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SHEAR ACOUSTIC WAVES IN PLATES

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In the paper the theoretical analysis and results of experimental investigations of SH waves propagation in plates are presented. The dimensions of different plates and methods of SH wave excitation were applied in order to attain mode separation and identification.

1. Introduction

A shear horizontal wave with polarization parallel to a plate surface can propagate in a plate with a definite thickness and other unlimited dimensions. Waves of this type can be generated by shear vibration transducers bonded to a plate edge. Such a wave is dispersive and multimode. A propagation analysis for this wave permits to obtain an exact analytical solution in the form of a relatively simple expression [1, 2]. However, the limitation of transverse dimension introduces new boundary conditions into the calculations. These conditions do not permit to obtain an exact analytical solution of the problem. This is due to reflections and the wave transformation (shear–longitudinal) on the plate edge. In this case oscillograms of pulses propagating in the plate are complicated. The separation of thickness plate modes from modes rising on plate edges is difficult. For more precise experimental investigations of SH wave propagation in plates it is necessary to choose boundary conditions and the method of wave excitation in such a way that the separation and identification of pulses are possible. It is not simple, as shown in the published photographs, owing to small velocity differences and superposition of pulses.

2. SH waves propagation in plate waveguides

2.1. Plates with limited thickness and infinite width

An elastic plate with infinite width and length can be treated as an approximation of real waveguide structures with limited dimensions. A thorough study of SH wave

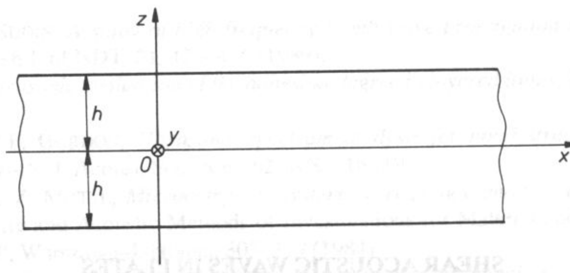

FIG. 1. Elastic plate infinitely extended in directions x and y, thickness 2h.SH wave with displacement in y direction propagates in x direction.

propagation in such a plate is helpful in investigations of two– and three–dimensional waveguides in which the acoustic beam is limited in the cross-section in relation to the propagation direction (x axis). We assume that the SH wave mode propagating in the x direction in the plate shown in Fig. 1 possesses the vibration component u only in the y axis direction. The medium is homogeneous and isotropic, nevertheless the propagating SH wave mode is dispersive as a result of multiple reflections at the boundaries $z = \pm h$ [1]. The problem of SH wave propagation in an infinite plate is described by the following differential problem:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{v_0^2} \cdot \frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\left. \frac{\partial u}{\partial z} \right|_{z = \pm h} = 0 \tag{2}$$

In the general case one can solve the problem of acoustic wave propagation in unlimited plates by applying the potential method, the partial wave method or the transverse resonance method [2]. It appears that the problem (1)–(2) possesses the finite analytical solution [1, 2].

$$u = B_1 \sin\left(\frac{n\pi}{2}\frac{z}{h}\right) \cdot \exp[i(kx - \omega t)] \tag{3}$$

$$n = 1, 3, 5, \ldots$$

for asymmetric modes, and

$$u = B_2 \cos\left(\frac{n\pi}{2}\frac{z}{h}\right) \cdot \exp[i(kx - \omega t)] \tag{4}$$

$$n = 0, 2, 4, \ldots$$

for symmetric modes, where B_1, B_2 – optimal constants, k – propagation constant of the SH plate wave.

The dispersive equation for both SH wave mode types is

$$\frac{\omega^2}{v_0^2} - k^2 = \frac{n^2 \pi^2}{4h^2} \tag{5}$$

From the relation (5) the phase velocity results

$$\upsilon_p = \upsilon_0 \frac{1}{\sqrt{1-\left(\frac{n\pi}{2hq}\right)^2}} \tag{6}$$

and the group velocity

$$\upsilon_g = \upsilon_0 \sqrt{1-\left(\frac{n\pi}{2hq}\right)^2} \tag{7}$$

where $q = \frac{\omega}{\upsilon_0}$ – propagation constant of the volume SH wave.

Except for the mode $n = 0$, all other modes are dispersive – Fig. 2.

