ARCHIVES OF ACOUSTICS
17, 2, 321–333 (1992)

ACOUSTIC POWER OF RADIATION OF A CIRCULAR PLATE FIXED ON THE RIM AND VIBRATING UNDER EXTERNAL PRESSURE

W. RDZANEK

Institute of Physics, Pedagogical University
(34-310 Rzeszów, Rejtana 16a)

The main scientific aim of our work was the realization of theoretical research on the problem of energy radiation of axially-symmetric forced vibrations of a circular plate. This research focussed on the determination of frequency characteristics of relative acoustic power. A thin plate, fixed on the rim in a rigid and flat acoustic baffle, radiating into a lossless and homogeneous fluid medium was considered. Dynamic interactions between the acoustic wave radiated by the plate and the form of the radiations as well as losses in the plate were neglected.

The active acoustic power of radiation was expressed with a single integral within finite limits and with elementary form in special cases, i.e., for high-frequency wave radiation and when the plate's thickness is sufficiently small in relation to its diameter. The results of calculations are also presented in graphical form.

1. Introduction

The practical application of a circular plate as a vibrating system in acoustic devices – sound emitting and sound receiving – has led to a comprehensive and more detailed description of acoustic properties and of the problem of radiation energy of axially symmetrical forces vibrations of a circular plate, in particular.

Besides theoretical considerations of the analysis of the field of acoustic radiation from supercifial sources with a "guessed" distribution of vibration velocity (which approximately satisfies the boundary conditions related with the shape of the source), this field was also theoretically analysed. Here the cause is taken into consideration assuming that the distribution of the force inducing vibrations of the source is known.

A detailed description of parameters which characterize a circular membrane as a sound source or receiver, with a stress on the problem of the output impedance frequency characteristic, can be found in HAJASAKI's paper [1]. The acoustic impedance of radiation of a circular membrane excited to vibrate with the neglect of losses in the membrane and the influence of the surroundings is known from the paper [5]. SZENDEROW [9] analysed sound radiation of an oscillating membrane without an acoustic baffle applying associated integral equations. The paper [8] analyzes the problem of radiation of an acoustic wave by a circular plate. However, the directional characteristic was only determined with the application of the classical Kirchoff–Love plate theory under the assumption that the surface distribution of the factor forcing vibrations is known.

Expressions for acoustic power radiated by a circular plate, concerning individual forms of vibrations, are presented in the papers [6] and [7]. Yet, their application was limited to high frequencies. This problem was expanded in LEVINE's and LEPPINGTON's paper [3] by including a correction for the "oscillating" character of radiated acoustic power. Furthermore the effective damping coefficient was calculated for frequencies comparable with resonance frequencies, including losses in the plate and the relation between the wave radiated by the plate and its vibrations.

However, there is no expression so far for acoustic power radiated by a circular plate when the surface distribution of the factor forcing vibrations is known.

This paper undertakes this problem under the assumption that the plate is sufficiently thin and the forcing pressure is strong enough to neglect the influence of losses, including vibration damping by the acoustic field. Frequency characteristics of active power were determined for a known surface distribution of the pressure forcing vibrations. Elementary forms of expressions were achieved for special cases, i.e., for high frequencies of radiated wave and for a plate thickness sufficiently small with respect to its diameter. It was also shown that expressions obtained for limiting cases are already known from previous papers. The results of numerical calculations are presented in graphical form.

2. Assumptions of the analysis

Let us consider the case of an acoustic wave radiated in a fluid medium with low self-resistance (e.g., air) by a thin homogeneous circular plate ($r \leq a$, $z = 0$) with a plane, as a rigid acoustic baffle ($r > a$, $z = 0$) behind it complete fixing of the plate results in the following boundary conditions: the deflection of the plate $\eta(r)$ and the derivative $\partial\eta(r)/\partial r$ are equal to zero for $r = a$. We assume that the plate is subject to external pressure $\mathrm{Re}\{f(r)\exp(-i\omega t)\}$ for $0 \leq r < a$.

The theoretical analysis of such a system is based on the equation of vibrations given by LEVINE and LEPPINGTON [3]

$$(k_0^{-4}\nabla^4 - 1)\,v + 2\varepsilon_1 k_0 \varphi = -i/(M\omega)f \tag{1}$$

for $z = +0$. The quantity $\varphi(r, z)\exp(-i\omega t)$ is the acoustic potential which fulfills the Helmholtz equation $(\nabla^2 + k_0^2)\,\varphi = 0$, $k = \omega/c$. From now on we will neglect the time factor exp $(-i\omega t)$. The normal component of vibration velocity of the plates surface $v(r) = -i\omega\eta(r)$ and acoustic pressure generated by the plate $p(r) = i\rho_0\omega\varphi(r)$, where ρ_0 is the density in rest stage of the fluid medium. The wave number of the plate is defined with $k_p^4 = M\omega^2/B$, where M is the mass of the plate per unit surface,

$B = B_0(1 - i\varepsilon')$ is the plates flexural rigidity with internal losses in the plate included [4], ε'– measure of plates damping. The quantity $\varepsilon_1 = \rho_0/(Mk_0)$ is the measure of density in rest stage of the fluid medium to material density of the plate ratio.

