

FULLWAVE THEORY OF $\Delta v/v$ SAW WAVEGUIDES AND COUPLERS

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This paper considers propagation of a surface acoustic wave (SAW) along a multi-periodic system of electrodes (note: a multiperiodic system is a system with a group of several equidistant electrodes with equal width occurring periodically with a certain period) distributed on the surface of a piezoelectric half-space. The boundary problem with homogeneous mechanical and mixed electric boundary conditions was solved on the basis of properties of the effective surface permittivity and Floquet's theorem for periodic structures. New functions satisfying adequate conditions in an assumed multiperiodic system were formulated. A dispersion relation for the velocity of a SAW guided along electrodes was derived.

The presented theory was applied in numerical analysis of SAW velocity dispersion in a $\Delta v/v$ single and two-electrode waveguide, assuming an adequately long repetition period of groups of electrodes. Velocities and field decay distributions for two modes — symmetric and asymmetric are given for a two-electrode waveguide. The coupling coefficient between both electrodes of a coupler constructed on the basis of such a waveguide was calculated.

1. Introduction

Most modern SAW devices apply a wide beam of surface waves. The application of such a beam involves certain undesirable effects: beam spreading which accompanies its propagation, difficulties with changes of direction of propagation and inefficient use of the piezoelectric substrates surface. The application of waveguides eliminates all these problems, because a previously excited SAW can be guided. However, there are certain difficulties with the general application of such solutions in SAW technology: high losses and ineffective excitation (small aperture of waveguides) [10, 20, 21, 24]. Nevertheless they are used mainly in constructions of:

- long delay lines, storing analog or digital signals [1].
- convolvers, performing nonlinear operations on signals e.g. Fourier transformation [11, 19],
- monolithic amplifiers on a semiconductor substrate [7, 8, 10]
- filters with high quality factor (a pair of coupled waveguides) [23].

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There are many methods of guiding a wave within a certain separated region. The guiding region has lower velocity of wave propagation in relation to the rest of the surface [6, 20, 22, 24]. Deposition of a thin conducting film on the surface of the piezoelectric substrate is one of the simplest methods in the SAW technology. A local electric field shorting constitutes a SAW guide, because as we know [10, 18] SAW velocity under a shorted surface, V_0 , is lower than SAW velocity on a free (adjoining vacuum) surface V_v .

These types of guides (in a certain specific configuration) called piezoelectric waveguides or $\Delta v/v$ waveguides, will be the subject of further fullwave analysis. The method of analysis applied in this paper was previously used in investigations of perpendicular SAW propagation in relation to the system of electrodes [2, 4, 5, 10, 12] and in the analysis of the guidance of a wave in a periodic electrode system (including the transition to the "rare" system, i.e. single $\Delta v/v$ waveguide) [10]. This paper is a generalization of the theory presented in [10, Chapter 3] for the case of a wave guided in a multiperiodic electrode system, i.e., system of electrodes where not one but a group of N electrodes repeats periodically (Fig. 1). In such a case the transition to the "rare" system leads to a description of a finite number of coupled waveguides. A two-electrode guide is discussed in detail.

The following part of this paper presents the boundary problem, simplifying assumptions and changes in the formulation of the problem suitable for the accepted method of analysis [2, 4, 5, 10]. The construction of the problem's solution on the basis of new special functions, described in the Appendix, is given in Section 3. Section 4 presents results of numerical analysis of a one- and two-electrode SAW guide.

2. Formulation of the problem

We are looking for the field of a surface acoustic wave propagating along a multiperiodical system of metal electrodes with width w , distributed on the surface of a piezoelectric half-space. A group of a definite number (N) of electrodes repeats periodically with period Λ . The distance between electrodes within a group is equal to $2p$. The structure is unlimited. The cross section of the analysed system is shown in Fig. 1. The structure is homogeneous in the direction perpendicular to the cross section (x_1).

From the mathematical point of view the problem can be reduced to the solution of a set of partial differential equations with homogeneous mechanical and

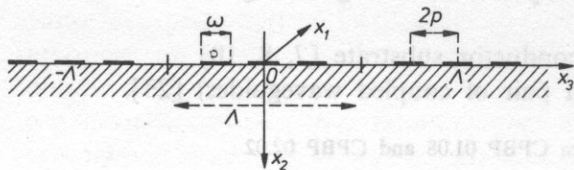


FIG. 1. Analysed system of electrodes (here: $N = 3$)

mixed electrical boundary conditions. The set of differential equations describing coupled mechanical-electrical wave processes in a linear and homogeneous medium has the following form for a case of electrostatic approximation [18] and for a solution harmonic in time:

$$\begin{cases} c_{ijkl} U_{k,jl} + e_{kij} \phi_{,jk} + \omega^2 d U_i = 0, & i, j, k, l = 1, 2, 3 \\ e_{ikl} U_{k,li} - \varepsilon_{ij} \phi_{,ji} = 0, & d - \text{mass density of the substrate} \end{cases} \quad (1)$$

where the vector of unknown quantities $U = [U_1, U_2, U_3, \phi]$ consists of components of the displacement vector U and electric potential ($E = -\nabla\phi$), c is the elastic tensor of the substrate e is the tensor of piezoelectric coefficients and ε is the permittivity tensor. The set of equations (1) has to be satisfied in the top ($x_2 < 0$) and bottom ($x_2 > 0$) half-space separately. Boundary conditions on the boundary of media ($x_2 = 0$) are described by the following equations:

a) mechanical

$$T_{2j} = c^{2jkl} U_{k,l} + e_{k2j} \phi_{,k} = 0 \quad \text{for } x_2 = 0 \quad (2)$$

b) electrical

$$\phi = 0 \quad \text{on the surface of electrodes}$$

$$\Delta D_2 = D_2^+ - D_2^- = 0 \quad \text{between electrodes} \quad (3)$$

The following additional idealizing assumptions were accepted:

- the metallization is a perfect conductor
- the thickness of the metallization is infinitely small
- metal electrodes do not load the surface of the piezoelectric otherwise condition (2) would not be fulfilled.

