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ACOUSTIC FILTERS WITH TWO PERFORATED TUBES

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This paper presents a physical and mathematical model of a filter. It was built on the basis of segmentation of its components and description of acoustic properties of these components with a transmission matrix. Also formulae for calculating transmission matrices for many specific solutions of the filter have been given in a convenient and generalized form. The presented model is limited to: the conditions of propagation of a harmonic plane acoustic wave, small dimensions of the holes in the perforation and laminar flow through these holes.

1. Introduction

Previously [3] a physical and mathematical model of an acoustic filter with one perforated tube was presented. Also cases of two parallel perforated tubes in a tube with rigid walls are encountered in practice applications.

Here we will outline the physical model and present a complete mathematical model, which can be a basis for an algorithm and computer programme for the last case.

The general conception of determining a mathematical model was given by SULLIVAN [5, 6]. It was developed by MUNJAL [4], who presented generalized forms of several equations. Yet, mathematical models given in the mentioned paper are too general and it is not possible to elaborate an algorithm for digital calculations on their basis. It took the author of this paper a considerable amount of time to determine detailed mathematical models when he was preparing such a programme. Therefore, the author considered it worth sharing the results of his work in order to save the time of readers who intended to create their own calculation programmes.

2. Outline of physical model

The physical model and accepted notations are presented in Fig. 1. As in the case of one perforated tube [3], this model is based on the assumption that the conditions of propagation of a plane wave are fulfilled in all wave-guides created by the tubes.

