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NONLINEARITY OF THE FREQUENCY-TEMPERATURE CHARACTERISTICS OF *LC*-CUT QUARTZ RESONATORS

E. ŻUCHOWSKI, M. ŁYSAKOWSKA

Tele–and Radio Research Institute
(03-450 Warszawa, Ratuszowa 11)

On the basis of the classical theory of thickness vibrations, the formulas for the temperature coefficients and their angular derivatives of the frequency-temperature characteristics of quartz resonators with *LC*-cut are derived. Approximation of these characteristics by straight lines enables the evaluation of nonlinearity effects in a given range of temperature. A practical example is given.

1. Introduction

Accurate temperature measurement represent an important branch of modern metrology. For these purposes many kinds of temperature sensors are used, for instance the quartz resonators of appropriate e.g. *Y*, *LC* or *NL* angle cuts. The best linearity of the frequency-temperature characteristic is obtained in the case of a *LC*-cut.

2. Fundamental formulae

From the classical theory of thickness vibrations of a thin ideal crystal plate we obtain two well-known formulae [1]: for the resonance frequency

$$f = \frac{N}{2h}(\bar{c}^{(m)}/\varrho)^{1/2} \qquad N = 1, 3, \ldots \qquad m = 1, 2, 3 \tag{1}$$

where h – thickness of the plate, $\bar{c}^{(m)}$ – effective striffness, ϱ – quartz density and the secular relation for the effective stiffness

$$|c_{ijkl}m_j m_k - \delta_{il}\bar{c}^{(m)}| = 0 \qquad i, j, k, l = 1, 2, 3, \tag{2}$$

where c_{ijkl} – piezoelectrically-stiffened elastic constants, m_k, m_j – components of a unit normal vector perpendicular to the plane of the plate, δ_{il} – Kronecker's delta.

From these two relations the values of the temperature coefficients and their angular derivatives of the frequency-temperature characteristics may be obtained.

3. Nonlinearity of the frequency-temperature characteristic

The most important parameter of temperature sensors is the frequency-temperature characteristic which is almost a straight line. Therefore, in the analysis we consider a new parameter, the nonlinearity of this characteristic, which gives more interesting results.

The real characteristic is given by the known relation

$$\frac{\Delta f}{f_0} = a(T-T_0)+b(T-T_0)^2+c(T-T_o)^3 \tag{3}$$

while the approximate equation for the characteristic is assumed in the linear form

$$\left(\frac{\Delta f}{f}\right)_l = a_l(T-T_0)+b_l. \tag{4}$$

In the case when $b_l = 0$, the optimum value of a_l derived by the method of least squares can be obtained from the condition

$$\frac{\partial \delta^2}{\partial a_l} = 0$$

If $b_l \neq 0$, then a_l and b_l are obtained from

$$\frac{\partial \delta^2}{\partial a_l} = 0, \quad \frac{\partial \delta^2}{\partial b_l} = 0$$

where

$$\delta^2 = \int_{x_d}^{x_g} (y_l - y)^2\, dx,$$

$$y = \frac{\Delta f}{f_0}, \quad y_l = \left(\frac{\Delta f}{f_0}\right)_l, \quad x_d = T_d - T_0, \quad x_g = T_g - T_0$$

T_d, T_g – lower and upper operating temperature, T_0 reference temperature.

Then the nonlinearity effect of the characteristic is defined as the differences $y_i - y$

$$\mathrm{N} = \mathrm{b}_l + (a_l - a)(T-T_0) - b(T-T_0)^2 - c(T-T_0{}^0)^2,$$

where b_l and $(a_l - a)$ are functions of b and c. The values of $a\ (T_f^{(1)})$, $b\ (T_f^{(2)})$ and $c(T_f^{(3)})$ and their angular derivatives will now be calculated.

4. Frequency-temperature coefficients and their derivatives

From Eq. (1), after differentiations with respect to the temperature, we obtain the following functional relation between the temperature coefficients of frequency and the temperature coefficients of $\bar{c}$, ϱ and h:

$$F(T_f^{(n)}) = \frac{1}{2}F(T_{\bar{c}}^{(n)}) - \frac{1}{2}F(T_\varrho^{(n)}) - F(T_h^{(n)}) \qquad n = 1, 2, 3$$

where

$$F(T_Y^{(n)}) = \frac{1}{n!}\frac{d^{n-1}\left(\frac{1}{Y}\frac{dY}{dT}\right)}{dT^{n-1}}$$

and

$$T_Y^{(1)} = \frac{1}{Y}\frac{dY}{dT}, \qquad Y \equiv f, \bar{c}, \varrho, h. \tag{7}$$