The problem in such a form is a three-dimensional problem. The method proposed in [4, 5] and developed in [2, 10, 12, 13, 14] was expanded in order to solve this problem. The mentioned method allows the problem to be reduced to a pseudoelectrostatic boundary problem on the boundary surface $x_2 = 0$ and its algebraization.

The expansion consist in the introduction of the effective surface permittivity [10, 18] which implicite contains the fulfilment of wave mechanical properties of SAW (1) and of the mechanical boundary condition (2). Therefore, the set of mixed electrical boundary conditions on the surface remains to be included.

The effective surface permittivity in the problem under consideration sufficiently characterizes the substrate. It is determined by the $\Delta D_{\perp}/E_{\parallel}$ relation where ΔD_{\perp} is the electric charge density on the substrate surface (it can be the charge on electrodes located on the surface), equal to the difference between the component of the electric displacement vector perpendicular to the surface $x_2 = 0$ considered from the substrate side ($x_2 = 0^+$) and from the vacuum side ($x_2 = 0^-$); E_{\parallel} is the electric

strength field on the substrate surface. In further text ΔD_{\perp} and E_{\parallel} denote complex amplitudes of equivalent harmonic waves:

$$\exp(j\omega t - \mathbf{k} \cdot \mathbf{x}) \quad (4)$$

where ω — angular wave frequency assumed in the paper as definite; factor $\exp(j\omega t)$ will be neglected further on, k — wave vector which assumes values from k_v on the free surface of the piezoelectric adjoining vacuum to k_0 on the metallized surface. In the considered case we assume that k_0 and k_v are real numbers and $k_0 \geq k_v$, $k_0 = \omega/V_0$, $k = \omega/V_v$, where V_0 and V_v are corresponding SAW velocities and the generally used in the SAW theory $\Delta v/v$ factor is defined as $\Delta v/v = (V_v - V_0)/V_v$.

In an approximation resulting from the neglect of acoustic bulk waves the relation between amplitudes ΔD_{\perp} and E_{\parallel} of harmonic waves in form (4) is expressed by [10, 18] (for $k > 0$)

$$\varepsilon(k) = -j \frac{\Delta D_{\perp}}{E_{\parallel}} = \varepsilon_0 \varepsilon_{ef} \frac{k^2 - k_v^2}{k^2 - k_0^2}, \quad (5)$$

where $k = |\mathbf{k}| = \sqrt{k_1^2 + k_3^2}$, ε_0 — permittivity of the vacuum.

As we know, a wave propagating in a periodically non-homogenous system can be described with the sum of harmonic components related to the period of the system, Λ . Consistently, in accordance to Floquet's theorem electric field E_{\parallel} and surface charge density ΔD_{\perp} on a the piezoelectric surface $x_1 x_3$ with a periodic electrode system have the following form

$$E_{\parallel} = \sum_{n=-\infty}^{\infty} E_n \exp(-j(\mathbf{k} + n\mathbf{K}) \cdot \mathbf{x}) \quad (6a)$$

$$\Delta D_{\perp} = \sum_{n=-\infty}^{\infty} D_n \exp(-j(\mathbf{k} + n\mathbf{K}) \cdot \mathbf{x}) \quad (6b)$$

where $\mathbf{x} = [x_1, x_3]$, \mathbf{K} — wave vector for considered system of electrodes with one component $\mathbf{K} = [0, K]$, $K = 2\pi/\Lambda$; and $\mathbf{k} = [k_1, k_3]$ is the wave vector of SAW propagation, e.g. of guiding SAW along electrodes $k_3 = 0$ (Fig. 1) in the considered case.

It can be easily checked that even when a wave propagates along electrodes, spatial harmonics into which the wave was decomposed have generally slant directions to the electrode system what, is expressed by the following

$$E_{\parallel} = \sum_{n=-\infty}^{\infty} E_n \mathbf{e}_n \exp(-j\kappa_n \xi_n)$$

$$\Delta D_{\perp} = \sum_{n=-\infty}^{\infty} D_n \exp(-j\kappa_n \xi_n), \quad (7)$$

where $\kappa_n = |\mathbf{k} + n\mathbf{K}| = \sqrt{k_1^2 + (nK)^2}$, ξ_n coordinate measured along the wave vector

κ_n , in such a manner that

$$\kappa_n \xi_n = (\mathbf{k} + n\mathbf{K}) \cdot \mathbf{x} \quad (\text{see Fig. 1}) \quad (8)$$

and \mathbf{e}_n is a versor in the direction of the wave vector $\mathbf{e}_n = (\mathbf{k} + n\mathbf{K})/\kappa_n$. Equivalent components of the potential have the following form:

$$\phi_n \exp(-j\kappa_n \xi_n), \quad \text{where} \quad \phi_n = -jE_n/\kappa_n, \quad (9)$$

whereas amplitudes E_n and D_n are related to each other when the effective surface permittivity is introduced for every n separately, as follows

$$D_n = j\epsilon_n E_n, \quad (10)$$

where

$$\epsilon_n = \epsilon(\kappa_n). \quad (11)$$

The boundary problem presented at the beginning can now be formulated as follows:

find amplitudes of the electric field E_n and wave number k_1 for given frequency ω , so that:

— electric field E_{\parallel} is zero on electrodes

$$\mathbf{E} = 0 \quad \text{on electrodes} \quad (12)$$

— distribution of surface charge density ΔD_{\perp} vanishes between electrodes

$$\Delta D_{\perp} = 0 \quad \text{between electrodes.} \quad (13)$$

3. Dispersion equations

We obtain the following conditions from (6a) for component e_3 and from (6b)

$$\mathbf{E} = \sum_{n=-\infty}^{\infty} \frac{nK}{\kappa_n} \cdot E_n e^{-jnKx_3} = 0 \quad \text{on electrodes} \quad (14)$$

$$\bar{\mathbf{D}} = \sum_{n=-\infty}^{\infty} \epsilon_n E_n e^{-jnKx_3} = 0 \quad \text{between electrodes}$$

We should notice that $\epsilon_n \rightarrow \epsilon_{\infty} = \epsilon_0 \epsilon_{ef}$ for $|n| \rightarrow \infty$ what more $\epsilon_n \cong \epsilon_{\infty}$ is fulfilled beginning from even small n for wave numbers K not too small in comparison to $k_{0,v}$. Similarly, $nK/\kappa_n \rightarrow \pm 1$ for $n \rightarrow \pm \infty$.

