

SPECTRAL THEORY OF NATURAL UNIDIRECTIONALITY OF SPUDT FOR BG WAVES

E. DANICKI

Institute of Fundamental Technological Research of the Polish Academy of Sciences
(00-049 Warszawa ul. Świętokrzyska 21)

A single-phase unidirectional interdigital transducer SPUDT is modelled by a system of periodic metal strips on a piezoelectric halfspace carrying Bleustein-Gulyaev BG wave. The dispersion relation for BG surface wave, as well as some relations between forward and backward waves in the system are discussed. Natural unidirectionality is explained in this way.

1. Introduction

In [1] a natural unidirectionality of a single-phase interdigital transducer was reported. The effect occurs for certain piezoelectric substrates only and only in the stop-band due to the cooperative mechanism of Bragg reflection of surface wave caused by electric, and mechanical interactions between the periodic strips of the transducer and the propagating surface acoustic wave.

In [2] a theory of propagation of BG wave was presented for quite general case of piezoelectric substrates, the strip property, and for frequencies including the stop-band. In fact, the theory includes the cases of both natural, and artificial [3] unidirectionality for BG waves, if only we put $\cos(\eta + 2\theta) = 0$ in relation (22) of [2]. The theory can be easily generalized to the case of Rayleigh surface acoustic wave (SAW).

2. Dispersion relation

Let's consider periodic metal strips arranged along x -axis on the surface of piezoelectric halfspace. The strip period is $\Lambda = 2\pi/K$, the strips have rectangular cross-section: thickness H and width $\Lambda/2$ and they are perfectly conducting, in this circumstance we have $\eta = 0$ [2]. The strip material is characterized by ρ and μ , and the mechanical interaction between strips and BG wave is characterized by a parameter $g = (\rho\omega^2 + k_s^2\mu)H/\pi$ [6], where $k_s = \omega/v_s$ approximates the wave vector of the wave, ω is its frequency.

The piezoelectric halfspace is characterized by the above mentioned cut-off wavenumber of bulk waves k_s and two dimensionless small parameters α and β , determining BG wave-numbers for metallized and free substrate surface, $k_s/\sqrt{(1-\alpha^2)}$ and $k_s/\sqrt{(1-\beta^2)}$, correspondingly. An effective surface permittivity of the substrate is $\varepsilon_0\varepsilon$, and effective surface stiffness is γ [2]. For free substrate surface the BG wave is coupled to the electric potential with complex amplitude φ and particle displacement with the amplitude u on the substrate surface, the relation between them is $\varphi/u = [\gamma(\alpha - \beta)/\varepsilon_0\varepsilon]^{1/2}\exp(j\vartheta)$. In the case of natural unidirectionality we should have $\vartheta = \pi/4$ [4], and this value will be assumed in what follows.

In the stopband, the dependence of wave-number of a wave propagating under the short-circuited strips can be approximated as follows [2]

$$r_0 = \frac{K}{2} + \frac{1}{v_s} \sqrt{(\omega - \omega'_1)(\omega - \omega_s)} \quad (1)$$

and similarly for free strips, where the wavenumber is r_∞ and stop-band frequencies ω_1 and ω_2 instead of ω'_1 and ω_3 above.

In the case considered in [2], where we put $\cos(\eta + 2\vartheta) = \pm 1$, it was $\omega'_1 = \omega_1$, but this does not take place here, where we have $\cos(\eta + 2\vartheta) = 0$. The stop-band frequencies ω'_1, ω_s are determined by two solutions of the following equation [2]

$$S^2 - \frac{3\alpha + \beta}{4} SR + \alpha(\alpha + \beta)R^2 - |g|^2\gamma^2 = 0 \quad (2)$$

where $R = K/2$ and $S = \sqrt{(R^2 - k_s^2)}$, $k_s = \omega/v_s$ (ω_1 and ω_2 are determined by (2) with replacement $\alpha \leftrightarrow \beta$).

The dependence of the stop-band frequencies is sketched in Fig. 1. As we see, the stop-bands are overlapping if $g \neq 0$, but they have not common stop-band frequency.

In fact, the theory includes the case of both natural and artificial [3] unidirectionality for BG waves. If only we put $\cos(\eta + 2\vartheta) = 0$ in relation (2), the theory can be easily generalized to the case of Rayleigh surface acoustic wave (SAW).