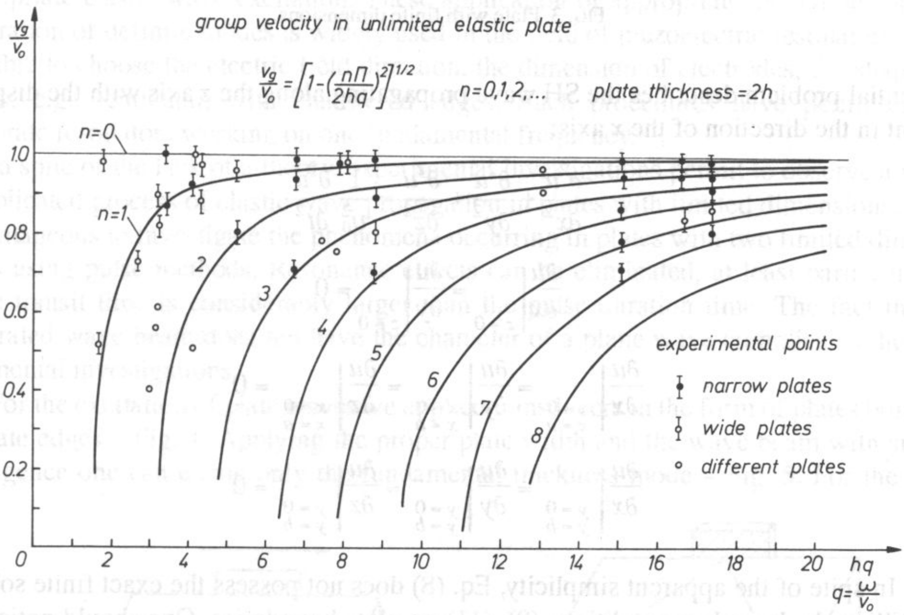

FIG. 2. Dispersion curves of SH modes group velocity in infinite elastic plate with thickness $2h$.

The structure considered hirtherto is infinite in the direction perpendicular to the wave propagation direction. In real waveguide structures one uses acoustic beams with limited width. This implies the application of waveguide structures with limited dimensions. Among these structures one can distinguish strip waveguides, topographic waveguides, ridge waveguides etc.

2.2. *Plates with limited thickness and width*

The general problem of acoustic SH wave propagation in plates with limited dimensions (Fig. 3), does not have, up till now, an analytical solution [2]. The following dif-

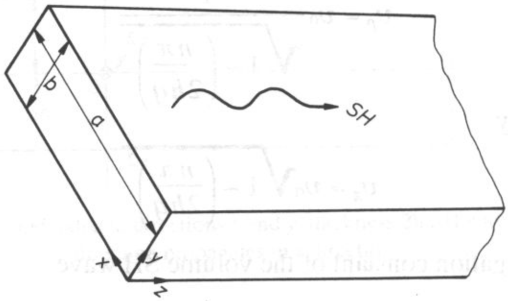

FIG. 3. Plate with finite dimensions.

ferential problem describes the SH wave propagating along the z axis with the displacement in the direction of the x axis:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{v_0^2}\frac{\partial^2 u}{\partial t^2} \tag{8}$$

$$\left.\frac{\partial u}{\partial x}\right|_{z=0}=\left.\frac{\partial u}{\partial y}\right|_{z=0}=0 \tag{9}$$

$$\left.\frac{\partial u}{\partial x}\right|_{\substack{x=0\\x=a}}=\left.\frac{\partial u}{\partial y}\right|_{\substack{x=0\\x=a}}=\left.\frac{\partial u}{\partial z}\right|_{\substack{x=0\\x=a}}=0 \tag{10}$$

$$\left.\frac{\partial u}{\partial x}\right|_{\substack{y=0\\y=b}}=\left.\frac{\partial u}{\partial y}\right|_{\substack{y=0\\y=b}}=\left.\frac{\partial u}{\partial z}\right|_{\substack{y=0\\y=b}}=0 \tag{11}$$

In spite of the apparent simplicity, Eq. (8) does not possess the exact finite solution fulfilling the boundary conditions (9)–(11) on plate boundaries. One should notice that the analogical problem of electromagnetic wave propagation in rectangular waveguides possesses an exact analytical solution.

The motion equation for the plate with limited dimensions can be solved only in the approximate way. To this end we can distinguish two procedures:

1) solving numerically the system of exact equations in the approximate way,

2) simplification of motion equations and exact solution of the obtained equations.

Mindlin's theory of vibrations of plates with finite dimensions [3–5] can be included in the second way of proceeding. We encounter similar problems in the case of strip waveguides [6] and topographic waveguides [7]. In both of these cases one can obtain field distributions and dispersive curves only numerically. A considerable limitation of the guided acoustic beam width has been achieved in these waveguides. It leads to the increase of acoustic wave power density, what can be employed in acoustoelectric devices utilizing nonlinear effects [8]. The complicated character of phenomena appears particularly strikingly in plates with three limited dimensions. Eventually one obtains standing waves using suitable excitation. This case occurs in resonators applied as ultrasonic transducers, constant frequency sources or electromechanical filter elements.

3. Experimental investigations of special cases of plate waves

3.1. Methods of plate wave excitation

Experimental investigations of plate waves aim at separating definite modes using appropriate elastic wave excitation. These application of appropriate excitation for the generation of definite modes is widely used in the field of piezoelectric resonators. It is possible to choose the electric field direction, the dimension of electrodes, the shape of plates, e.g., lenticular, with chamfered edges. Such procedures have permitted to elaborate resonators working on one fundamental frequency.

In spite of the lack of a theory, experimental investigations permit to observe a very complicated process of elastic wave propagation in plates with limited dimensions. It is advantageous to investigate the phenomena occurring in plates with two limited dimensions using pulse methods. Resonance effects can be eliminated, at least partly, if the pulse transit time is considerably larger than the pulse duration time. The fact that a generated wave beam does not have the character of a plane wave complicates the experimental investigations.