Conditions (14) ensure vanishing on the electrodes of one component of the electric field-component E_3 perpendicular to electrodes. These conditions should be supplemented with an additional relationship, namely vanishing condition for the

second component of the field along the electrodes in an arbitrary point of the electrodes. It is convenient to relate this condition to the axis of electrodes [10]. In accordance with (6a) we obtain

$$\sum_{n=-\infty}^{\infty} \frac{k_1}{\kappa_n} E_n e^{-jnK(2l-N-1)p} = 0, \quad l = 1, 2, \dots, N. \quad (15)$$

There are as many conditions as there are electrodes in a group.

The set of equations (14) will be solved jointly in the first place, and afterwards condition (15) will be satisfied.

As we know [2, 3], the set of two functions

$$g(\theta) = \sum_{n=-\infty}^{\infty} S_n P_n(\cos \Delta) e^{-jn\theta} \quad (16)$$

$$f(\theta) = \sum_{n=-\infty}^{\infty} P_n(\cos \Delta) e^{-jn\theta}$$

where $\Delta = Kw/2$, $\theta = Kx_3$, $S_n = n/|n|$ is the function of sign n and P_n are Legendre's polynomials, satisfies the following conditions

$$g(\theta) = \begin{cases} 0, & 0 \leq |\theta| \leq \Delta \\ -j \frac{\theta}{|\theta|} 2^{1/2} e^{j\theta/2} (\cos \Delta - \cos \theta)^{-1/2}, & \Delta < |\theta| \leq \pi \end{cases} \quad (17)$$

$$f(\theta) = \begin{cases} 2^{1/2} e^{j\theta/2} (\cos \theta - \cos \Delta)^{-1/2}, & 0 \leq |\theta| \leq \Delta \\ 0, & \Delta < |\theta| \leq \pi \end{cases}$$

Indeed, functions $g(\theta)$ and $f(\theta)$ are solutions to a corresponding electrostatic problem for a dielectric (i.e. for $k_0 = k_v$ in expression (5)). However, we have $k_0 \cong k_v$ for piezoelectrics and expression (5) differs significantly from a similar expression for a dielectric in a narrow range of wave numbers $k \in [k_v, k_0]$. This has been noticed and utilized in [2, 4, 5, 10] to construct a solution to the boundary problem (12), (13) for a piezoelectric substrate ($k_0 \neq k_v$) with a periodic system of electrodes.

Following a similar procedure as in [2, 12, 13, 14], functions which are analogical to (16) solutions to an electrostatic problem for the considered system of groups of electrodes from Fig. 1 are introduced

$$G_N(\theta; \alpha) = \sum_{n=-\infty}^{\infty} S_n X_n^N e^{-jn\theta} \quad (18)$$

$$F_N(\theta; \alpha) = \sum_{n=-\infty}^{\infty} X_n^N e^{-jn\theta},$$

where parameter related to the distance between electrodes in a group $\alpha = Kp$, while

conditions

$$G_N(\theta; \alpha) = 0 \quad \text{on electrodes} \quad (19)$$

$$F_N(\theta; \alpha) = 0 \quad \text{between electrodes}$$

are satisfied. Forms and properties of these functions are given in the Appendix.

Applying method [2, 10] quantities \bar{E} and \bar{D} were expressed by functions X_m as follows

$$\begin{aligned} \bar{E} &= \sum_{m=-\infty}^{\infty} \alpha_m G_N(Kx_3; Kp) e^{-jmKx_3} \\ \bar{D} &= j \sum_{m=-\infty}^{\infty} \beta_m F_N(Kx_3; Kp) e^{-jmKx_3} \end{aligned} \quad (20)$$

In this case boundary conditions (14) are reduced to:

$$\begin{aligned} \frac{nK}{\chi_n} E_n &= \sum_{m=M_1}^{M_2} \alpha_m S_{n-m} X_{n-m}^N \\ \epsilon_n E_n &= \sum_{m=M_1}^{M_2} \beta_m X_{n-m}^N, \end{aligned} \quad (21)$$

where infinite summation according to m is substituted with summation with in the range $M_1 \leq m \leq M_2$. This corresponds to the cut-off of an equivalent infinite set of equations, what in this case is possible [2, 10, 14], because nK/χ_n as well as ϵ_n quickly achieve constant values for $n \rightarrow \pm\infty$. α_m are determined from a comparison between left and right sides of (21) for $N_1 \leq n \leq N_2$ (we will discuss the choice of such limits below; but it is generally known that all terms for which nK/χ_n as well as ϵ_n differ significantly from their fixed values for $n \rightarrow \pm\infty$ have to be taken into account). Expressions (21) should have the form of a compatible set of equations for $n < N_1$ and $n > N_2$ for α_m calculated as described above. This determinates the choice of M_1 and M_2 , depending on N_1 and N_2 . Additionally [2, 10, 14] the compatibility condition of this set of equations imposes the method of selecting N_1 and N_2 namely for $n < N_1$ and $n > N_2$ terms on the left side in equation (21) have to have practically constant values (approximately equal to values for $|n| \rightarrow \infty$).

We should notice that the number of variables α_m within range $M_1 \leq m \leq M_2$ has to exceed the number $N_2 - N_1 + 1$ (i.e. number of equations resulting from a comparison between the right and left side of (21), because N more equations (15) resulting from field vanishing along the axis of electrodes remain to be satisfied.

