

TRANSFER IMPEDANCE OF A THREE-LAYER VISCOELASTIC ROD

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The present work is an attempt to apply the transfer impedance method to the investigation of systems composed of several layers of a material with known viscoelastic properties. A multilayer rod excited to longitudinal vibrations is examined theoretically. The end of the rod is stiffly connected with an optional mass. We assume that the component layers are homogeneous whereas the specific wave impedance undergoes jump-like changes at the border of the layers. In order to find the expression for the transfer impedance of the rod considered, the electromechanical analogies are used. The formulae given in the work allow to find the transfer impedance modulus which well describes the ability of a given structure to energy transmission. So, knowing the sound velocity c_n and loss factor η_n for a given layer of the material, one can determine the components of propagation constant α_n and β_n which subsequently allow to find the modulus of the transfer impedance for an optional frequency, at the fulfilled condition that the length of the longitudinal wave in the rod is much bigger than the lateral dimension of the rod. To illustrate the usefulness of the introduced formulae, a number of numerical investigations for vibroisolating materials are made. The influence of the mutual configuration of layers and their properties as components on the transfer impedance of the composed rod-like specimen is discussed.

Praca stanowi próbę zastosowania impedancji przenoszenia do badania układów złożonych z kilku warstw materiału o znanych własnościach lepkosprężystych. Rozpatrzono teoretycznie przypadek pręta lepkosprężystego złożonego z trzech różnych warstw i pobudzanego do drgań harmoniczných. Założono, że warstwy składowe są jednorodne, przy czym na granicy warstw impedancja falowa zmienia się skokowo. Wyprowadzono wzory na część rzeczywistą i urojoną impedancji przenoszenia pręta połączanego sztywno z dowolną masą oraz podano przykładowe wyniki obliczeń numerycznych, pokazujących wpływ konfiguracji warstw na charakter zmian modułu impedancji przenoszenia rozpatrywanego pręta.

1. Introduction

Designing of sandwich type partitions of given insulating properties requires determination of acoustic impedance not only for separate layered components but also for complex structures as their effective values. There are papers related to

evaluation of transmission and reflection coefficients of layered media [4, 5]. In [8] a partition of the impedance gradient distribution across its thickness was suggested. Such a system may be constructed as consisting of many layers differing among each other of a small jump of impedance. Construction of layered elements requires measurements of impedance variations during joining layers together. Works [6, 1, 2, 3] have shown that when applying measurements of mechanical impedance (input or transfer ones), it is easy to determine the viscoelastic properties of rubber-like materials. The present work is an attempt of applying the transfer impedance method to the investigation of systems composed of several layers of a material with known viscoelastic properties.

In this paper the general equation of transfer impedance of a three-layer viscoelastic rod, stiffly connected with an optional mass, has been derived and the influence of the mutual configuration of layers and their properties as components on the transfer impedance modulus of the composed rod-like specimen have been presented.

2. Theory

Let us consider the situation shown in Fig. 1 which presents a theoretical model of a three-layer rod excited to longitudinal vibrations at point $x = 0$. The end of the

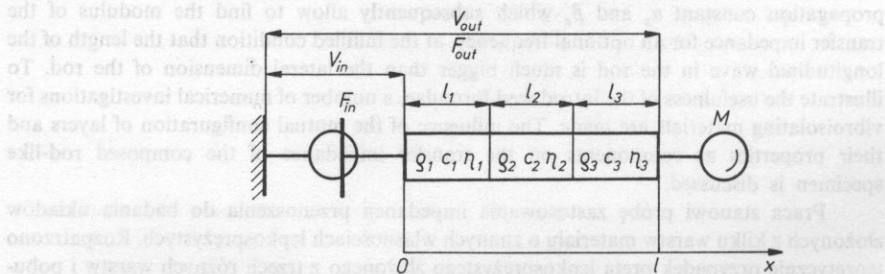


FIG. 1. Theoretical model of a three-layer rod-like specimen excited to longitudinal vibrations by a shaker and loaded statically by mass M

rod is stiffly connected with an optional mass M . It is assumed that the component layers are homogeneous whereas the specific wave impedance undergoes jump-like changes at the border of the layers. The considered rod is, thus, a combination of three rods of different wave impedances Z_{01} , Z_{02} and Z_{03} . Besides it is assumed that the cross-section is constant along the rod and its lateral dimension is much smaller than the length of the longitudinal wave propagating in the rod. In order to find the expression for the transfer impedance of the rod considered, one makes use of electromechanical analogies. The analogue of the rod presented in Fig. 1 is the wave-guide loaded by inductivity which can be regarded as a chain connection of