For the excitation of plate waves we applied transducers in the form of plates bonded to plate edges – Fig. 4. Applying the proper plate width and the wave beam with small divergence one can excite only the fundamental thickness mode – Fig. 5. For the low

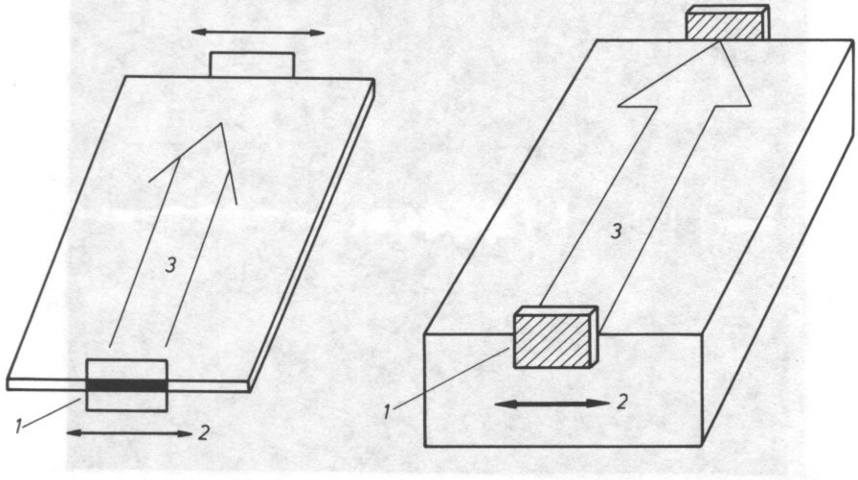

FIG. 4. Methods of plate waves excitation. 1 - piezoelectric transducer, 2 - direction of transducer vibrations, 3 - direction of wave propagation.

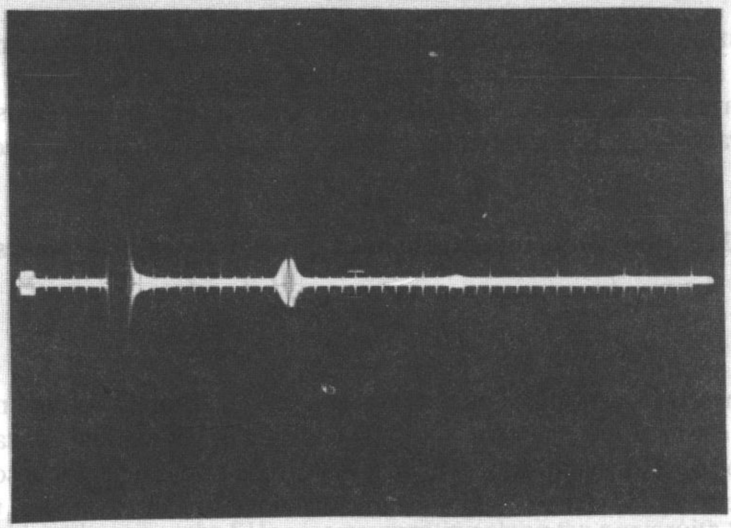

FIG. 5. Plate 15 × 14.9 × 0.5 mm excited as in Fig. 4a, f = 10 MHz, 5 μs/div, 100 mV/div.

frequency, when the beam is strongly divergent, transverse (plate thickness) and width SH wave modes are excited. This effect seriously complicates the image of received pulses – Fig. 6. One should also take into consideration shear wave transformations into a longitudinal wave. Mode separation becomes impossible. Narrowing of the plate should eliminate transverse modes but then flexural symmetric and asymmetric modes arise. All these effects can be observed in the photos of pulses.

The edge excitation (Fig. 4b) permits to privilege higher thickness mode but width modes are not eliminated.