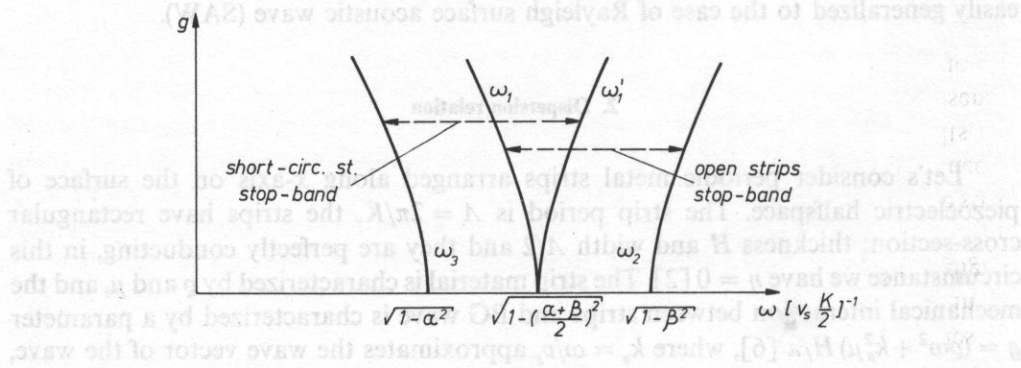


FIG. 1

3. Field components

As known, in periodic system two fundamental components of wave exist, namely forward, and backward propagating waves with wave-numbers r_0 and $r_0 - K$, correspondingly ($Re(r_0) > 0$ here). On the substrate surface, the amplitudes of particle displacements of these two waves are u_0 and u_{-1} , where

$$u_{-1}/u_0 = \Gamma(r_0) = \frac{S_0 - R_0(\alpha + \beta)/4 + g^*\gamma \exp(-j2\vartheta)}{S_1 - R_1(\alpha + \beta)/4 + g\gamma \exp(j2\vartheta)} \exp(j2\vartheta) \quad (3)$$

where $R_0 = |r_0|$, $R_1 = |r_0 - K|$, $S_n = \sqrt{(R_n^2 - k_s^2)}$, $n = 0, 1$.

It can be easily shown, that for certain frequency f_0 in the stopband, where $S_n - R_n(\alpha + \beta)/4$ is imaginary valued quantity, $\Gamma(r_0) = 0$. Similar relation for $\Gamma(-r_0)$ exists for surface wave propagating in opposite direction, where forward and backward waves have wave numbers $-r_0$ and $-r_0 + K$. In this case the angle ϑ should be replaced by $-\vartheta$ and g by g^* . As the consequence, $\Gamma(-r_0) = \infty$ at frequency f_0 .

The above relations are very important for explanation of the natural unidirectionality. Firstly note, that surface wave Poynting vector magnitude Π is determined by the magnitude of u as follows

$$\Pi = \frac{\omega |u|^2}{4(r/k_s)^2 \gamma \sqrt{(r^2 - k_s^2)}} \quad (4)$$

where r is the wavenumber and u is the amplitude of one of the above mentioned partial waves (the Poynting vector and the wave-vector have the same direction in the considered case).

It follows from the above, that the surface wave cannot propagate in $-x$ direction at frequency f_0 , because there are not energy transported by partial waves with wave-numbers $r_0 - K$ and $-r_0$, simply because their amplitudes are zero at f_0 .

Consider now a system, where all strips are grounded except one, which is supplied from an external voltage source. There is surface wave generated under this strip (see [5] for details of similar considerations for Rayleigh waves). The conclusion from the above reasoning is, that at the frequency f_0 the strip generate surface waves only in one direction (there are two partial waves generated, having wave-numbers r_0 and $K - r_0$). For frequencies different from f_0 , but still in the stopband, surface waves are generated in both directions, however the generation in one direction still prevails ($|\Gamma(r_0)| < 1$ in stopband).

Outside the stopband, where $S_n - R_n(\alpha + \beta)/4$ has real value while $g\gamma \exp(-j2\vartheta)$ is imaginary valued, the symmetry of both directions are fully restored and the surface wave powers generated under the supplied strip in both directions are equal.

4. Some remarks on strip admittance

A surface wave propagating under the grounded strip induces electric current I flowing to it. The relation can be obtained

$$I \sim \sqrt{[\varepsilon_0 \varepsilon (\alpha - \beta) / \gamma]} (s u_0 e^{j\vartheta} + (1-s) u_{-1} e^{-j\vartheta}) \quad (5)$$

where $s = r_0 / K$. It is seen, that in the stopband where $s \approx 1/2$, both backward and forward waves contribute equally to the current, if only $|u_0| = |u_{-1}|$, so that there is not any further mechanism of unidirectionality.