We will limit our considerations to the case of acoustic power radiated by the plate to which the theory of bending of thin plates applies, accepting that the plates thickness h satisfies the inequality (e.g., [2]).

$$h \le 0.1\,D, \tag{2}$$

where $D = 2a$ is the diameter of the plate. In accordance with the assumptions, the plate is surrounded by a fluid medium with low self-resistance and the following condition is fulfilled:

$$\varepsilon_i k_0 = \rho_0/M << 1, \tag{3}$$

Hence, instead of Eq. (1) we have

$$(\nabla^4 - k^4)\,\eta(r) = f(r)/B_0, \tag{4}$$

where $k^2 = \omega\sqrt{M/B_0}$. The disregard of term $2\varepsilon_1 k_0 \varphi$ in Eq. (1) means that the influence of the acoustic wave radiated by both surfaces of the plate on the form of vibrations is disregarded.

Moreover, we accept that the amplitude of the factor inducing vibrations is as follows:

$$f(r) = \begin{cases} f_0 & \text{for} \quad 0 < r < a_0, \\ 0 & \text{for} \quad a_0 < r < a. \end{cases} \tag{5}$$

where f_0 = const. Accepted simplifications lead to a limitation of the range of application of the solution to Eq. (4) depending on the frequency of the factor inducing vibrations. The solution to Eq. (4) should not be applied to frequencies close or equal to resonance frequencies.

In practice such a type of vibration excitation can be realized with, for example, two flat circular electrodes with a radius $a_0 < a$, parallel to the surface of the plate [1]. The solution to Eq. (4) for a plate excited to vibrate by the factor (5) is as follows [8]:

$$\eta_1(r)/\eta_0 = 1 - \frac{\gamma_0}{2S(\gamma)}\left\{\frac{1}{\gamma}I_1(\gamma_0) + \frac{\pi}{2}I_1(\gamma)\,[\,J_1(\gamma_0)N_0(\gamma) + \right.$$
$$\left. - J_0(\gamma)N_1(\gamma_0)\,] - \frac{\pi}{2}I_0(\gamma)\,[\,J_1(\gamma)N_1(\gamma_0) - J_1(\gamma_0)N_1(\gamma)\,]\right\} \times$$
$$\times J_0(kr) + \frac{\gamma_0}{2S(\gamma)}\left\{\frac{1}{\gamma}J_1(\gamma_0) + J_1(\gamma)\,[\,I_1(\gamma_0)K_0(\gamma) + I_0(\gamma)K_1(\gamma_0)\,] + \right.$$
$$\left. - J_0(\gamma)\,[\,I_1(\gamma_0)K_1(\gamma) - I_1(\gamma)K_1(\gamma_0)\,]\right\} I_0(kr) \tag{6}$$

for $0 < r < a_0$,

$$\eta_2(r)/\eta_0 = -\frac{\gamma_0}{2S(\gamma)}\left\{\frac{1}{\gamma}I_1(\gamma_0)+\frac{\pi}{2}J_1(\gamma_0)\left[N_0(\gamma)I_1(\gamma)+\right.\right.$$

$$\left.\left.+N_1(\gamma)I_0(\gamma)\right]\right\}J_0(kr)-\frac{\gamma_0}{2S(\gamma)}\left\{\frac{1}{\gamma}J_1(\gamma_0)+I_1(\gamma_0)\left[K_0(\gamma)J_1(\gamma)+\right.\right.$$

$$\left.\left.-K_1(\gamma)J_0(\gamma)\right]\right\}I_0(kr)+\frac{\gamma_0}{2}\left[I_1(\gamma_0)K_0(kr)+\frac{\pi}{2}J_1(\gamma_0)N_0(kr)\right] \quad (7)$$

for $a_0 < r < a$

where $J_n(x)$ is a Bessel function, $I_n(x)$ – modified Bessel function, $N_n(x)$ – Neumann function, $K_n(x)$ – cylindrical MacDonald function, all are of the n-order.
The following notation was introduced:

$$\gamma = ka, \quad \gamma_0 = ka_0, \quad S(\gamma) = J_0(\gamma)I_1(\gamma) + I_0(\gamma)J_1(\gamma) \quad (8)$$

and

$$\eta_0 = -\frac{f_0}{B_0 k^4}. \quad (9)$$

The relative amplitude of the transverse displacement of points on the plates surface can be expressed in a much simpler way in a special case when the whole surface of the plate is excited to vibrate with a factor different from zero. If we accept $a_0 = a$ ($\gamma_0 = \gamma$), then instead of the solution (6) and (7) we have

$$\eta_1(r)/\eta_0 = 1 - \frac{1}{S(\gamma)}\left[I_1(\gamma)J_0(kr) + J_1(\gamma)I_0(kr)\right] \quad (10)$$