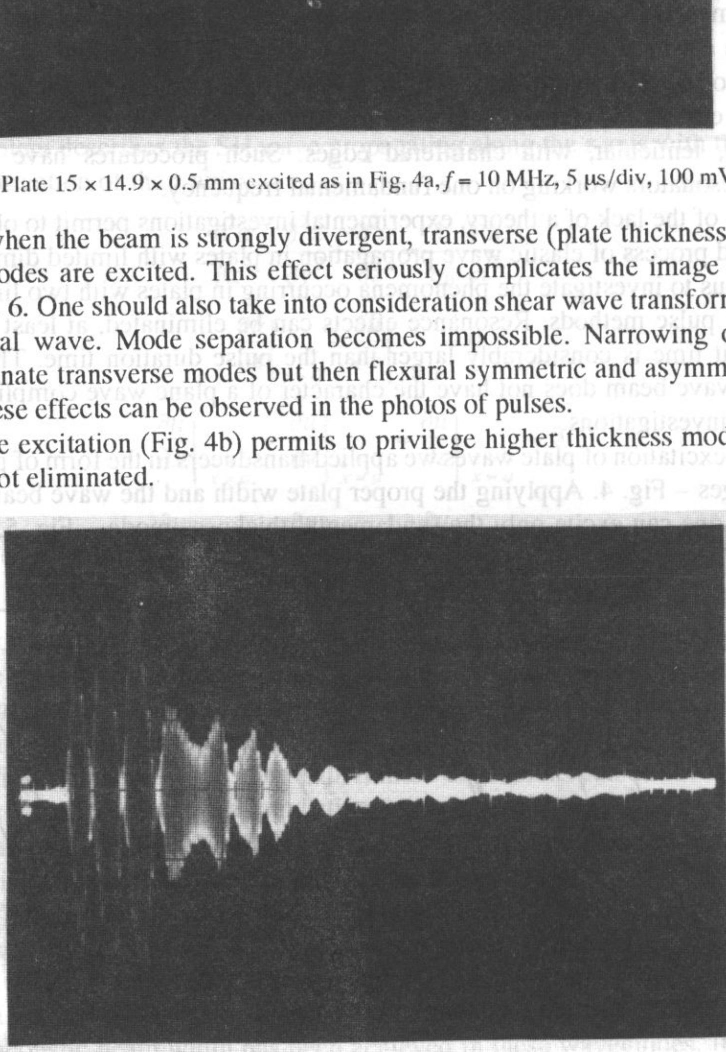

FIG. 6. Plate 15 × 14.9 × 0.5 mm excited as in Fig. 4a, f = 1 MHz, 10 μs/div, 0.2 V/div.

3.2. SH waves in plates with large width in relation to length

In order to eliminate SH wave reflections from the plate sides, we applied plates with large dimension in a direction perpendicular to the wave propagation direction. The widths of these plates were greater than their lengths or comparable. In this way we tried to approach unlimited space conditions. The SH wave was excited as in Fig. 4a or Fig.

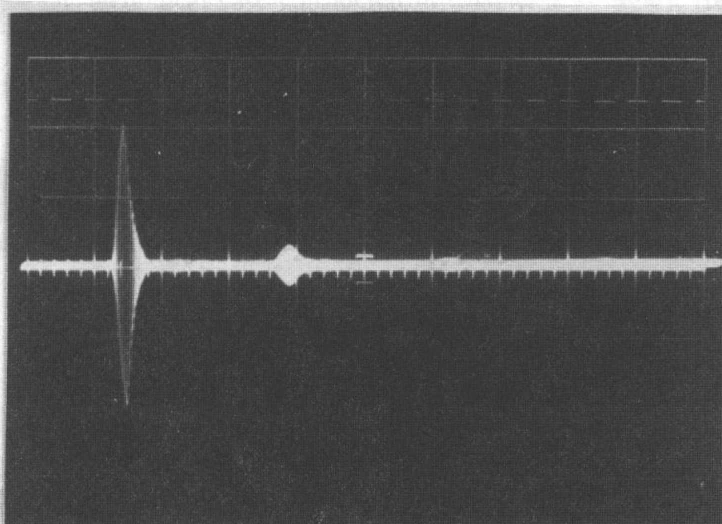

FIG. 7. Plate 16.5 × 15.8 × 1 mm excited as in Fig. 4a, f = 5 MHz, 5 μs/div, 0.5 V/div.

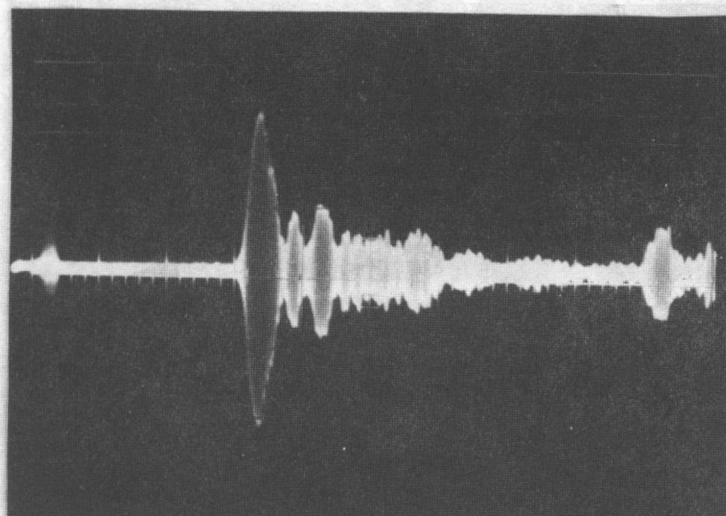

FIG. 8. Plate 16.5 × 15.8 × 1 mm excited as in Fig. 4b, f = 5 MHz, 2 μs/div, 20 mV/div.