Analogous theory as that presented in [5] for Rayleigh waves can be developed for the case considered. The main result is that the strip admittance Y is

$$Y \sim \frac{\alpha \omega + (\alpha^2 - \beta^2) / 2}{[(\omega - \omega'_1)(\omega - \omega_3)]^{1/2}} \quad (6)$$

as compared to $Y \sim \sqrt{[(\omega - \omega'_1) / (\omega - \omega_3)]}$ for bidirectional case [5].

The admittance of interdigital transducer having finite number of electrodes is similar to the above-mentioned strip admittance in the infinite system of periodic strips [5], but it is smoothed so that the transducer admittance does not exhibit singularities like pole or zero. The similarity of function (6) and the admittance of SPUDT presented in [4] may be noticed.

5. Conclusions

From the above spectral theory we may conclude that

- for cases where $\vartheta \neq \pi/4$, we can obtain unidirectionality if we apply complex value of g , by using the strips with trapezium cross-section [2], or split strips made from different metals [3],
- the natural unidirectionality takes place only in stopband, but strict unidirectionality takes place only for one frequency,
- the thickness of the strips does not effect the unidirectionality, however note that the average loading of the substrate by the strips can change ϑ [2].

References

- [1] P. V. WRIGHT, *The natural single-phase unidirectional transducer, a new low-loss SAW transducer*, *Ultras. Symp. Proc.*, 59–63 (1985).
- [2] E. DANICKI, *Propagation of transverse surface acoustic waves in rotated Y-cut quartz substrates under heavy periodic metal electrodes*, *IEEE Trans.*, SU-30, 5, 304–312 (1983).
- [3] C. S. HARTMANN, P. V. WRIGHT, R. J. KANSY, E. M. GARBER, *An analysis of SAW interdigital transducer with internal reflections and the application to the design of single-phase unidirectional transducers*. *Ultras. Symp. Proc.*, 40–45, (1982).

- [4] T. THORVALDSSON, *Analysis of the natural single-phase unidirectional SAW transducer*, *Ultras. Symp.* (1989).
- [5] E. DANICKI, *Unified theory of interdigital transducers and SAW reflectors*, *J. Tech. Phys.*, **21**, 3, 387-403 (1980).
- [6] E. DANICKI, *Perturbation theory of surface acoustic wave reflection from a periodic structure with arbitrary angle of incidence*, *Arch. Mech.*, **36**, 5-6, 623-638 (1984).

INFLUENCE OF A BIASING STRESS ON THE SAW VELOCITY ON PIEZOELECTRIC SUBSTRATE

D. GAFKA

Institute of Fundamental Technological Research of the Polish Academy of Sciences
(00-049 Warsaw ul. Świeżokrzaska 21)

The nonlinear differential equations and boundary conditions in small-field variables, for small fields superposed on large static biasing states, are obtained from general invariant nonlinear electroelastic equations. The influence on Surface Acoustic Wave (SAW) velocity of prestress (six independent tensor coefficients) is analyzed. Biasing stress tensor is involved directly into equations of motion, and the linearization about the static state is done. Change of mass density as well as the effective elastic constants of the piezoelectric substrate as a consequence of prestress is taken into account. Numerical results, given SAW velocity on prestressed substrate for different direction of stress and wave propagation are obtained for lithium niobate.

Developed theory and numerical program may be useful for analyzing and designing SAW sensors of temperature, pressure, acceleration and stress.

1. Introduction

In recent years, a number of workers [3, 5, 6, 7, 9-12, 15, 17, 20, 21] have analyzed and measured the change in acoustic surface wave (SAW) velocity due to applied static biasing stresses. The problem is interesting from three points of view. First, the effect can be easily used in SAW sensors of pressure, acceleration, force, temperature, gas existence and others [6, 12, 15, 16, 17]. Second, the main sources of frequency instabilities of SAW oscillators and other devices are temperature and piezo effects [6, 16, 20]. Therefore it can be useful to describe stress sensitivity of SAW velocity. Third, the stress or strain may be utilized to control the performance of SAW devices for example selectivity of filters [8, 9] or temperature compensation of the devices.

In this paper a system of nonlinear electroelastic equations for small fields superposed on a bias [1, 2, 3, 13, 18, 21] is applied to the determination of the velocity of SAW in prestressed piezoelectric substrate. The influence of the biasing stress appears in the boundary conditions as well as the differential equations. The equations in the paper are written in the reference frame, connected with the material coordinates to make the boundary conditions easy to formulate, so the stress tensor is the Piola-Kirchhoff