and $\eta_2(r)/\eta_0 = 0$

2. Acoustic power

The calculation of active acoustic power radiated by the vibrating plate will be based on the definition

$$N = \frac{1}{2}\int_\sigma p(\overline{r})\, v^*(\overline{r})\, d\sigma \quad (11)$$

where p is the pressure radiated by the plate and v^* is a quantity conjugate with the complex quantity of vibration velocity v. In the case of a circular plate with its vibrations presented by the formulae (6) and (7), i.e., a circular source with an axially-symmetrical distribution of vibration velocity the Hankel representation (6) for acoustic power is applied:

$$N = \rho_0 c\, \pi k_0^2 \int_0^{\pi/2} M(\vartheta)M^*(\vartheta)\sin\vartheta\, d\vartheta, \quad (12)$$

where

$$M(\vartheta) = i\omega\left[\int_0^{a_0} \eta_1(r)J_0(k_0 r\sin\vartheta)r\,dr + \int_{a_0}^{a} \eta_2(r)J_0(k_0 r\sin\vartheta)r\,dr\right] \tag{13}$$

is the characteristic function of the source. We calculate integrals from Eq. (13)

$$N = \rho_0 c\pi a^2\varepsilon_0^4\left(\frac{f_0 k_0 a}{M\omega}\right)^2\int_0^{\pi/2}\left\{\frac{1}{1-\left(\dfrac{w}{\gamma}\right)^4}\left[\frac{J_1(\varepsilon_0 w)}{\varepsilon_0 w} + \right.\right.$$

$$\left.\left. - U(\varepsilon_0,\gamma)\frac{w}{\gamma}J_1(w) - W(\varepsilon_0,\gamma)J_0(w)\right]\right\}^2 \sin\vartheta\, d\vartheta, \tag{14}$$

where $\varepsilon_0 = a_0/a$, $W = k_0 a\sin\vartheta$ and

$$U = U(\varepsilon_0,\gamma) = \frac{1}{\varepsilon_0}\gamma S\left[J_1(\varepsilon_0\gamma)I_0(\gamma) - I_1(\varepsilon_0\gamma)J_0(\gamma)\right], \tag{15}$$

$$W = W(\varepsilon_0\gamma) = \frac{1}{\varepsilon_0}\gamma S\left[J_1(\varepsilon_0\gamma)I_1(\gamma) + I_1(\varepsilon_0\gamma)J_1(\gamma)\right]. \tag{16}$$

It is convenient to use the notion of relative acoustic power $N/N^{(\infty)}$ in numerical calculations $N^{(\infty)}$ is the active power of the source for $k_0 = 2\pi/\lambda \to \infty$. If $k_0 \to \infty$ then $p(\bar{r}) = \rho c\, v(\bar{r})$. On the basis of the formula (11), we reach

$$N^{(\infty)} = \lim_{k_0\to\infty} N = \frac{1}{2}\rho_0 c\int_\sigma v^2(\bar{r})\,d\sigma. \tag{17}$$

In the case of vibrations of the plate (6) and (7) the quantity $N^{(\infty)}$ is equal to

$$N^{(\infty)} = \rho_0 c\pi a^2\left(\frac{f_0}{M\omega}\right)^2\frac{\varepsilon_0^2}{4S(\gamma)}\left\{2S(\gamma) + \varepsilon_0 S(\gamma_0) + \right.$$

$$+\frac{\pi}{2}I_0(\gamma)\,T_1\left[\gamma_0 J_0(\gamma_0) - 3J_1(\gamma_0)\right] - J_0(\gamma)T_2\left[\gamma_0 I_0(\gamma_0) - 3I_1(\gamma_0)\right] +$$

$$+\frac{\pi}{2}I_1(\gamma)J_1(\gamma_0)\left[\gamma_0 T_3 - 3W_1\right] + J_1(\gamma)I_1(\gamma_0)\left[\gamma_0 T_4 - 3W_2\right] +$$

$$\left. -\frac{6}{\gamma}I_1(\gamma_0)J_1(\gamma_0) + \frac{1}{S(\gamma)}\left[J_1(\gamma_0)I_0(\gamma) - I_1(\gamma_0)J_0(\gamma)\right]^2\right\}, \tag{18}$$

where

$$\begin{aligned}
W_2 &= J_1(\gamma_0)N_0(\gamma) - J_0(\gamma)J_1(\gamma_0),\\
W_2 &= I_1(\gamma_0)K_0(\gamma) + I_0(\gamma)K_1(\gamma_0),\\
T_1 &= J_1(\gamma_0)N_1(\gamma) - J_1(\gamma)N_1(\gamma_0)\\
T_2 &= I_1(\gamma_0)K_1(\gamma) - I_1(\gamma)K_1(\gamma_0),\\
T_3 &= J_0(\gamma_0)N_0(\gamma) - J_0(\gamma)N_0(\gamma_0),\\
T_4 &= I_0(\gamma_0)K_0(\gamma) - I_0(\gamma)K_0(\gamma_0).
\end{aligned} \tag{19}$$