Hence, we obtain

$$T_f^{(1)} = G_1,\ T_f^{(2)} = G_2 + \frac{1}{2}G_1^2,\ T_f^{(3)} = G_3 + G_1 G_2 + \frac{1}{6}G_1^3 \tag{8}$$

where

$$G_n = \frac{1}{2}F(T_c^{(n)}) - \frac{1}{2}F(T_\varrho^{(n)}) - F(T_n^{(n)}). \tag{9}$$

The value of $T_c^{(n)}$ can be calculated on the basis of the formula (2). Further we can easily find that

$$T_\varrho^{(n)} = -(2\alpha_x^{(n)} + \alpha_z^{(n)}), \tag{10}$$

where α_x, α_z — linear thermal expansion coefficients of quartz. To obtain $T_h^{(h)}$ we write the plate thickness in the form of a vector

$$\mathbf{h} = ih_x + jh_y + kh_z. \tag{11}$$

Then for thickness vibrations

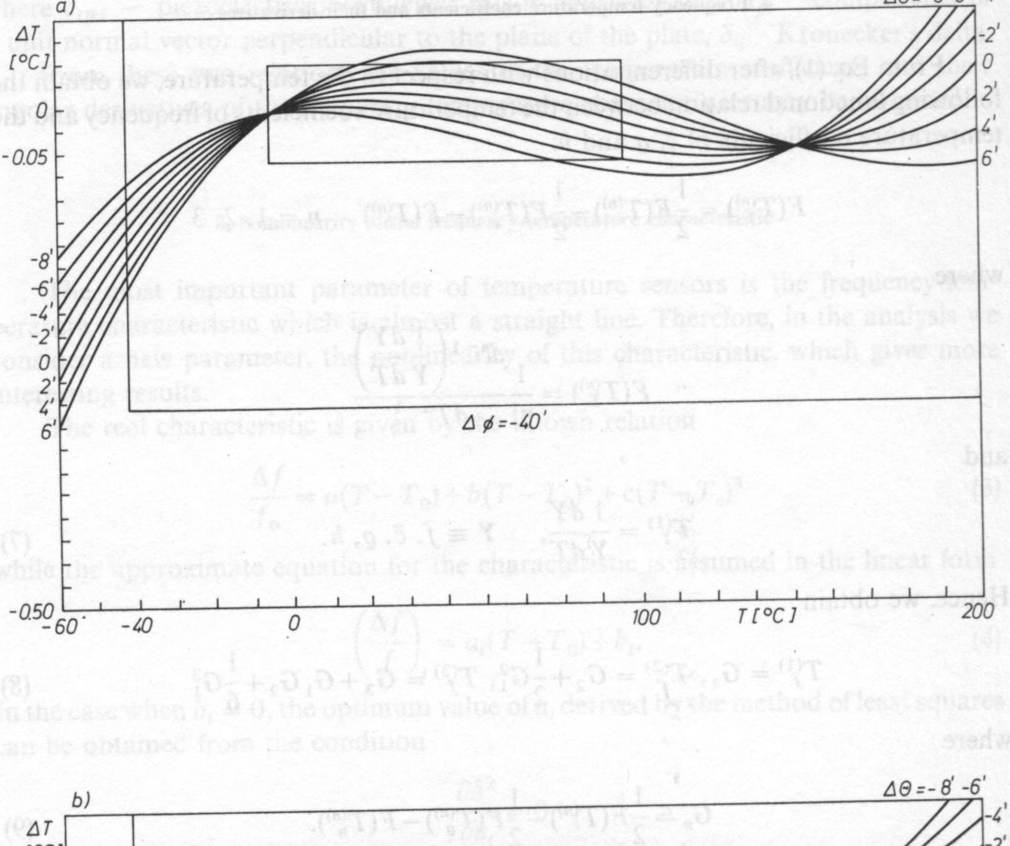
$$\mathbf{h} = h\mathbf{m} = h(im_x + jm_y + km_z) \tag{12}$$

where

$$h^2 = h^2x + h^2y + h^2z. \tag{13}$$

After calculations we obtain

$$T_h^{(n)} = \alpha_x^{(n)} + (\alpha_z^{(n)} - \alpha_x^{(n)})n_z^2, \qquad \alpha_y^{(n)} = \alpha_x^{(n)} \tag{14}$$


a)
ΔT
[°C]
ΔΘ=-8' -6' -4'
-2'
0
2'
4'
6'
0
-0.05
-8'
-6'
-4'
-2'
0
2'
4'
6'
Δφ=-40'
-0.50
-60
-40
0
100
T [°C]
200
b)
ΔT
[°C]
ΔΘ=-8' -6'
-4'
-2'
0
2'
4'
6'
0
-8'
-6'
-4'
-2'
0
2'
4'
6'
-0.30
Δφ=-20'
-0.50
-60
-40
0
100
T [°C]
200