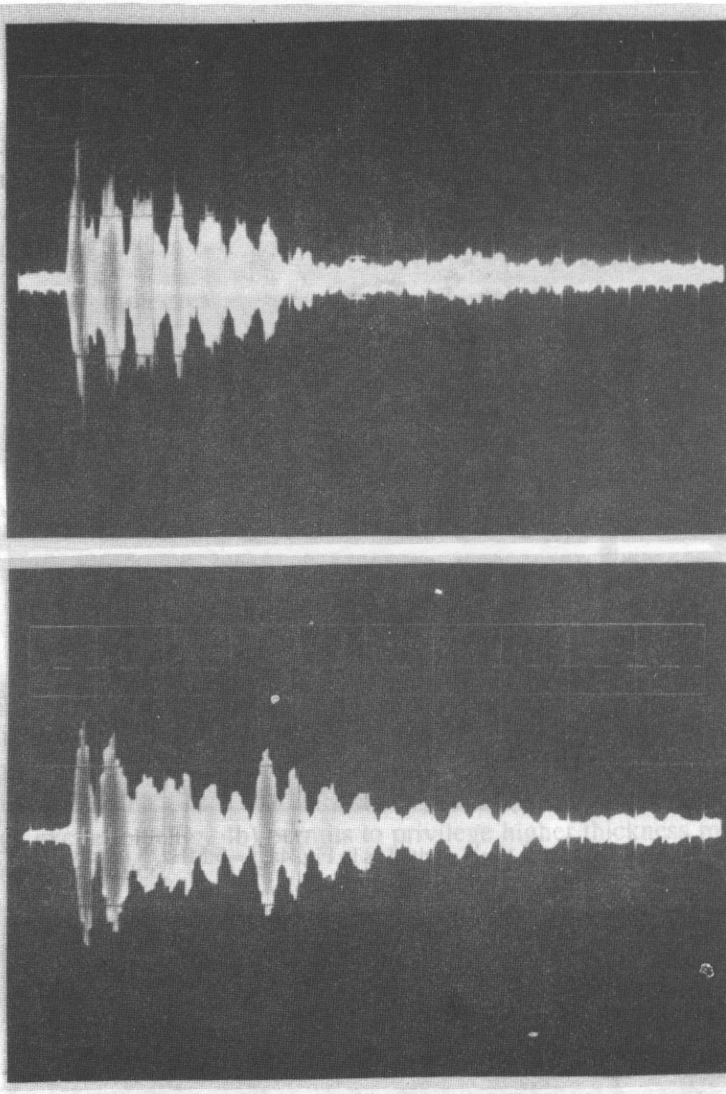

FIG. 9. Plate 16,5 × 15,8 × 1 mm, f = 1 MHz, 10 μs/div
a) excited as in Fig. 4b, 100 mV/div,
b) excited as in Fig. 4a, 0.5 V/div.

4b. The obtained oscillograms of pulses are presented in Fig. 7 and 8. In Fig. 7 the fundamental mode pulse is visible, higher modes are almost imperceptible, whereas in Fig. 8 higher modes are clearly noticeable. The superposition of visible modes and the deformation of transmitted pulses come out here, what makes velocity calculations impossible. The third echo observation does not improve the situation. Difficulties of the separation of pulses appear specially for lower frequencies, e.g., 1 MHz, when the wave beam is more divergent – Figs. 9a and 9b. As one can see in the figures, it is not simple to obtain measurement conditions close to the theoretical conditions because the generated wave is not the plane wave and the generated pulses are expanded in time.

3.3. SH waves in plates with transverse dimension smaller than the dimension in the wave propagation direction

This case of SH wave propagation in a plate is the most complicated and it corresponds with the theoretical case considered in Chapter 2.2. Figure 10 presents the

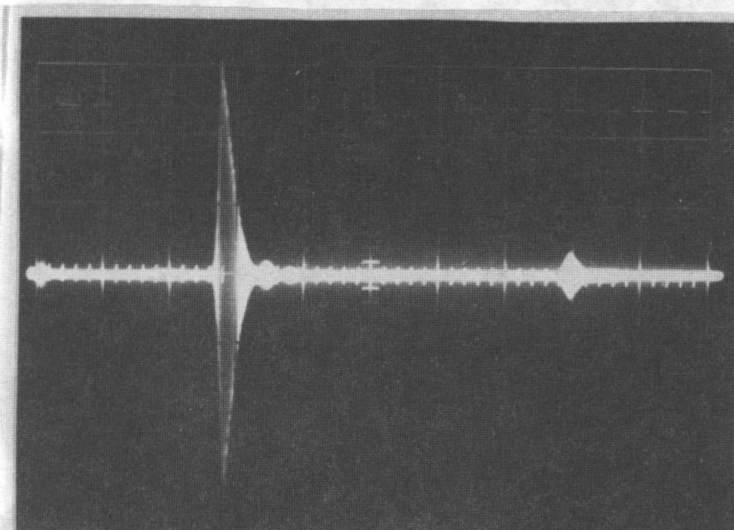

FIG. 10. Plate 26.4 × 17 × 1.1 mm excited as in Fig. 4a. $f = 5$ MHz, 5 μs/div, 100mV/div.