The form of the quantity $N^{(\infty)}$ is more elementary when the whole surface of the plate is excited to vibrate with a factor different from zero. For $a_0 = a$, we have $W_1 = 2/(\pi\gamma)$, $W_z = 1/\gamma$, $T_1 = T_2 = T_3 = T_4 = 0$ and instead of Eq. (18) we have

$$N^{(\infty)} = \rho_0 c \pi a^2 \left(\frac{f_0}{M\omega}\right)^2 \beta, \tag{20}$$

where

$$\beta = 1 - \frac{J_1(\gamma) I_1(\gamma)}{S(\gamma)} \left[\frac{3}{\gamma} + \frac{J_0(\gamma) I_0(\gamma)}{S(\gamma)}\right]. \tag{20a}$$

4. Acoustic power for high frequencies

The relative acoustic power $\sigma_0 = N/N$ for the case of $a_0 = a\,(\gamma_0 = \gamma)$ is calculated on the basis of the relations (14) and (20). We substitute $\xi = \sin\vartheta$ and introduce the following notations

$$\alpha = k_0 a, \; \delta = k/k_0 = \gamma/a, \tag{21}$$

$$W_0 = (2/S) J_1(\gamma) I_1(\gamma),$$
$$U_0 = (1/S)[J_1(\gamma) I_0(\gamma) - I_1(\gamma) J_0(\gamma)]. \tag{22}$$

We achieve

$$\sigma_0 = \delta^6 \beta^{-1} \int_0^1 \frac{\varepsilon}{\sqrt{1-\xi^2}} \times$$
$$\times \frac{\{(\delta/\xi) J_1(\alpha\xi) - (\xi/\delta) U_0 J_1(\alpha\xi) - W_0 J_0(\alpha\xi)\}^2}{(\xi^4 - \delta^4)^2} d\xi. \tag{23}$$

The integral (23) is converted into a form making σ_0 analysis for high frequencies easier on the basis of the LEVINE and LEPPINGTON method [3]. We introduce the function of a complex variable

$$F(z) = [\delta/z - (z/\delta) U_0]^2 J_1(\alpha z) H_1^{(1)}(\alpha z) +$$
$$+ 2(z/\delta) U_0 W_0 J_0(\alpha z) H_1^{(1)}(\alpha z) + W_0 [W_0 J_0(\alpha z) +$$
$$- 2(\delta/z) J_1(\alpha z)] H_0^{(1)}(\alpha z) \tag{24}$$

for

$$\operatorname{Re} F(\xi) = \{[\delta/\xi - (\xi/\delta) U_0] J_1(\alpha\xi) - W_0 J_0(\alpha\xi)\}^2. \tag{25}$$

The calculation begins from a contour integral

$$\int_C \frac{z F(z)\, dz}{\sqrt{1-z^2}\,(z^4 - \delta^4)^2} = 0. \tag{26}$$

The integral is calculated for contour C (Fig. 1) inside which the integrand is single-valued and regular. We assume that $\delta < 1$ $(k_0 > k)$. The contour integral is noted as follows

$$\int_0^1 + \int_1^\infty + \int_{R_\infty} + \int_\infty^0 = \frac{1}{2}\operatorname{Re} z(z=\delta) + \frac{1}{2}\operatorname{Re} z(z=i\delta) + \frac{1}{4}\operatorname{Re} z(z=0). \tag{26a}$$

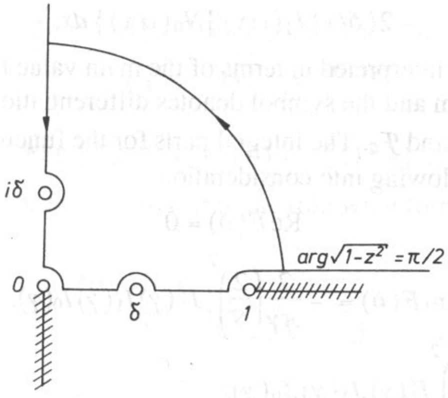

FIG. 1. Integration path (see [3]) for the expression (26)

Contributions to the value of the integral (26) from small semicircles around singular points with second order poles $z = \delta, +i\delta$ and 1/4 arc of the small circle arround the breakway point $z = 0$, which is a first order pole at the same time are calculated with the following auxiliary taken into consideration:

$$\mathcal{F}_1(z) = \frac{zF(z)}{\sqrt{1-z^2}\,(z+\delta)^2(z^2+\delta^2)^2},$$

$$\mathcal{F}_2(z) = \frac{zF(z)}{\sqrt{1-z^2}\,(z+i\delta)^2(z^2-\delta^2)^2},$$

$$\mathcal{F}_0(z) = \frac{z^2F(z)}{\sqrt{1-z^2}\,(z^4-\delta^4)^2}. \tag{27}$$

The contribution during integration over the big circle R_∞ disappears when its radius oncreases infinitely. If we also take into consideration the fact that $\operatorname{Re} F(i\tau) = 0$ for real values of τ, we obtain from the expression (26)