Let us recapitulate. A comparison between the pair of equations (21) for $N_1 \leq n \leq N_2$ results in the following expression

$$\sum_{m=M_1}^{M_2} \left(\frac{nK}{\chi_n} \cdot \epsilon_n S_{n-m} \right) \alpha_m X_{n-m}^N = 0, \quad (22)$$

which has to be fulfilled for every $n \in [N_1, N_2]$, $N_1 < 0$, $N_2 > 0$.

Additionally from (15) and (21) we obtain

$$\sum_{m=M_2}^{M_2} \alpha_m \left(\sum_{n \neq 0} \frac{S_{n-m} X_{n-m}^N}{nK} e^{-jnK(2l-N-1)p} + \frac{\varepsilon_{ef}}{\varepsilon_0} X_{-m}^N \right) = 0, \quad (23)$$

where $l \in [1, N]$.

M_1 and M_2 have to be chosen after N_1 and N_2 are determined. They are chosen as shown below in order to secure an equal number of unknown quantities and equations

$$M_1 = N_1 - \text{int}(N/2) \quad (N_1 < 0) \quad (24)$$

$$M_2 = N_2 + \text{int}((N+1)/2).$$

The total number of equations and unknowns is equal to $N_2 - N_1 + N + 1$ in such a case. The condition for the existence of a solution to the set of equations (22) and (23), i.e. when the equivalent characteristic determinant $\Delta(k_1)$ is equal to zero, gives the sought for dispersion relation

$$\Delta(k) = 0 \quad (25)$$

where k is the value of wave number k_1 which determines the guiding velocity of SAW in the waveguide under consideration

$$V = \omega/k. \quad (26)$$

4. Numerical results

This Section presents numerical results of the analysis of SAW guidance along a system of electrodes deposited on a LiNbO_3 YZ. A propagation constant k was sought which would fulfill the following condition

$$k_v \leq k \leq k_0 \quad (27)$$

There are many solutions for the wave number in a periodic system, but it was accepted that k is a solution of an equivalent dispersion equation (25) from Brillouin's I zone [10]. K is large enough (Λ small) in numerically analysed structures that there is only one n which fulfills condition (27).

Presented below results have been divided into two groups according to the size of the parameters:

a) the distance between groups of electrodes greatly exceeds the distance between electrodes within a group and the width of electrodes

$$\Lambda \gg w, p$$

b) mentioned above distances are comparable

$$\Lambda \cong w, p$$

(a) type cases have an easy physical interpretation and practical application. Condition $\Lambda \gg w, p$ means that approximately there is no interaction between electrodes belonging to neighbouring groups of electrodes. Therefore, results obtained for such parameters can be accepted as propagation constants of waveguides with SAW ($\Delta v/v$ waveguide for $N = 1$, pair of coupled waveguides – SAW waveguide coupler for $N = 2$). It should be noted that a method of the fullwave analysis of an isolated waveguide with SAW does not exist; whereas when we assume $\Lambda \gg w, p$, then such an analysis is possible with certain approximation within the framework of the theory of periodic systems.

(b) type cases do not have a simple physical interpretation, because it is a problem of guiding a wave along a coupled infinite number of groups of electrodes. However, chosen relationships have been described here in order to make this elaboration complete.

a) SAW waveguides

The case of a one-electrode insulated approximately waveguide has been already analysed in papers [9, 10, 17, 23, 24]. A system of two coupled electrodes is considered below. The width of electrodes was accepted at $w = 0.015$ mm and repetition period of the pair of electrodes was accepted at $\Lambda = 3$ mm, considered as sufficiently large. Figure 2 presents the $1/V(w)$ dependence (V wave propagation velocity, $V = \omega/k$) in enlarged scale for $\Lambda = 3$ and 6 mm, and $N = 1$ in order to illustrate the influence of neighbouring groups of electrodes.

When the distance between neighbouring electrodes is doubled the velocity of a wave propagating in the structure changes by about 0.09%. The distance of 3 mm between electrodes with width equal to $w = 0.015$ mm was considered as sufficiently

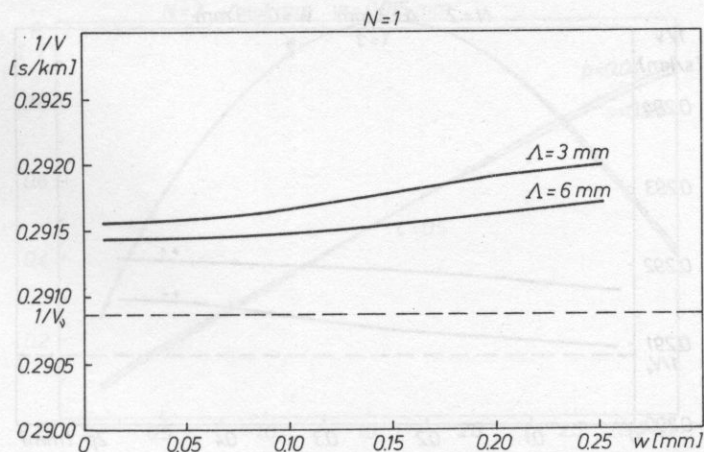


FIG. 2. Comparison of SAW guidance under the electrode in terms of W for different Λ .

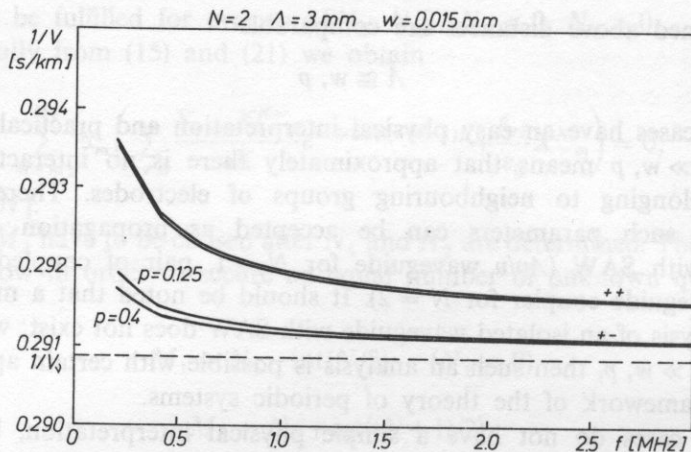


FIG. 3. SAW velocity dispersion in a two-electrode guide for two modes

to satisfy the condition of insulation of the waveguide from neighbouring electrodes.