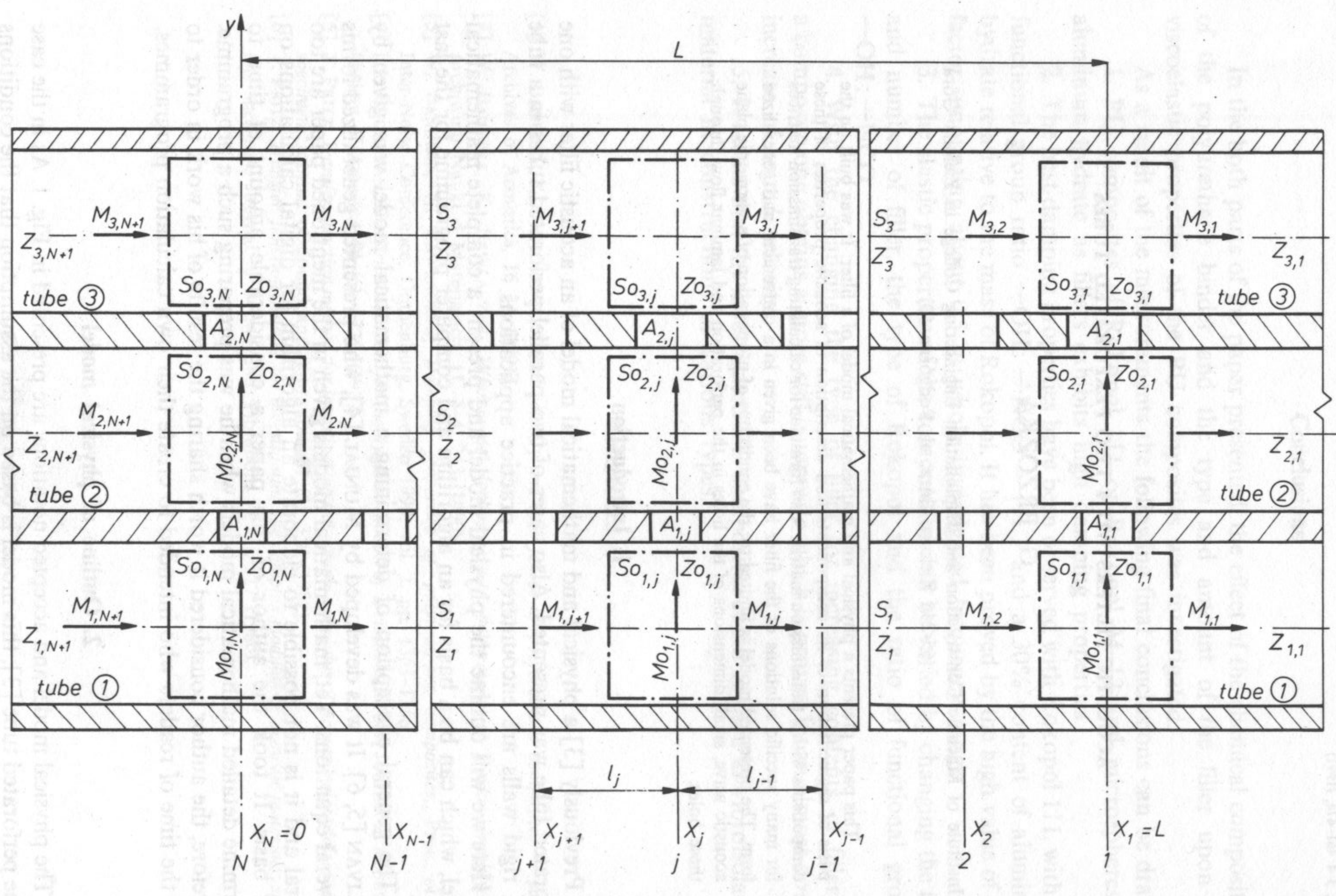


FIG. 1. Physical model and acoustic filter with two perforated tubes

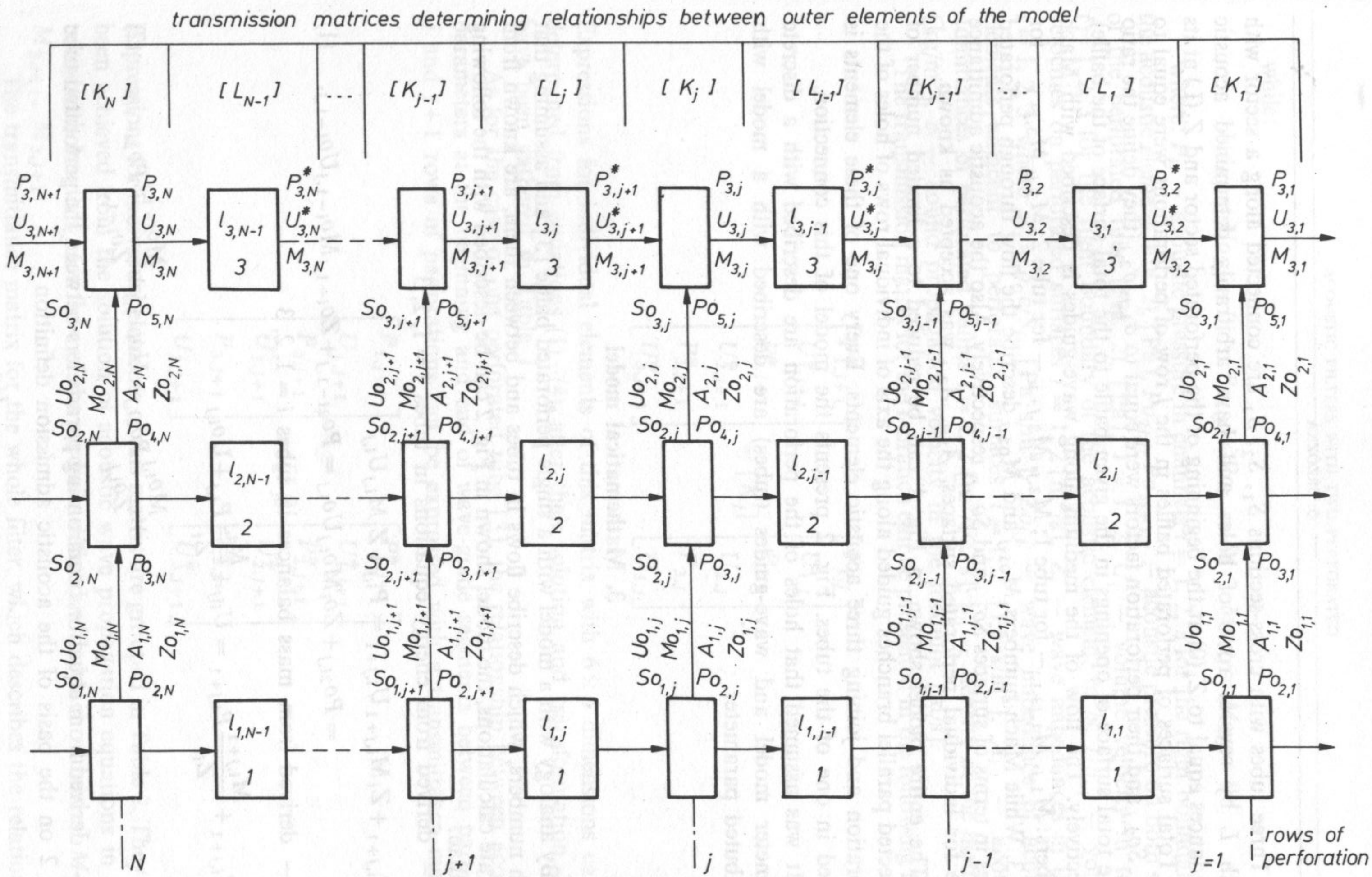


FIG. 2. Diagram of an analogue system of a filter with two perforated tubes

Three tubes with cross-sections S_1, S_2, S_3 are connected along a sector with length L by several rows of holes and have arbitrarily determined acoustic impedances equal to $Z_k(0)$ at the beginning of the perforated sector and $Z_k(\mathrm{L})$ at its end. Total surfaces of perforated baffles in the j row of perforations were equal to $So_{1,j}$, $So_{2,j}$ and their perforation factors were equal to $\sigma_{1,j}$, $\sigma_{2,j}$ (they define the ratio of the total surface of openings in the given baffle to the total surface of the baffle), respectively. The flow of the medium along wave-guides is described with Mach numbers: $M_{1,j}$, $M_{1,j+1}$ – for tube 1; $M_{2,j}$, $M_{2,j+1}$ – for tube 2; $M_{3,j}$, $M_{3,j+1}$ – for tube 3. While Mach numbers $M_{01,j}$ and $M_{02,j}$ describe the flow through perforated baffles in terms of surfaces $So_{1,j}$ and $So_{2,j}$, respectively. Also the acoustic admittance value for individual perforated surfaces. $A_{1,j}$, $A_{2,j}$ was accepted as known.