$$\operatorname{Re}\!\!\fint_0^1 \frac{xF(x)}{\sqrt{1-x^2}(x^4-\delta^4)^2}\,dx = \int_0^1 \frac{x}{\sqrt{1-x^2}} \times$$

$$\times \frac{\{\,[\,\delta/x - (x/\delta)U_0\,]J_1(\alpha x) - W_0 J_0(\alpha x)\,\}^2}{(x^4-\delta^4)^2}\,dx =$$

$$= \operatorname{Re}\left\{\pi i\left[\frac{1}{2}\mathcal{F}_0(0) + \mathcal{F}_1'(\delta) + \mathcal{F}_2'(i\delta)\right]\right\} + \tag{28}$$

$$+\int_1^{\infty} \frac{x}{\sqrt{x^2-1}\,(x^4-\delta^4)^2} \{ [\, \delta/x - (x/\delta) U_0 \,]^2 J_1(\alpha x) N_1(\alpha x) +$$
$$+ 2(x/\delta) U_0 W_0 J_0(\alpha x) N_1(\alpha x) + W_0 [\, W_0 J_0(\alpha x) +$$
$$- 2(\delta/x) J_1(\alpha x) \,] N_0(\alpha x) \} \, dx, \qquad \text{[cont.] (28)}$$

where the first integral is interpreted in terms of the main value the second integral is its expansion to explicit form and the symbol denotes differentiation for an adequate argument of the functions $\mathcal{F}_1$ and $\mathcal{F}_2$. The integral parts for the functions $\mathcal{F}_1'(\delta)$, $\mathcal{F}_2(i\delta)$ are found after taking the following into consideration:

$$\mathrm{Re} F(\delta) = 0$$

$$\mathrm{Im} F(\delta) = -\frac{2}{\pi\gamma}\left(\frac{2}{S}\right)^2 J_1^2(\gamma) I_1(\gamma) I_0(\gamma),$$

$$F(i\delta) = \frac{2i}{\pi\gamma}\left(\frac{2}{S}\right)^2 I_1^2(\gamma) J_1(\gamma) J_0(\gamma),$$

$$\mathrm{Im}\, \delta F'(\delta) = \frac{2}{\pi}\left(\frac{2}{S}\right)^2 \left\{ \frac{2}{\gamma} S J_1(\gamma) I_1(\gamma) - I_1^2(\gamma) [\, J_0^2(\gamma) + J_1^2(\gamma) \,] \right.,$$

$$\mathrm{Re}\, \delta F'(i\delta) = -\frac{2}{\pi}\left(\frac{2}{S}\right)^2 \left\{ \frac{2}{\gamma} S J_1(\gamma) I_1(\gamma) + J_1^2(\gamma) [\, I_0^2(\gamma) - I_1^2(\gamma) \,] \right., \qquad (29)$$

while $\mathrm{Im}\mathcal{F}_0(0) = -\delta^{-6}$ is obtained immediately.

The integral from the formula (28) is calculated within the limits $(1, \infty)$ on the basis of known asymptotic expressions:

$$J_1(\alpha x) N_1(\alpha x) \approx -J_0(\alpha x) N_0(\alpha x) \approx (\pi\alpha x)^{-1} \cos 2\alpha x,$$
$$J_0(\alpha x) N_1(\alpha x) \approx -(\pi\alpha x)^{-1} (1 + \sin 2\alpha x),$$
$$J_1(\alpha x) N_0(\alpha x) \approx (\pi\alpha x)^{-1} (1 - \sin 2\alpha x), \qquad (30)$$

for $\alpha \to \infty$, $x > 1$. Since the "non-oscillating" part of the integral is equal to

$$-(\pi\alpha)^{-1} \int_1^{\infty} \frac{2U_0 W_0 x/\delta + 2W_0 \delta/x}{\sqrt{x^2-1}\,(x^4-\delta^4)^2} dx =$$

$$= \frac{2J_1(\gamma) I_1(\gamma)}{\gamma S \delta^6} \left\{ -1 + \frac{1}{2\sqrt{1-\delta^2}} + \frac{1}{2\sqrt{1+\delta^2}} + \right.$$
$$\left. - \frac{\delta^2}{4}\left[\frac{1}{2(1-\delta^2)^{3/2}} - \frac{1}{2(1+\delta^2)^{3/2}} \right] \right\}, \qquad (31)$$

then when we apply the asymptotic calculation method to the "oscillating" part of the integral

$$(\pi\alpha)^{-1}\int_{1}^{\infty}\left\{\frac{[\,(\delta/x)^2+(x/\delta)^2U_0^2-2U_0-W_0^2\,]\cos 2\alpha x}{\sqrt{x^2-1}\,(x^4-\delta^4)^2}+\right.$$

$$\left.+\;\frac{[2(\delta/x)\,W_0-2(x/\delta)\,U_0W_0]\sin 2\alpha x}{\sqrt{x^2-1}\,(x^4-\delta^4)^2}\right\}dx, \tag{32}$$