In the case of a two-electrode waveguide, there are two, orthogonal with respect to each other modes with different SAW propagation velocities. Figure 3 presents dispersion characteristics $1/V(f)$ for both modes even, symmetric $++$ and odd, antisymmetric $+-$ for $p = 0.04$ and 0.125 mm. Figure 4 illustrates the influence of the distance between electrodes $2p$ on $1/V$ changes for $f = 1$ MHz. The odd mode for $2p$ close to w should be cut-off just as the first higher mode with odd charge distribution in a one-electrode waveguide [17]. The curve in Fig. 4 only approaches the cut-off value $1/V_0$. In order to closely investigate the characteristic near the cut-off, the complex value of the SAW wave number with damping should be permitted.

Taking advantage of the difference of wave velocity a SAW coupler can be

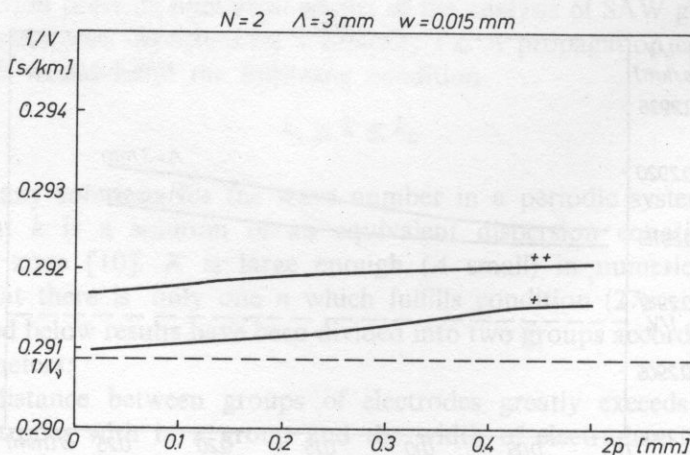


FIG. 4. SAW velocity in terms of distance between coupled electrodes

constructed, because of the existence of two modes with different velocities in a two-electrode waveguide. Power transmission from one port to the other of an ideal coupler is expressed by:

$$\tau = \sin^2\left(\frac{1}{2} \cdot (k_e - k_o)Z\right) \quad (28)$$

where τ — power transmission factor, k_e — value of even mode wave vector, k_o — value of odd mode wave vector, l — length of coupler.

Figure 5 presents the required length of couplers 3 dB and 10 dB in terms of

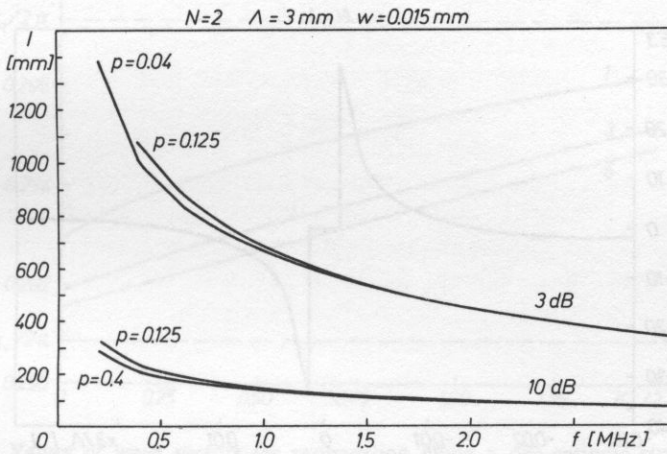


FIG. 5. Required lengths of SAW couplers — 3 dB and 10 dB for various frequencies

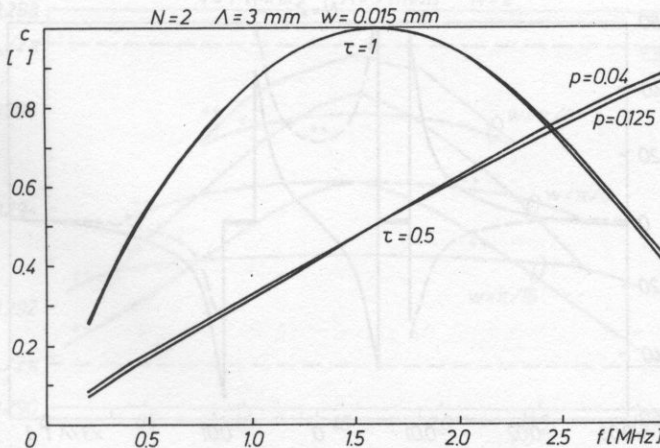


FIG. 6. Frequency characteristics of couplers with central frequency of 1.6 MHz ($\tau = 0.5, 1$)

frequency for $p = 0.04$ and $p = 0.125$ mm. For comparison the wavelength of a SAW is equal to about 3.4 mm for $f = 1$ MHz. A frequency coupling characteristic of couplers designed for frequency $f = 1.6$ MHz are shown in Fig. 6 for $\tau = 1$ full power transmission from one port to the other and $\tau = 0.5$ (3 dB coupler).