three mechanical four-poles [9]. The connection between the force and vibration velocity at the ends of the rod can be thus expressed by the following matrix equation:

$$\begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} = \begin{bmatrix} \operatorname{ch} \gamma_1 l_1 & Z_{01} \operatorname{sh} \gamma_1 l_1 \\ Z_{01}^{-1} \operatorname{sh} \gamma_1 l_1 & \operatorname{ch} \gamma_1 l_1 \end{bmatrix} \begin{bmatrix} \operatorname{ch} \gamma_2 l_2 & Z_{02} \operatorname{sh} \gamma_2 l_2 \\ Z_{02}^{-1} \operatorname{sh} \gamma_2 l_2 & \operatorname{ch} \gamma_2 l_2 \end{bmatrix} \begin{bmatrix} F_{out} \\ V_{out} \end{bmatrix} \quad (1)$$

where F_{in} , V_{in} and F_{out} , V_{out} are the complex forces and complex vibration velocities at input and output of the specimen, Z_0 is the wave impedance, l is the component rod length, γ is the complex propagation constant, $\gamma = \alpha + i\beta$ where α and β are the attenuation constant and phase constant, respectively. These latter can be expressed in terms of the material parameters [7] as

$$\beta = \beta_0 \sqrt{D+1} / \sqrt{2D}, \quad \alpha = \beta_0 \sqrt{D-1} / \sqrt{2D}, \quad (2, 3)$$

$$\beta_0 = 2\pi f(\rho/E_d)^{1/2}, \quad D = \sqrt{1+\eta^2}, \quad (4, 5)$$

in which E_d is the dynamic Young's modulus, η is the loss factor, ρ is the density and f is the frequency. The chain of three four-poles presented in Fig. 2 can be reduced to

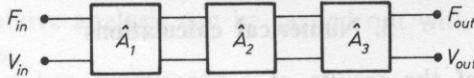


FIG. 2. Three-layer rod-like specimen regarded as a chain connection of three mechanical four-poles

an equivalent four-pole whose chain matrix is equal to the product of the chain matrixes of the component fourpoles [9], and equation (1) can be put down as

$$\begin{bmatrix} F_{in} \\ V_{in} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} F_{out} \\ V_{out} \end{bmatrix} \quad (6)$$

from which one obtains the expression for force F_{in}

$$F_{in} = AF_{out} + BV_{out} \quad (7)$$

Substituting $F_{out} = i\omega M V_{out}$ for (7) one gets

$$F_{in} = (i\omega MA + B)V_{out} \quad (8)$$

and finally the complex transfer impedance of the three-layer rod is found from the ratio of the exciting force F_{in} to the vibration velocity V_{out}

$$Z_t = \frac{F_{in}}{V_{out}} = i\omega MA + B \quad (9)$$

where elements A and B of the equivalent matrix (6) are expressed by the viscoelastic parameters of each rod layer as follows:

$$A = \text{ch} \gamma_1 l_1 \text{ch} \gamma_2 l_2 \text{ch} \gamma_3 l_3 + Z_{01} \text{sh} \gamma_1 l_1 Z_{02}^{-1} \text{sh} \gamma_2 l_2 \cdot \text{ch} \gamma_3 l_3 + \\ + \text{ch} \gamma_1 l_1 \cdot Z_{02} \text{sh} \gamma_2 l_2 Z_{03}^{-1} \text{sh} \gamma_3 l_3 + Z_{01} \text{sh} \gamma_1 l_1 \cdot \text{ch} \gamma_2 l_2 \cdot Z_{03}^{-1} \text{sh} \gamma_3 l_3 \quad (10)$$

$$B = \text{ch} \gamma_1 l_1 \text{ch} \gamma_2 l_2 \cdot Z_{03} \text{sh} \gamma_3 l_3 + Z_{01} \text{sh} \gamma_1 l_1 \cdot Z_{02}^{-1} \text{sh} \gamma_2 l_2 \cdot Z_{03} \text{sh} \gamma_3 l_3 + \\ + \text{ch} \gamma_1 l_1 \cdot Z_{02} \text{sh} \gamma_2 l_2 \text{ch} \gamma_3 l_3 + Z_{01} \cdot \text{sh} \gamma_1 l_1 \cdot \text{ch} \gamma_2 l_2 \cdot \text{ch} \gamma_3 l_3 \quad (11)$$

The real and imaginary parts of the complex transfer impedance written in an explicit form are given in the Appendix.