and take the value $\mathrm{Re}\{\,\pi i\,[\,\frac{1}{2}\mathcal{F}_0(0)+\mathcal{F}_1'(\delta)+\mathcal{F}_2'(i\delta)]\}$ into consideration, we will finally reach a formula for relative power in the following form:

$$\sigma_0=\beta^{-1}\left\{\frac{1}{2}-\frac{J_1(\gamma)I_1(\gamma)}{\gamma S}\left(2+\frac{1}{2\sqrt{1-\delta^2}}+\frac{1}{2\sqrt{1+\delta^2}}+\right.\right.$$

$$+\frac{1}{S^2}\left[\frac{I_1^2(\gamma)[\,J_0^2(\gamma)+J_1^2(\gamma)\,]}{2\sqrt{1-\delta^2}}+\frac{J_1^2(\gamma)[\,I_0^2(\gamma)-I_1^2(\gamma)\,]}{2\sqrt{1+\delta^2}}\right]+$$

$$+\frac{\delta^4}{\pi^{1/2}\alpha^{3/2}(1-\delta^4)^2}\left\{\left[\frac{1}{2}(1+\delta^4)-\frac{2J_1(\gamma)I_1(\gamma)}{S^2}(J_0(\gamma)I_0(\gamma)+\delta^2J_1(\gamma)I_1(\gamma))+\right.\right.$$

$$\left.-\frac{\delta^2}{S}(J_1(\gamma)I_0(\gamma)-I_1(\gamma)J_0(\gamma)\right]\cos(2\alpha+\pi/4)+$$

$$\left.\left.+\;2\frac{\delta}{S}\left[\,\delta^2-\frac{J_1(\gamma)I_1(\gamma)}{S}(J_1(\gamma)I_0(\gamma)-I_1(\gamma)J_0(\gamma))\,\right]\sin(2\alpha+\pi/4)\right\}\right\} \tag{33}$$

with the error $0(\delta^4\alpha^{-3/2})$.

For the case of frequency of a factor which forces the plate to vibrate equal to the frequency of free vibrations, that is for

$$\delta=\delta_n,\quad \gamma=\gamma_n,\quad a_n=\frac{J_1(\gamma_n)}{J_0(\gamma_n)},\quad S(\gamma_n)=0 \tag{34}$$

we obtain the formula

$$\sigma_n=\lim_{k\to k_n}\sigma_0(k)=\frac{1}{2}\,\frac{(1+a_n)^2}{(1-\delta_n^2)^{1/2}}+\frac{1}{2}\,\frac{(1-a_n)^2}{(1+\delta_n^2)^{1/2}}+$$

$$\frac{2S_n^4}{\pi^{1/2}\alpha^{3/2}(1-\delta_n^4)^2}\{\,(1-a_n^2\delta_n^2)\cos(2\alpha+\pi/4)+2a_n\delta_n\sin(2\alpha+\pi/4)\}. \tag{35}$$

identical with this published in [3].

5. Acoustic power in specific case

For $(k_0/k)^4 = (a/\gamma)^4 << 1$ a simplification in the formula (14) was accepted

$$\left[1-(k_0/k)^4 \sin^4\vartheta\right]^{-2} \approx 1. \tag{36}$$

The condition $(k_0/k)^4 << 1$ can also be substituted with another one, namely

$$\left(\frac{h}{2a}\right)^2 \left(\frac{\omega}{\omega_1}\right) << \frac{3\rho(1-\nu^2)}{E}\left(\frac{c}{\gamma_1}\right)^2, \tag{37}$$

where γ_1 is the root of the frequency equation $S(\gamma_1)=0$, corresponding with the pulsation ω_1, ρ is the volumetric density of the material of the plate, E – Young modulus, ν – Poissons ratio. As opposed to the inequality $(k/k_0)^4 << 1$ the inequality (37) contains an explicit dependence between the quantities $h/2a$, ω/ω_1 and the so-called "material constants". Accepting $a_0 = a$, we have

$$\sigma_0 = (\alpha/\gamma)^2 \beta^{-1} \int_0^{\pi/2} \left[\frac{\gamma}{\alpha}\frac{J_1(\alpha\sin\vartheta)}{\sin\vartheta} - \frac{\alpha}{\gamma} U_0 \sin\vartheta J_1(\alpha\sin\vartheta) + \right. \tag{38}$$