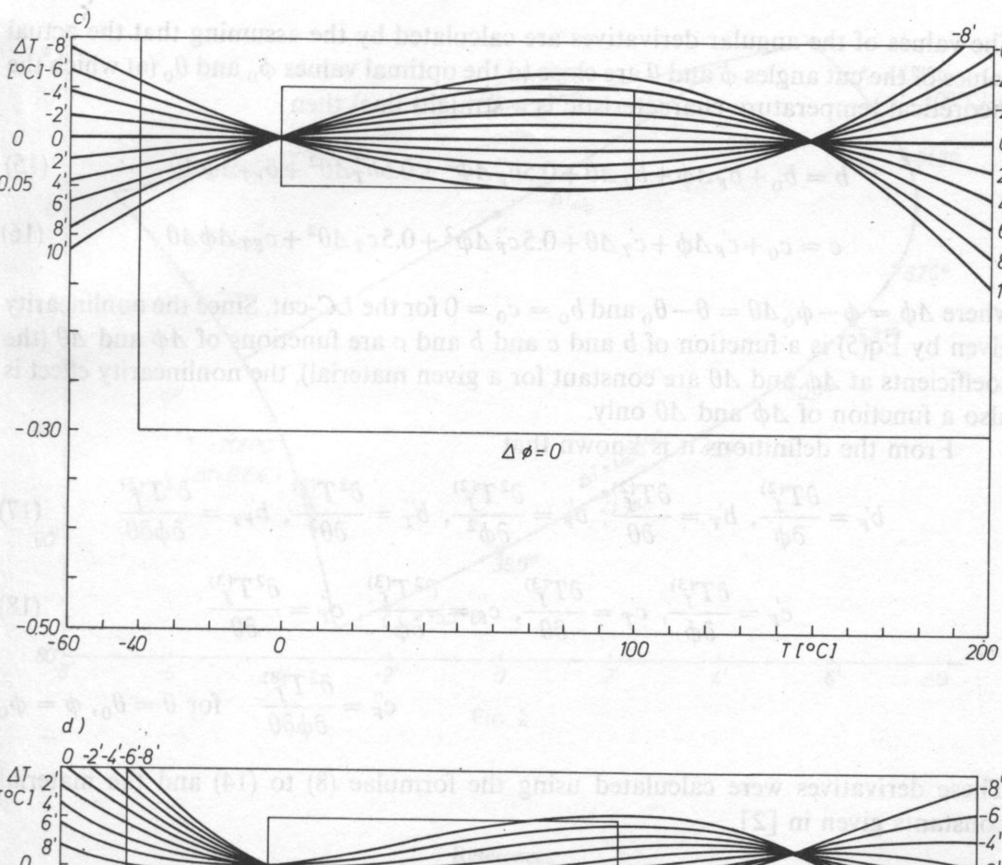

c)
ΔT
[°C]
-8'
-6'
-4'
-2'
0
2'
4'
6'
8'
10'
0
-0.05
-0.30
-0.50
Δφ = 0
-60
-40
0
100
T [°C]
200
d)
0 -2' -4' -6' -8'
ΔT
[°C]
2'
4'
6'
8'
0
-0.05
-0.30
-0.50
Δφ = 20'
-8'
-6'
-4'
-2'
0
2'
4'
6'
8'
-60
-40
0
100
T [°C]
200

FIG. 1

The values of the angular derivatives are calculated by the assuming that the actual values of the cut angles ϕ and θ are close to the optimal values ϕ_0 and θ_0 (at which the theoretical temperature characteristic is a straight line) then

$$b = b_0 + b'_F \Delta\phi + b'_T \Delta\theta + 0.5 b''_F \Delta\Phi^2 + 0.5 b''_T \Delta\theta^2 + b''_{FT} \Delta\phi \Delta\theta \tag{15}$$

$$c = c_0 + c'_F \Delta\phi + c'_T \Delta\theta + 0.5 c''_F \Delta\phi^2 + 0.5 c''_T \Delta\theta^2 + c''_{FT} \Delta\phi \Delta\theta \tag{16}$$

where $\Delta\phi = \phi - \phi_0 \Delta\theta = \theta - \theta_0$ and $b_0 = c_0 = 0$ for the *LC*-cut. Since the nonlinearity given by Eq(5) is a function of b and c and b and c are functions of $\Delta\phi$ and $\Delta\theta$ (the coefficients at $\Delta\phi$ and $\Delta\theta$ are constant for a given material), the nonlinearity effect is also a function of $\Delta\phi$ and $\Delta\theta$ only.