Formulas (9), (10) and (11) allow to find real and imaginary part of the transfer impedance for a given frequency if only one knows viscoelastic properties, i.e. sound velocity c and loss factor η for each layer of the material. The calculations can be carried out at optional boundary conditions accepting different values of loading mass M . Finally the transfer impedance modulus which describes well the ability of a structure to energy transmission is calculated. Putting in calculations $l_3 = 0$ one can proceed in a simple way to a two-layer system or when accepting $l_3 = 0$ and $l_2 = 0$ one obtains expressions for transfer impedance for a homogeneous rod, given earlier in [2].

3. Numerical calculations

Some examples of the results of a three-layer rod calculations versus the sequence of the layers are shown in Fig. 3. The calculations were made in the

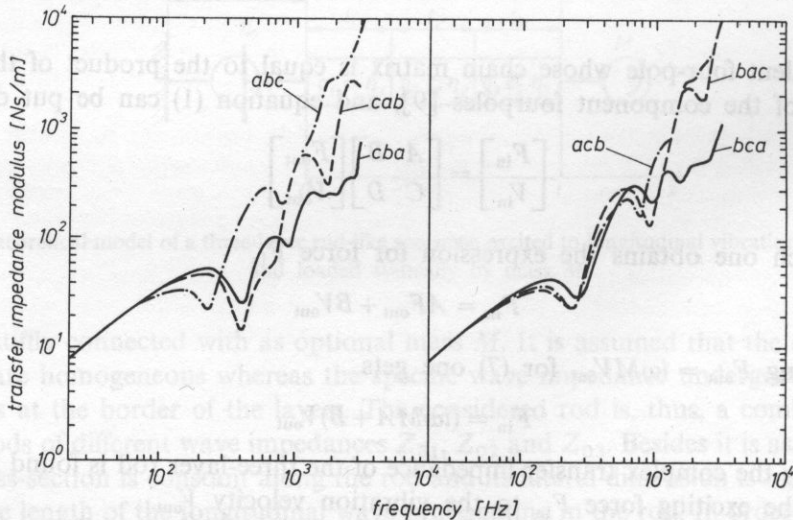


FIG. 3. Transfer impedance modulus for the different configuration of layers; a—Elastomer Z-22, b—Elastomer Z-7, c—Polyurethane Syntactic Foam. Calculations were made for $M/m = 1$, where M is the loading mass and m is the total mass of the three-layer specimen

frequency range from 0 Hz up to 3000 Hz. The following values have been accepted for the calculations: $c_1 = 60$ m/s, $\eta_1 = 0.35$, $\varrho_1 = 936$ kg/m³, $c_2 = 95$ m/s, $\eta_2 = 0.14$, $\varrho_2 = 1129$ kg/m³, $c_3 = 270$ m/s, $\eta_3 = 0.13$, $\varrho_3 = 1313$ kg/m³ appearing for real rubber-like materials i.e. Polyurethane Syntactic Foam, Elastomer Z-7 and Elastomer Z-22, respectively. Six different configurations abc, cab, cba, acb, bac and bca were numerically examined. As seen from the figure, the character of the changes of transfer impedance modulus of the regarded layer system is similar to the configuration of layers abc and acb as well as bca and cba. The influence of the mutual configuration of layers on the transfer impedance modulus is evident.

4. Conclusions

The introduced formulas enable a quick numerical analysis of transfer impedance modulus for an optional two- or three-layer system with required viscoelastic properties. The method enables to control how the transfer impedance modulus is varied depending on the sequence of layers and their viscoelastic properties. In the author's opinion, the presented method of numerical analysis of transfer impedance should be helpful in the estimation of the properties of viscoelastic layer systems and may enable their right selection for the vibration minimization. It is worth emphasizing that the above analysis can be carried out with the help of relatively simple microcomputers.