The entire model shown in Fig. 1 can be reduced to a limited number of connected parallel branches guided along the axis of individual rows of holes of the perforation and joining three acoustic elements. Every one of these elements is located in one of the tubes. Fig. 2 presents the model of this connection.

It was assumed that holes of the perforation are described with a discrete parameter model and wave-guides (tubes) are described with a model with distributed parameter.

3. Mathematical model

By analogy with a model with a single perforated baffle [3] and assuming that Mach numbers, which describe flows in tubes and between them, are known from separate calculations, the model shown in Fig. 2 can be described with the following formulae:

– derived from energy equations in tubes $i = 1, 2, 3$

$$P_{i,j+1}+Z_iM_{i,j+1}U_{i,j+1} = P_{i,j}+Z_iM_{i,j}U_{i,j}$$
$$= Po_{2i,j}+Zo_iMo_{i,j}Uo_{i,j} = Po_{2i-1,j}+Zo_{i-1,j}Mo_{i-1,j}Uo_{i-1,j}, \tag{1}$$

– derived from mass balances in tubes $i = 1, 2, 3$

$$U_{i,j+1}+\frac{M_{i,j+1}}{Z_i}P_{i,j+1} = U_{i,j}+\frac{M_{i,j}}{Z_i}P_{i,j}+Uo_{i,j}$$
$$+\frac{Mo_{i,j}}{Zo_{i,j}}Po_{2i,j}-Uo_{i-1,j}+\frac{Mo_{i-1,j}}{Zo_{i-1,j}}Po_{2i-1,j}, \tag{2}$$