To illustrate the decay rate of the electric field around the electrodes, E_3 in terms of x_3/Λ was plotted for the following systems:

Fig. 7 $N = 1$, $w = 0.015$ mm, $\Lambda = 3$ mm

Fig. 8 $N = 2$, $w = 0.015$ mm, $p = 0.04$ mm

$\Lambda = 3$ mm both modes

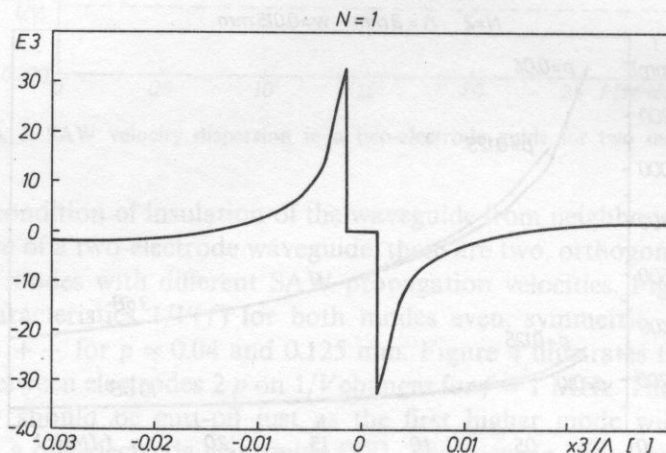


FIG. 7. Electric field decay of SAW guided under the electrode

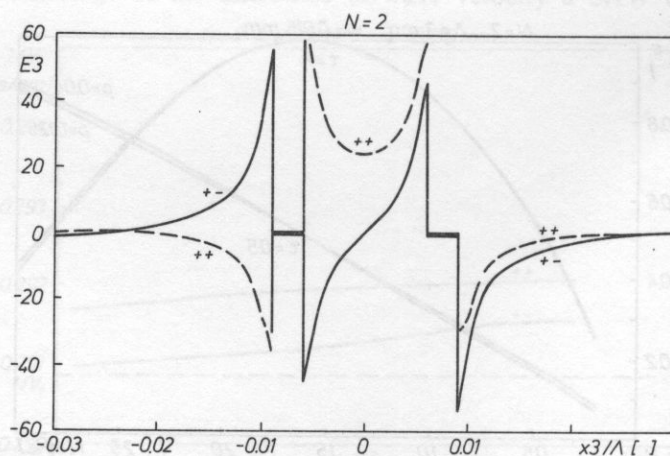


FIG. 8. Electric field decay of SAW guided under coupled electrodes

b) SAW propagation along a multiperiodic electrode system

Two more dependences are presented to make the elaboration complete:

Fig. 9 $k/2\pi$ ($Kw/2$) for $N = 1$, $f = 1$ MHz, $\Lambda = 1, 3, 6$ mm

Fig. 10 $k/2\pi$ (Kp) for $N = 2$, $f = 1$ MHz, $\Lambda = 3$ mm, $w = \Lambda/4, \Lambda/8$

When the metallized surface is increased the value of the wave vector approximates k_0 . Curves in Fig. 10 are symmetrical in respect to $Kp = \pi/2$, because

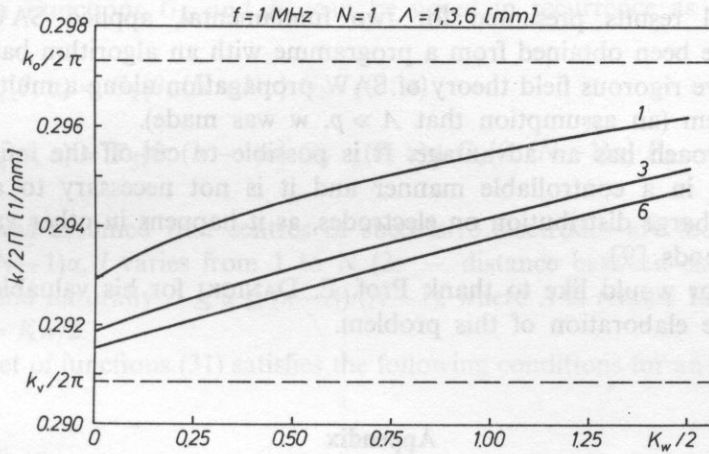


FIG. 9. Values of wave vectors for propagation along a one-periodic structure

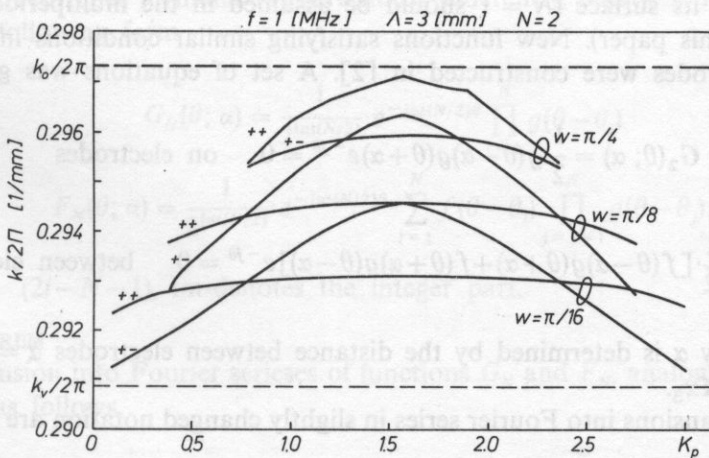


FIG. 10. Values of wave vectors for propagation along a two-periodic structure

for greater p electrodes interact with outer electrodes from the neighbouring group, i.e. $k(Kp) = k(\pi - Kp)$. As it can be seen for a case of SAW propagation along a multiperiodic electrode system there is a strong interaction between all electrodes for such parameters. There are p values for which velocities of both types i.e.: $++$ $++$ $++$ etc. and $+-$ $+-$ $+-$ etc. are equal.

5. Final remarks

Numerical results presented for two fundamental, applied SAW guiding structures have been obtained from a programme with an algorithm based on the discussed above rigorous field theory of SAW propagation along a multiperiodical electrode system (an assumption that $\lambda \gg p$, w was made).

This approach has an advantage. It is possible to cut-off the infinite set of equations (14) in a controllable manner and it is not necessary to assume an approximate charge distribution on electrodes, as it happens in other variation or Galerkin methods [9].