APPENDIX

In order to write down expression for real and imaginary parts of the complex transfer impedance in an explicit form, elements A and B (see eq. (10) and (11)) of matrix (6) should be expressed by the viscoelastic parameters of each rod layer. To do this, the following substitutions can be introduced:

$$Z_{on} = x_n + iy_n, \quad x_n = \frac{S\omega\varrho_n\beta_n}{\alpha_n^2 + \beta_n^2}, \quad y_n = \frac{S\omega\varrho_n\alpha_n}{\alpha_n^2 + \beta_n^2} \quad (A1)$$

$$Z_{on}^{-1} = u_n - iw_n, \quad u_n = \frac{\beta_n}{\omega\varrho_n S}, \quad w_n = \frac{\alpha_n}{\omega\varrho_n S}, \quad E_n = \operatorname{ch} \alpha_n l_n \cos \beta_n l_n \quad (A2)$$

$$F_n = \operatorname{sh} \alpha_n l_n \sin \beta_n l_n, \quad H_n = \operatorname{sh} \alpha_n l_n \cos \beta_n l_n, \quad K_n = \operatorname{ch} \alpha_n l_n \sin \beta_n l_n \quad (A3)$$

where Z_{on} is the wave impedance, α_n is the attenuation constant, β_n is the phase constant, ϱ_n is the material density, S is the cross-sectional area of the rod, l_n is the component rod length, n is the number of successive layer ($n = 1, 2, 3$), $i = \sqrt{-1}$ is the imaginary unit. The attenuation constant α_n and the phase constant β_n can be

expressed in terms of dynamic characteristics as

$$\beta_n = \frac{\omega}{c_n}, \quad \alpha_n = \beta_n \frac{\sqrt{D_n - 1}}{\sqrt{D_n + 1}}, \quad D_n = \sqrt{1 + \eta_n^2} \quad (\text{A4})$$

where c_n is the sound velocity and η_n is the loss factor.

Using the above given substitutions, after the calculations according to (10), (11), (9) and by applying several mathematical manipulations and transformations one obtains the final form of the expression for the real and imaginary part of transfer impedance of three-layer rod in the following form:

$$\begin{aligned} \text{Re}(Z_t) = & \omega M [E_3(E_1 F_2 + F_1 E_2) + F_3(E_1 E_2 - F_1 F_2) + \\ & + (H_1 K_2 + K_1 H_2) \cdot (x_1 w_2 F_3 + x_1 u_2 E_3 + y_1 w_2 E_3 - y_1 u_2 F_3) + \\ & + (H_1 H_2 - K_1 K_2) (x_1 u_2 F_3 + y_1 u_2 E_3 + y_1 w_2 F_3 - x_1 w_2 E_3) + \\ & + (E_1 K_2 + F_1 H_2) (x_2 u_3 H_3 + x_2 w_3 K_3 + y_2 w_3 H_3 - y_2 u_3 K_3) + \\ & + (E_1 H_2 - F_1 K_2) (x_2 u_3 K_3 + y_2 u_3 H_3 + y_2 w_3 K_3 - x_2 w_3 H_3) + \\ & + (H_1 F_2 + K_1 E_2) (x_1 u_3 H_3 + x_1 w_3 K_3 + y_1 w_3 H_3 - y_1 u_3 K_3) + \\ & + (H_1 E_2 - K_1 F_2) (x_1 u_3 K_3 + y_1 u_3 H_3 + y_1 w_3 K_3 - x_1 w_3 H_3)] + \\ & + E_1 E_2 (x_3 H_3 - y_3 K_3) - E_1 F_2 (y_3 H_3 + x_3 K_3) - F_1 E_2 (y_3 H_3 + x_3 K_3) - \\ & - F_1 F_2 (x_3 H_3 - y_3 K_3) + E_1 E_3 (x_2 H_2 - y_2 K_2) - E_1 F_3 (x_2 K_2 + y_2 H_2) - \\ & - F_1 E_3 (x_2 K_2 + y_2 H_2) - F_1 F_3 (x_2 H_2 - y_2 K_2) + E_2 E_3 (x_1 H_1 - y_1 K_1) + \\ & + F_2 F_3 (y_1 K_1 - x_1 H_1) - E_2 F_3 (x_1 K_1 + y_1 H_1) - \\ & - F_2 E_3 (x_1 K_1 + y_1 H_1) + (x_3 H_3 - y_3 K_3) (x_1 u_2 H_1 H_2 + \\ & + x_1 w_2 H_1 K_2) + (x_3 K_3 + y_3 H_3) (x_1 w_2 H_1 H_2 - x_1 u_2 H_1 K_2) + \\ & + (x_3 H_3 - y_3 K_3) (x_1 w_2 K_1 H_2 - x_1 u_2 K_1 K_2) - (x_3 K_3 + y_3 H_3) \cdot \\ & \cdot (x_1 K_1 u_2 H_2 + x_1 K_1 w_2 K_2) - (x_3 K_3 + y_3 H_3) (y_1 H_1 u_2 H_2 + \\ & + y_1 H_1 w_2 K_2) + (x_3 H_3 - y_3 K_3) (y_1 H_1 w_2 H_2 - y_1 H_1 u_2 K_2) + \\ & + (x_3 K_3 + y_3 H_3) (y_1 K_1 u_2 K_2 - y_1 K_1 w_2 H_2) - \\ & - (x_3 H_3 - y_3 K_3) (y_1 K_1 u_2 H_2 + y_1 K_1 w_2 K_2). \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} \text{Im}(Z_t) = & \omega M [E_3(E_1 E_2 - F_1 F_2) - F_3(E_1 E_2 - F_1 F_2) + (H_1 H_2 - \\ & - K_1 K_2) (x_1 u_2 E_3 + x_1 w_2 F_3 + y_1 w_2 E_3 - y_1 u_2 F_3) + (H_1 K_2 + \\ & + K_1 H_2) (x_1 w_2 E_3 - x_1 u_2 F_3 - y_1 u_2 E_3 - y_1 w_2 F_3) + (E_1 H_2 - \\ & - F_1 K_2) (x_2 u_3 H_3 + x_2 w_3 H_3 + y_2 w_3 H_3 - y_2 u_3 K_3) + (E_1 K_2 + \end{aligned} \quad (\text{A6})$$