From the definitions it is known that

$$b'_F = \frac{\partial T_f^{(2)}}{\partial\phi}, \quad b'_T = \frac{\partial T_f^{(2)}}{\partial\theta}, \quad b''_F = \frac{\partial^2 T_f^{(2)}}{\partial\phi^2}, \quad b''_T = \frac{\partial^2 T_f^{(2)}}{\partial\theta^2}, \quad b''_{FT} = \frac{\partial^2 T_f^{(2)}}{\partial\phi\partial\theta} \tag{17}$$

$$c'_F = \frac{\partial T_f^{(3)}}{\partial\phi}, \quad c'_T = \frac{\partial T_f^{(3)}}{\partial\theta}, \quad c''_F = \frac{\partial^2 T_f^{(3)}}{\partial\phi^2}, \quad c''_T = \frac{\partial^2 T_f^{(3)}}{\partial\theta} \tag{18}$$

$$c''_F = \frac{\partial^2 T_f^{(3)}}{\partial\phi\partial\theta} \qquad \text{for } \theta = \theta_0,\ \phi = \phi_0$$

These derivatives were calculated using the formulae (8) to (14) and the material constants given in [2]

5. Example

At first it must be said that, in practice, instead of nonlinearity in frequency, nonlinearity in temperature is used. Then from the relation (3), by assuming that $b_l = 0$ and $T_0 = 0$ we obtain

$$\Delta T = \frac{f_0}{S}(\bar{a}T - bT^2 - cT^3)$$

where f_0 – frequency for $T = 0$, S – mean value of sensor sensitivity in the temperature range.

Figure 1 presents the results of calculations concerning the temperature nonlinearity in a practical example. From Fig. 1 or analytically we can obtain the tolerance map of cut angles ϕ and θ, as shown in Fig. 2.

For the values $\Delta\phi$ and $\Delta\theta$, given by the line $\Delta T_{\min}$ we obtain the minimum value of nonlinearity (for a given value of $\Delta\phi$ or $\Delta\theta$).

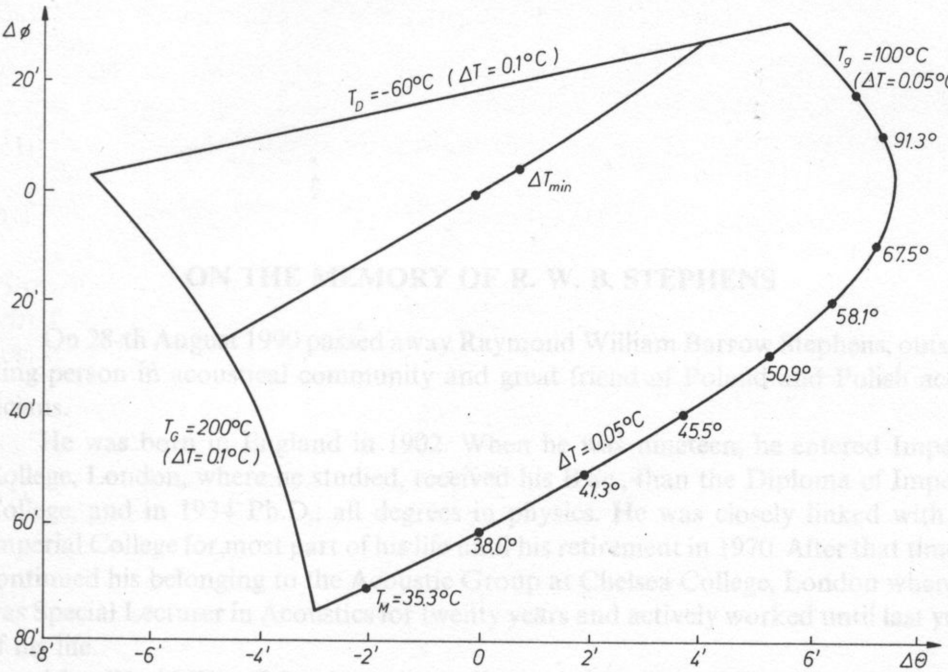


FIG. 2

References

[1] M. NAKAROVA et al., *A study of quartz temperature sensors characterized by ultralinear frequency-temperature responses*, IEEE Trans. of Sonics and Ultrasonics SU-32, 828–834 (1985).

[2] C. A. ADAMS et al., *Selected topics in quartz crystal research*, Proc. AFCS, 24, 55–63 (1970).