The author would like to thank Prof. E. DANICKI for his valuable remarks concerning the elaboration of this problem.

Appendix

The set of functions $g(\theta)$ and $f(\theta)$, described with equations (16) and given in [3, 16] is a solution of a canonical electrostatic problem with mixed boundary conditions for a dielectric with a system of metal electrodes with period 2π , distributed on its surface ($N = 1$ should be assumed in the multiperiodic system presented in this paper). New functions satisfying similar conditions in a case of separate electrodes were constructed in [2]. A set of equations was given there

$$G_2(\theta; \alpha) = \frac{1}{2} \cdot g(\theta - \alpha)g(\theta + \alpha)e^{-j\theta} = 0 \quad \text{on electrodes} \quad (29)$$

$$F_2(\theta; \alpha) = \frac{1}{2} \cdot [f(\theta - \alpha)g(\theta + \alpha) + f(\theta + \alpha)g(\theta - \alpha)]e^{-j\theta} = 0 \quad \text{between electrodes}$$

where quantity α is determined by the distance between electrodes $\alpha = Kp$, while variable $\theta = Kx_3$.

Their expansions into Fourier series in slightly changed notation are as follows:

$$G_2(\theta; \alpha) = \sum_{n=-\infty}^{\infty} X_n^2 S_n e^{-jn\theta} \quad F_2(\theta; \alpha) = \sum_{n=-\infty}^{\infty} X_n^2 e^{-jn\theta}, \quad (30)$$

where

$$X_0^2 = 0; \quad X_n^2 = \sum_{m=0}^{n-1} P_m(\cos \Delta) \cdot P_{n-m-1}(\cos \Delta) \cdot \cos(2m-n+1)\alpha$$

$$X_{-n}^2 = -X_n^2, \quad \pm \alpha = \pm Kp \text{ are centres of electrodes.}$$

Taking advantage of results of (29) and (30) a generalized set of functions was constructed below. It fulfills analogic mixed boundary conditions in a system with a group of an arbitrary number (N) of electrodes repeated with a 2π period (in variable θ). Functions G_N and F_N can be noted in recurrence as follows

$$G_N(\theta; \alpha) = G_2(\theta; (N-1)\alpha) \cdot G_{N-2}(\theta; \alpha) \quad (31)$$

$$F_N(\theta; \alpha) = F_2(\theta; (N-1)\alpha) \cdot G_{N-2}(\theta; \alpha) + G_2(\theta; (N-1)\alpha) \cdot F_{N-2}(\theta; \alpha),$$

where it was assumed that centres of successive electrodes are located at points $\theta_l = (2l-N-1)\alpha$, l varies from 1 to N (2α — distance between electrodes within a group) and naturally $\Delta \leq \alpha \leq (\pi-\Delta)/(N-1)$, where Δ is related to the electrodes width $\Delta = Kw/2$.

The set of functions (31) satisfies the following conditions for an N — electrode group:

$$G_N(\theta; \alpha) = 0 \text{ on electrodes} \quad (32)$$

$$F_N(\theta; \alpha) = 0 \text{ between electrodes}$$

After recurrence formulas (31) are noted differently, functions F_N and G_N will have the following form

$$G_N(\theta; \alpha) = \frac{1}{2^{\text{int}(N/2)}} \cdot e^{-\text{int}(N/2)\theta} \prod_{l=1}^N g(\theta - \theta_l) \quad (33)$$

$$F_N(\theta; \alpha) = \frac{1}{2^{\text{int}(N/2)}} \cdot e^{-\text{int}(N/2)\theta} \cdot \sum_{l=1}^N f(\theta - \theta_l) \prod_{j=1, j \neq l}^N g(\theta - \theta_j),$$

where $\theta_l = (2l-N-1)\alpha$, int-denotes the integer part.

THEOREM

Expansion into Fourier serieses of functions G_N and F_N , analogical to (16) and (30), are as follows

$$G_N(\theta; \alpha) = \sum_{n=-\infty}^{\infty} S_n X_n^N e^{-jn\theta} \quad F_N(\theta; \alpha) = \sum_{n=-\infty}^{\infty} X_n^N e^{-jn\theta}, \quad (34)$$

while for $n \geq 0$

$$\begin{cases} X_n^1 = P_n(\cos \Delta) \\ X_n^2 = \sum_{m=0}^{n-1} P_m P_{n-m-1} \cos[(2m-n+1)\alpha]; X_0^2 = 0 \\ \dots \dots \dots \\ X_n^N = 2 \sum_{m=1}^n X_m^2 X_{n-m}^{N-2} \end{cases} \quad (35)$$

where $X_m^2|_\beta$ means that X_m^2 are expansion coefficients of function $F_2(\theta; \beta)$

For $n \leq -1$ the following symmetry relations are fulfilled

$$\begin{aligned} X_{-n-1}^N &= X_n^N \quad \text{for odd } N \\ X_{-n}^N &= -X_n^N \quad \text{for even } N \end{aligned} \quad (36)$$

Relationships (35) and (36) require an explanation. Below they are proved on the basis of principles of mathematical induction.

PROOF.

- 1) For $N = 1$, (35) is trivial
- 2) For $N = 2$, (35) was proved in [2], different notation.
- 3) Assuming that the expansion for $N = 2$ (30) and for $N - 2$ is given

$$G_{N-2}(\theta; \alpha) = \sum_{n=-\infty}^{\infty} X_n^{N-2} S_n e^{-jn\theta} \quad F_{N-2}(\theta; \alpha) = \sum_{n=-\infty}^{\infty} X_n^{N-2} e^{-jn\theta} \quad (37)$$

while X_n^{N-2} fulfills relationships (35)–(36).