$$\left. - W_0 J_0(\alpha\sin\vartheta)\right]^2 \sin\vartheta d\vartheta.$$

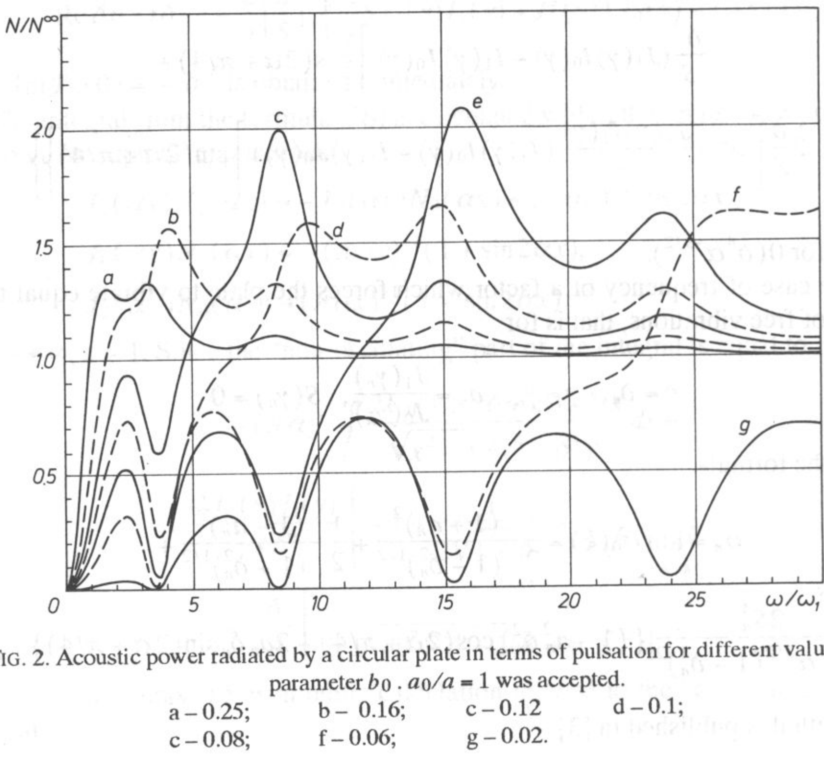

FIG. 2. Acoustic power radiated by a circular plate in terms of pulsation for different values of the parameter b_0. $a_0/a = 1$ was accepted.

a – 0.25; b – 0.16; c – 0.12 d – 0.1;
c – 0.08; f – 0.06; g – 0.02.

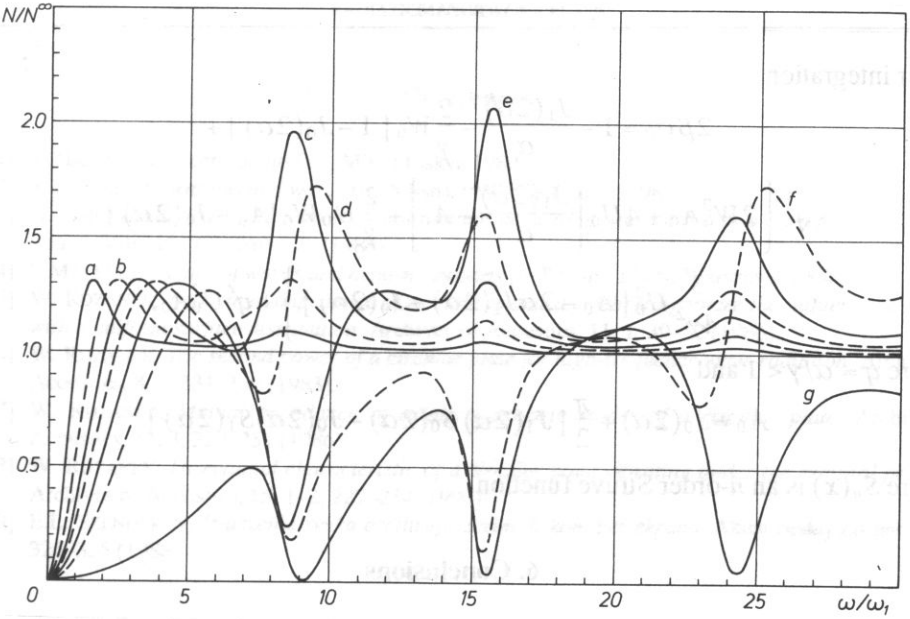

FIG. 3. Acoustic power radiated by a circular plate in terms of pulsation for different values of the parameter b_0 . $a_0/a = 0.5995$ was accepted.

a – 0.25; b – 0.16; c – 0.12; d – 0.1;
c – 0.08; f – 0.06; g – 0.02.

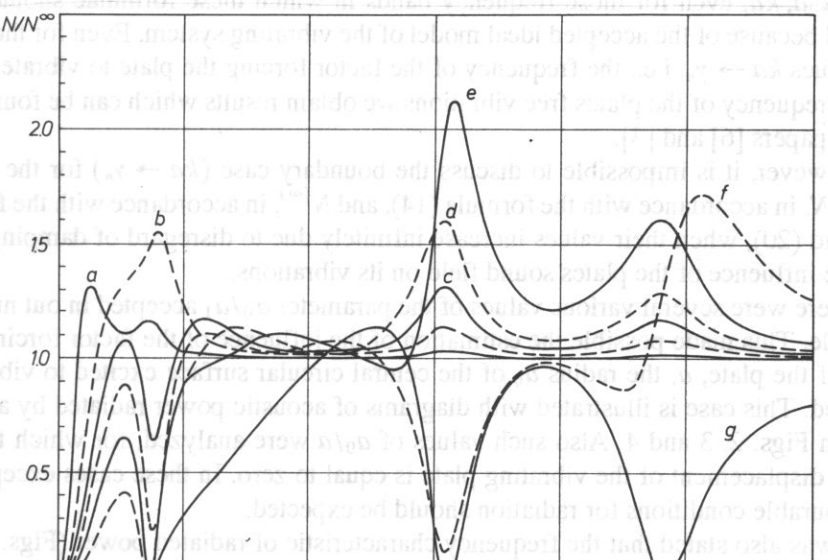

FIG. 4. Acoustic power radiated by a circular plate in terms of pulsation for different values of the parameter b_0 . $a_0/a = 0.7373$ was accepted.

