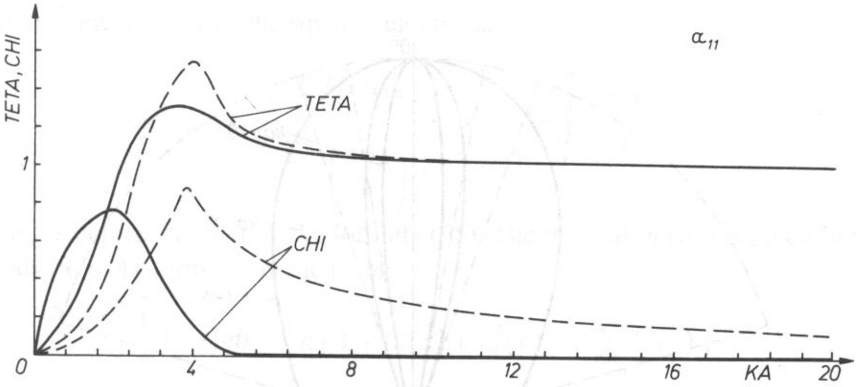

FIG. 5. Real θ_m and imaginary χ_m parts of the specific impedance for α_{11}, α_{12} and α_{13} versus *ka*. Continuous line – accurate solution, dashed line – approximate one.

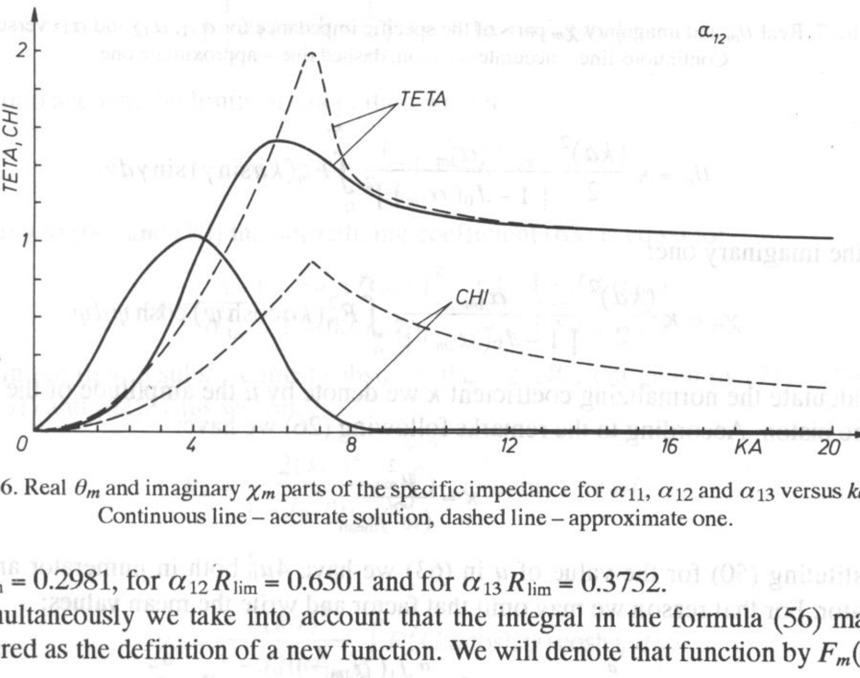

FIG. 6. Real θ_m and imaginary χ_m parts of the specific impedance for α_{11}, α_{12} and α_{13} versus *ka*. Continuous line – accurate solution, dashed line – approximate one.

$\alpha_{11} R_{\lim} = 0.2981$, for $\alpha_{12} R_{\lim} = 0.6501$ and for $\alpha_{13} R_{\lim} = 0.3752$.

Simultaneously we take into account that the integral in the formula (56) may be considered as the definition of a new function. We will denote that function by $F_m(x)$

$$F_m(x) = \int_0^1 J_1(\alpha_{1m} t) J_0(xt)\, dt. \tag{59}$$

Examination of that function must be considered as a separate subject; in the present paper we will only use the values $F_m(x)$ calculated numerically. Applying (56), the directivity index takes now the form:

$$R_m = \frac{\alpha_{1m}}{1 - J_0(\alpha_{1m})} F_m(ka \sin\gamma). \tag{60}$$

According to (26) we can write the real part of the specific impedance in the following form:

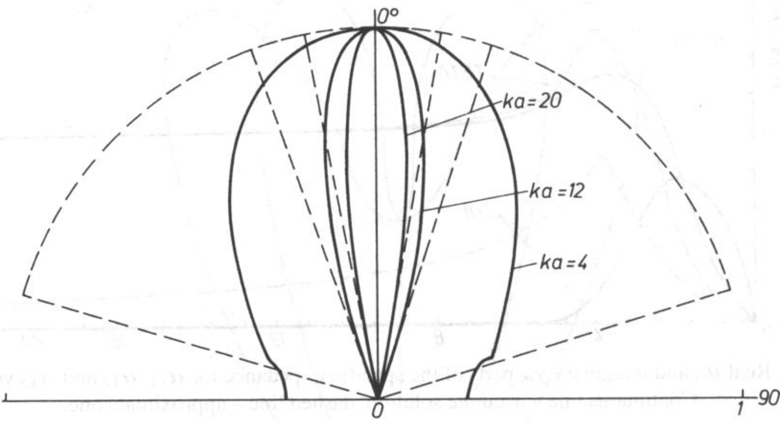

FIG. 7. Real θ_m and imaginary χ_m parts of the specific impedance for α_{11}, α_{12} and α_{13} versus ka. Continuous line – accurate solution, dashed line – approximate one.