- 4) It should be proved that coefficients X_n^N in expansions (34) fulfill the last equation (35) and relation (36). Taking advantage of (31) and substituting (30) and (37), for G_N as first, we obtain

$$G_N = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} S_n S_m X_m^2 X_n^{N-2} e^{-j(n+m)\theta} = \sum_{n=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} S_{n-m} S_m X_m^2 X_n^{N-2} \right] e^{-jn\theta} \quad (38)$$

Comparing (34) with (38) we have

$$S_n X_n^N = \sum_{m=-\infty}^{\infty} S_{n-m} S_m X_m^2 X_n^{N-2}, \quad (39)$$

what for different n can be noted as

$$\begin{aligned} S_n X_n^N &= -C_n + 2 \sum_{m=0}^n X_m^2 X_{n-m}^{N-2} & \text{for } n \geq 0 \\ S_n X_n^N &= -C_n & \text{for } n = -1 \\ S_n X_n^N &= -C_n + 2 \sum_{m=n+1}^{-1} X_m^2 X_{n-m}^{N-2}, & \text{for } n \leq -2 \end{aligned} \quad (40)$$

where

$$C_n = \sum_{m=-\infty}^{\infty} X_m^2 X_{n-m}^{N-2} \equiv 0, \quad (41)$$

are coefficients of Fourier series expansion of the function

$$F_2 F_{N-2} = \sum_{n=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} X_m^2 X_{n-m}^{N-2} \right] e^{-jn\theta}, \quad (42)$$

equal identically to zero [2].

When we take advantage of equation $X_0^2 = 0$ in (40), we have

$$\begin{aligned} S_n X_n^N &= 2 \sum_{m=1}^n X_m^2 X_{n-m}^{N-2} & \text{for } n \geq 1 \\ S_n X_n^N &= 0 & \text{for } n = 0, -1 \\ S_n X_n^N &= 2 \sum_{m=n+1}^{-1} X_m^2 X_{n-m}^{N-2} & \text{for } n \leq -2. \end{aligned} \quad (43)$$

Similarly for F_N , from (31), (30) and (37) we have

$$\begin{aligned} F_N &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (S_m X_m^{N-2} X_n^2 + S_m X_n^{N-2} X_m^2) e^{-j(n+m)\theta} = \\ &= \sum_{n=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} S_m (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) \right] e^{-jn\theta}. \end{aligned} \quad (44)$$

Comparing (44) with (34)

$$X_n^N = \sum_{m=-\infty}^{\infty} S_m (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2), \quad (45)$$

what for different ranges of n results in:

for $n \geq 0$

$$\begin{aligned} X_n^N &= \sum_{m=0}^{\infty} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) - \sum_{m=-\infty}^{-1} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) = \\ &= \sum_{m=0}^{\infty} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) - \sum_{m=n+1}^{\infty} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) = \\ &= \sum_{m=0}^n (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) = 2S_n \sum_{m=0}^n X_m^{N-2} X_{n-m}^2 = 2S_n \sum_{m=0}^n X_m^2 X_{n-m}^{N-2} \end{aligned}$$

for $n = -1$

$$X_{-1}^N = \sum_{m=-\infty}^{\infty} S_m (X_m^{N-2} X_{-1-m}^2 + X_{-1-m}^{N-2} X_m^2) = \sum_{m=-\infty}^{\infty} X_m^2 X_{-1-m}^{N-2} (S_m + S_{-m-1}) \equiv 0$$

for $n \leq -2$

$$\begin{aligned}
 X_n^N &= \sum_{m=0}^{\infty} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) - \sum_{m=-\infty}^{-1} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) = \\
 &= \sum_{m=-\infty}^n (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) - \sum_{m=-\infty}^{-1} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) = \\
 &= - \sum_{m=n+1}^{-1} (X_m^{N-2} X_{n-m}^2 + X_{n-m}^{N-2} X_m^2) = 2S_n \sum_{m=n+1}^{-1} X_m^2 X_{n-m}^{N-2},
 \end{aligned}$$

or finally

$$X_n^N = 2S_n \sum_{m=0}^n X_m^2 X_{n-m}^{N-2} \quad \text{for } n \geq 1$$

$$X_n^N = 0 \quad \text{for } n = -1, 0 \quad (46)$$

$$X_n^N = 2S_n \sum_{m=n+1}^{-1} X_m^2 X_{n-m}^{N-2} \quad \text{for } n \leq -2$$

Obtained relations (43), and (46), prove that expansions (34) with coefficients X_n^N defined by (35) are correct. Symmetry relations (36) remain to be proved:

If $n \leq -2$ ($k = -n$), then from (46) we have

$$X_{-k}^N = 2S_{-k} \sum_{m=-k+1}^{-1} X_m^2 X_{-k-m}^{N-2}, \quad (47)$$

when the index is changed, $m = p - k$, we have

$$X_{-k}^N = -2 \sum_{p=1}^{k-1} X_{p-k}^2 X_{-p}^{N-2} = -2 \sum_{p=1}^k X_{p-k}^2 X_{-p}^{N-2}. \quad (48)$$

For even N ($X_p^{N-2} = -X_p^{N-2}$ from assumption (3)) we have

$$X_{-k}^N = 2 \sum_{p=1}^k X_{-(k-p)}^2 X_p^{N-2} = -2 \sum_{p=1}^k X_{k-p}^2 X_p^{N-2}. \quad (49)$$

A comparison between this expression and first equation (46) results in:

$$X_{-n}^N = -X_n^N. \quad (50)$$

For odd N ($X_{-p}^{N-2} = X_p^{N-2}$) we have

$$X_{-k}^N = 2 \sum_{p=1}^k X_{k-p}^2 X_{-p}^{N-2} = 2 \sum_{p=1}^k X_{k-p}^2 X_{p-1}^{N-2} = 2 \sum_{m=0}^{k-1} X_{k-m-1}^2 X_m^{N-2} = X_{k-1}^N,$$

and by analogy

$$X_n^N = X_{n-1}^N \quad (51)$$

what brings to the end the proof for the correctness of expansions (34) with (35) and (36).

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