$$\begin{aligned}
& + F_1 H_2)(x_2 w_3 H_3 - x_2 u_3 K_3 - y_2 u_3 H_3 - y_2 w_3 K_3) + (H_1 E_2 - \\
& - K_1 F_2)(x_1 u_3 H_3 + x_1 w_3 K_3 + y_1 w_3 H_3 - y_1 u_3 K_3) + (H_1 F_2 + \\
& + K_1 E_2)(x_1 w_3 H_3 - x_1 u_3 K_3 - y_1 u_3 H_3 - y_1 w_3 K_3)] + \\
& + E_1 E_2 (y_3 H_3 + x_3 K_3) + E_1 F_2 (x_3 H_3 - y_3 K_3) + F_1 E_2 (x_3 H_3 - \\
& - y_3 K_3) - F_1 F_2 (y_3 H_3 + x_3 K_3) + E_1 E_3 (x_2 K_2 + y_2 H_2) + \\
& + E_1 F_3 (x_2 H_2 - y_2 K_2) + F_1 E_3 (x_2 H_2 - y_2 K_2) - F_1 F_3 (x_2 K_2 + \\
& + y_2 H_2) + E_2 F_3 (x_1 H_1 - y_1 K_1) + F_2 E_3 (x_1 H_1 - y_1 K_1) + \\
& + E_2 E_3 (x_1 K_1 + y_1 H_1) - F_2 F_3 (x_1 K_1 + y_1 H_1) + (x_3 K_3 + \\
& + y_3 H_3)(x_1 H_1 u_2 H_2 + x_1 H_1 w_2 K_2) + (x_3 H_3 - y_3 K_3)(x_1 H_1 u_2 K_2 - \\
& - x_1 H_1 w_2 H_2) + (x_3 H_3 - y_3 K_3)(x_1 K_1 u_2 H_2 + x_1 K_1 w_2 K_2) + \\
& + (x_3 K_3 + y_3 H_3)(x_1 K_1 w_2 H_2 - x_1 K_1 u_2 K_2) + (x_3 H_3 - \\
& - y_3 K_3)(y_1 H_1 u_2 H_2 + y_1 H_1 w_2 K_2) + (x_3 K_3 + y_3 H_3)(y_1 H_1 w_2 H_2 - \\
& - y_1 H_1 u_2 K_2) + (x_3 H_3 - y_3 K_3)(y_1 K_1 w_2 H_2 - y_1 K_1 u_2 K_2) - \\
& - (x_3 K_3 + y_3 H_3)(y_1 K_1 u_2 H_2 + y_1 K_1 w_2 K_2).
\end{aligned}
\tag{A6}$$

[cont.]

The transfer impedance modulus is calculated from well-known formula

$$|Z_t| = [Re^2(Z_t) + I_m^2(Z_t)]^{1/2}. \tag{A7}$$

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