$$\theta_m = \kappa \frac{(ka)^2}{2} \frac{\alpha_{1m}^2}{[1 - J_0(\alpha_{1m})]^2} \int_0^{\pi/2} F_m^2(ka \sin\gamma) \sin\gamma d\gamma \tag{61}$$

and the imaginary one:

$$\chi_m = \kappa \frac{(ka)^2}{2} \frac{\alpha_{1m}^2}{[1 - J_0(\alpha_{1m})]^2} \int_0^{\infty} F_m^2(ka \cosh\psi) \cosh\psi d\psi \tag{62}$$

To calculate the normalizing coefficient κ we denote by u the amplitude of the velocity on the piston. According to the remarks following (26) we have:

$$\kappa = \frac{u_{\text{mean}}^2}{(u^2)_{\text{mean}}}. \tag{63}$$

Substituting (50) for the value of u in (63) we have $4u_0^2$ both in numerator and denominator. For that reason we may omit that factor and write the mean values:

$$u_{\text{mean}} = \frac{2}{a^2} \int_0^a u(r) r dr = \frac{2}{a^2} \int_0^{\alpha} \frac{J_1(\alpha_{1m} \frac{r}{a})}{\frac{\alpha_{1m}}{a}} dr = \frac{2}{\alpha_{1m}^2} \int_0^{\alpha_{1m}} J_1(x) dx. \tag{64}$$

where:

$$x = \alpha_{1m} \frac{r}{a}. \tag{65}$$

The integral in the formula (64) is a simple one and we get:

$$u_{\text{mean}} = \frac{2}{a^2} [1 - J_0(\alpha_{1m})] \tag{66}$$

We obtain the mean value of the square velocity as:

$$(u^2)_{\text{mean}} = \frac{2}{a^2}\int_0^{a} \frac{J_1^2(\alpha_{1m}\frac{r}{a})}{\left(\frac{\alpha_{1m}}{a}\right)^2 r}\, dr = \frac{2}{\alpha_{1m}^2}\int_0^{\alpha_{1m}} \frac{J_1^2(x)}{x}\, dx, \tag{67}$$

where x is, as before, defined by the formula (65). The integral in (67) is given in tables of integrals [3, p. 41 form 17] in the form:

$$\int_0^x \frac{1}{t} J_n^2(t)\, dt = \frac{1}{2} n\,[\, 1 + J_0^2(x) + J_n^2(x) - 2\sum_{k=0}^{n} J_k^2(x)\,] \tag{68}$$

In our case we have $n = 1$ and

$$\int_0^x \frac{1}{t} J_1^2(t)\, dt = \frac{1}{2}[\, 1 - J_0^2(x) - J_1^2(x)\,]. \tag{69}$$

Taking into account the limits of integration we get

$$(u^2)_{\text{mean}} = \frac{1}{\alpha_{1m}^2}[\, 1 - J_0^2(\alpha_{1m})\,]. \tag{70}$$

According to (66) and (70) the normalizing coefficient (63) is equal to:

$$\kappa = \frac{4}{\alpha_{1m}^2}\frac{[\, 1 - J_0(\alpha_{1m})\,]^2}{1 - J_0^2(\alpha_{1m})} = \frac{4}{\alpha_{1m}^2}\frac{1 - J_0(\alpha_{1m})}{1 + J_0(\alpha_{1m})}. \tag{71}$$

To obtain the final results we must substitute the κ coefficient given by (71) to the formulae (61) and (62). Thus we get

$$\theta_m = \frac{2(ka)^2}{1 - J_0{}^2(\alpha_{1m})}\int_0^{\pi/2} F_m^2(ka\sin\gamma)\sin\gamma\, d\gamma \tag{72}$$

and:

$$\chi_m = \frac{2(ka)^2}{1 - J_0^2(\alpha_{1m})}\int_0^{\infty} F_m^2(ka\cosh\psi)\cosh\psi\, d\psi. \tag{73}$$

The enclosed figures present the values of θ_m and χ_m (72), (73) continuous line and the approximate values dashed line for $m = 1, 2, 3$ versus ka. The results were obtained numerically.

Conclusions

The approximate method of the considered velocity distribution gives us the directivity coefficient equal to 1 in a certain angle, when $ka > \alpha_{1m}$. The results of the accurate method have a better directivity. Of course, the cut-off of the directivity coefficient does not occur, but practically the pattern is sharper and the lateral lobes are so small, that they can be neglected – they are invisible in the figure. The real part of the specific impedance is practically equal to 1 for $ka > 1.5\alpha_{1m}$, and the imaginary part is then equal to 0. So we may say that the source adjusts well to the medium. If we compare the results for different values of α_{1m} we see that the source for α_{11} has the best properties. Since this is the case the easiest to be realised we have no nodal lines, this value should be chosen.

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