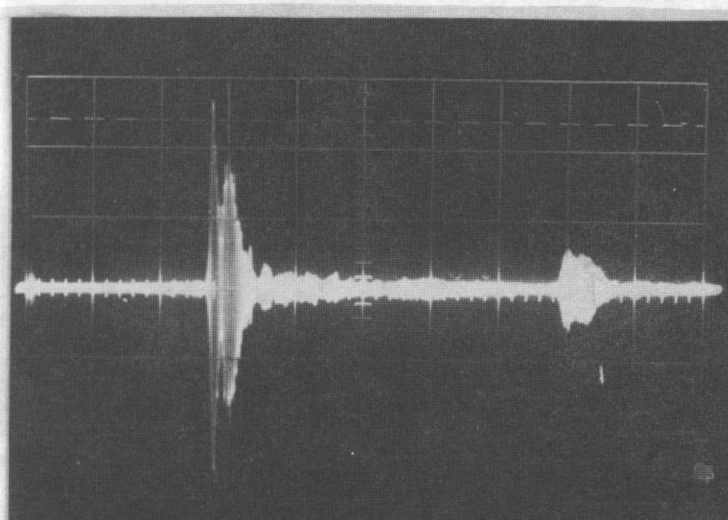

FIG. 11. Plate 26.4 × 17 × 1.1 mm excited as in Fig. 4b, $f = 5$ MHz, 5 μs/div, 20mV/div.

image of pulses rising with the excitation as in Fig. 4a. Besides the main pulse, one can see additional pulses rising as a result of reflections at side edges of the plate. Edge excitation (Fig. 4b) complicates furthermore the image of pulses – Fig. 11. Consequently, the identification of SH wave modes is not possible. The image becomes simplified for higher frequency (10 MHz) when the wave beam is narrow and the side reflections are eliminated. This case does not possess an exact analytical solutions, it is also complicated experimentally.

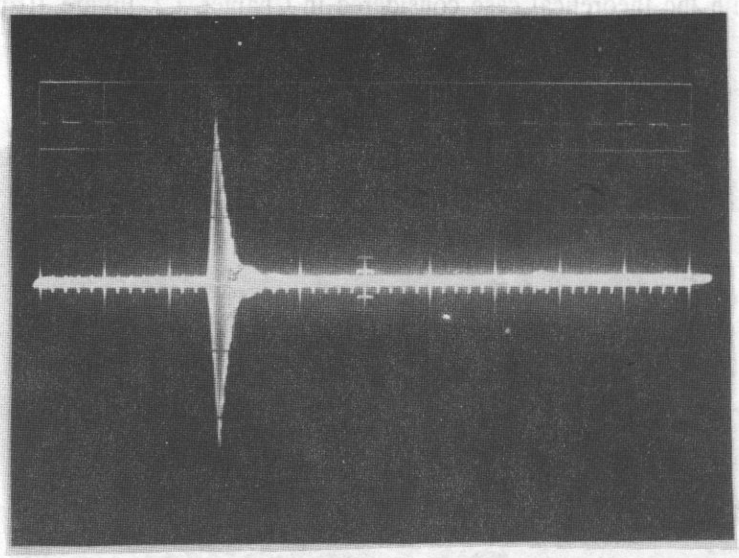

FIG. 12. Plate 25 × 5 × 1.1 mm excited as in Fig. 4a, $f = 5$ MHz, 5 μs/div, 100 mV/div.

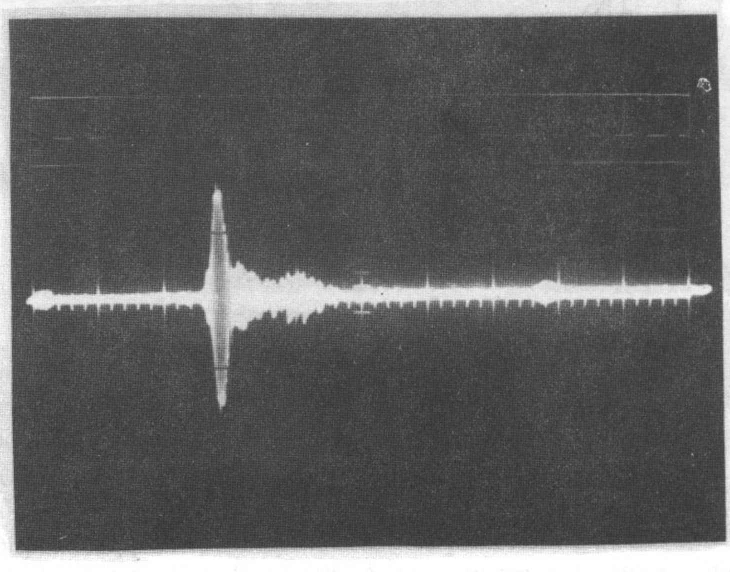

FIG. 13. Plate 25 × 5 × 1.1 mm excited as in Fig. 4b, $f = 5$ MHz, 5 μs/div, 20 mV/div.

3.4. SH wave in narrow plates

The elimination or the reduction of side reflections of SH waves is possible in narrow plates. Figure 12 presents SH wave pulses excited by a plate transducer as in Fig. 4a, while Fig. 13 – as in Fig. 4b.