a – 0.3; b – 0.16; c – 0.12; d – 0.1;
c – 0.08; f – 0.06; g – 0.02.

After integration

$$2\beta\sigma_0 = 1 - \frac{J_1(2\alpha)}{\alpha} - \frac{2}{\gamma}W_0[1 - J_0(2\alpha)] +$$

$$+ q^2\left\{2W_0^2A_0 + 4U_0\left[\frac{J_1(2\alpha)}{\alpha} - A_0\right] + \frac{2}{\gamma}U_0W_0[A_0 - J_0(2\alpha)] +\right.$$

$$\left. + \frac{3}{4\gamma^2}U_0^2[A_0 - 2\alpha J_1(2\alpha) - J_0(2\alpha)]\right\} + q^4U_0^2A_0, \tag{39}$$

where $q = \alpha/\gamma < 1$ and

$$A_0 = J_0(2\alpha) + \frac{\pi}{2}[J_1(2\alpha)S_0(2\alpha) - J_0(2\alpha)S_1(2\alpha)], \tag{40}$$

where $S_n(x)$ is an n-order Struve function.

6. Conclusions

Our theoretical analysis has resulted in an expression for acoustic radiated by a circular plate, including the factor forcing the plate to vibrate.

The performed calculations indicate that the relative radiation power $N/N^{(\infty)}$, in accordance with the formulae (23) and (33), accepts finite values for all values of the parameter k_0a, ka, even for those frequency bands in which these formulae should not be applied because of the accepted ideal model of the vibrating system. Even for the boundary values $ka \to \gamma_n$, i.e., the frequency of the factor forcing the plate to vibrate is equal to the frequency of the plates free vibrations we obtain results which can be found in the earlier papers [6] and [3].

However, it is impossible to discuss the boundary case ($ka \to \gamma_n$) for the radiated power N, in accordance with the formula (14), and $N^{(\infty)}$, in accordance with the formulae (18) and (20), when their values increase infinitely due to disregard of damping effects and the influence of the plates sound field on its vibrations.

There were several various values of the parameter a_0/a_1 accepted in out numerical example. This made possible the estimation of the influence of the factor forcing vibrations of the plate, a, the radius a_0 of the central circular surface excited to vibrate was changed. This case is illustrated with diagrams of acoustic power radiated by a circular plate in Figs. 2, 3 and 4. Also such values of a_0/a were analyzed, for which the volumetric displacement of the vibrating plate is equal to zero. In these cases exceptionally unfavourable conditions for radiation should be expected.

It was also stated that the frequency characteristic of radiated power (Figs. 2, 3 and 4) significantly depends on the plates thickness h, its diameter 2a and material constants. To this end calculations of radiated power were carried out also for various values of the following dimensionless parameter

$$b_0 = k_0a/(ka)^2 = (h/2a)\sqrt{E/[3\rho c^2(1 - \nu^2)]},$$

including the material constants, and is proportional to the quantity $h/2a$.

References

[1] T. HAJASAKA, *Electroacoustics* „Mir" Moskva 1982.
[2] S. KALISKI, *Vibrations and waves* (in Polish), PWN, Warszawa 1985.
[3] H. LEVINE and F.G. LEPPINGTON, *A note on the acoustic power output of a circural plate*, Journ. Sound. Vib., **121** (2), 269–275 (1988).
[4] I. MALECKI, *Theory of waves and acoustic systems* (in Polish), PWN, Warszawa 1964.
[5] W. RDZANEK, *Acoustical impedance of circular membrane vibrating under the influence of a force with a uniform surface distribution*. Archives of Acoustics, **11**, 1, 39–52 (1986).
[6] W. RDZANEK, *The second power of a circular plate for high-frequency wave radiation*, Archives of Acoustics, **8**, 4, 331–340 (1983).
[7] W. RDZANEK, *Mutual impedance of axially-symmetric modes of a circular plate*, Archives of Acoustics, **11**, 3, 329–251 (1986).
[8] W. RDZANEK, *Directional characteristic of a circular plate vibrating under the external pressure*, Archives of Acoustics, **15**, 1–2, 227–234 (1990).
[9] E.L. SZENDEROV, *Izłuczenie zvuka oscilirujuszczim diskom bez ekrana*, Akusticeskij Żurnał, **34**, 2, 326–335 (1988)

Received December 1, 1990, English version June 27, 1991