3.5. SH waves in plates with different thickness

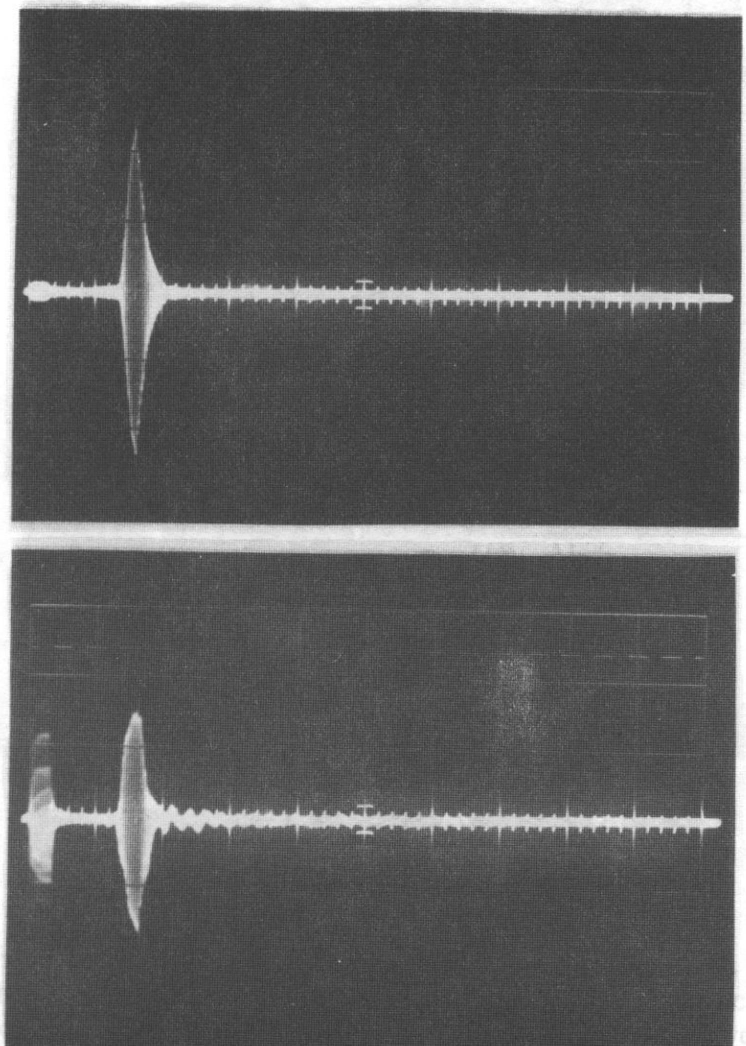

FIG. 14. Plate 25 × 12.1 × 0.5 mm,5 μs/div,

a) excited as in Fig. 4a, f = 5 MHz, 100 mV/div,

b) excited as in Fig. 4b, f = 10 MHz, 10 mV/div.

One can see from the analytical solution for the plate wave that for an unlimited plate the frequency limitation appears in the form of cut-off frequency. Below this frequency plate modes do not exist. The cut-off frequency is defined by the formula

$$f_{\mathrm{cut}} = \frac{v_0}{4h}$$

The experimental investigations of plates with different thickness and also for different frequencies should enable to observe the cut-off frequency. Measurements have

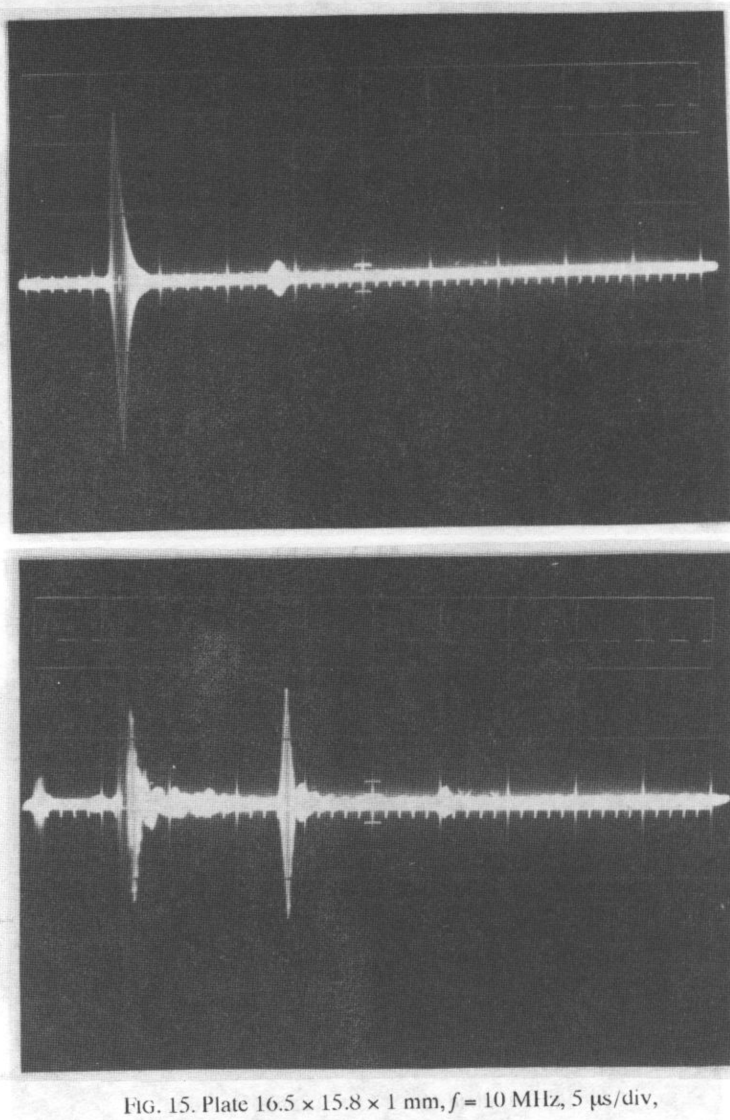

FIG. 15. Plate 16.5 × 15.8 × 1 mm, f = 10 MHz, 5 μs/div,

a) excited as in Fig. 4a, 0.2 V/div,

b) excited as in Fig. 4b, 20 mV/div.

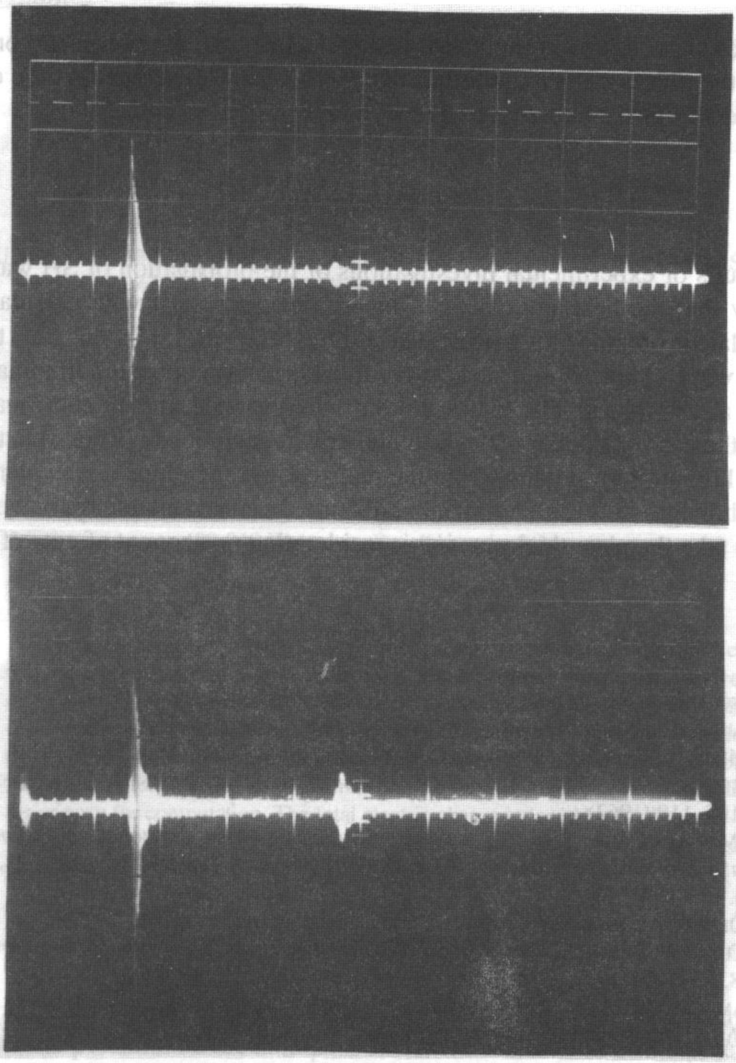

FIG. 16. Plate 33.5 × 18 × 2 mm, $f = 5$ MHz, 10 μs/div,
a) excited as in Fig. 4a, 0.2 V/div,
b) excited as in Fig. 4b, 20 mV/div.

been performed for plates with thicknesses of 0.5 mm, 1 mm, 2 mm. Figures 14a, b, 15a, b and 16a, b present the results of measurements. The results of velocity calculations for SH plate modes are shown in Fig. 2. In spite of the mentioned difficulties, we succeeded in identifying plate modes, especially for frequencies higher than 5 MHz. For this range we succeded in separating the modes particularly for edge excitation. Instead, for the frequencies 1 MHz and 2 MHz, where the considerable differences of mode velocities caused by the cut-off frequency existence should be visible, measurements and observation of pulses show a very complicated image. In this case we succeeded only partly in identifying the pulses corresponding to SH wave modes. It seems that the existence of flexural modes is possible in this frequency range.

4. Conclusion

The results of experimental investigations of the propagation of SH wave pulses in plates show that the best agreement of measurements results with calculations for an unlimited plate is obtained for narrow plates (ratio 5/20 = 0.25). In this case the SH wave has a plane wave character and shear wave transformation into a longitudinal wave does not appear at the edges. The plates are sufficiently wide and flexural waves are not present. In the case of thicker plates and higher wavenumbers it is possible to excite and to separate higher order modes. From the realized investigations it is evident that plates with a width/length ratio smaller than 0.4–0.5 are the most suitable for waveguides. One should also take the plate thickness into consideration on account of cut-off frequency.

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