– derived from equations confronting parameters between the perforated tube $i = 1, 2$ on the basis of the acoustic admission definition

$$Po_{2i+1,j} = Po_{2i,j}+\frac{Uo_{i,j}}{A_{i,j}}, \tag{3}$$

while:

$$Uo_{0,j} = Uo_{3,j} = 0; \quad Po_{1,j} = Po_{6,j} = 0; \quad Mo_{0,j} = M_{3,j} = 0. \tag{4}$$

where: P_{ij} – acoustic pressure in channels $i = 1, 2, 3$ for the right boundary of the acoustic element (Fig. 2), [Pa], $Po_{2i,j}$, $Po_{2i+1,j}$ – acoustic pressure on both sides of the perforated tube (Fig. 2), [Pa]; $i = 1, 2$, U_i – volume velocity in tubes $i = 1, 2, 3$ for the right boundary of the acoustic element, m^3/s, $Uo_{i,j}$ – volume velocities on perforated surfaces $i = 1, 2$ [m^3/s], Z_i – wave impedances in tubes $i = 1, 2, 3$ where $Z_i = \varrho_0 c/S_i$, [$Pa \cdot s/m^3$], $Zo_{i,j}$ – wave impedance of acoustic elements adjacent to perforated surfaces $i = 1, 2$ [$Pa \cdot s/m^3$], $A_{i,j}$ – acoustic admittances of perforated baffles $i = 1, 2$ [$m^3/Pa \cdot s$], ϱ_0 – density of the medium, [kg/m^3], c – sound propagation velocity in the medium [m/s].

The transmission matrix for the *i*–branch can be presented in the following form

$$\begin{bmatrix} P^*_{1,j+1} \\ U^*_{1,j+1} \\ P^*_{2,j+1} \\ U^*_{2,j+1} \\ P^*_{3,j+1} \\ U^*_{3,j+1} \end{bmatrix} = [K_j] \begin{bmatrix} P_{1,j} \\ U_{1,j} \\ P_{2,j} \\ U_{2,j} \\ P_{3,j} \\ U_{3,j} \end{bmatrix}. \tag{5}$$

Expressions for individual elements of this matrix with 6×6 dimensions can be determined from equations (1)–(4), after their solution and transformation. They have been gathered in Table 1.

As for the case of one perforated tube [3], the relationship between acoustic parameters at the beginning and end of wave-guide segments between particular j and $j+1$ rows of perforations can be expressed with a transmittance matrix

$$\begin{bmatrix} P_{1,j+1} \\ U_{1,j+1} \\ P_{2,j+1} \\ U_{2,j+1} \\ P_{3,j+1} \\ U_{3,j+1} \end{bmatrix} = [L_j] \begin{bmatrix} P^*_{1,j+1} \\ U^*_{1,j+1} \\ P^*_{2,j+1} \\ U^*_{2,j+1} \\ P^*_{3,j+1} \\ U^*_{3,j+1} \end{bmatrix}. \tag{6}$$

Expressions for individual elements of this matrix are grouped in Table 2. They have been achieved from the solution of acoustic wave propagation equations in three tubes with length $l_{i,j}$ for flow of the medium defined with Mach numbers: $M_{1,j+1}$, $M_{2,j+1}$, $M_{3,j+1}$.

The transmittance matrix for the whole filter, which describes the relationship between acoustic parameters in the last N-row and first row of perforations ($j = 1$),

Table 1. Formulae for elements of the transmission matrix $[K_j]$

K_{kl}	Calculation formula	K_{kl}	Calculation formula
K_{11}	$1-B_3G_1/E_1$	K_{21}	G_1/E_1
K_{12}	$B_1-B_3[1-B_1(B_2-G_1)]/E_1$	K_{22}	$[1-B_1(B_2-G_1)]/E_1$
K_{13}	B_3G_1/E_1	K_{23}	G_1/E_1
K_{14}	$B_3C_1G_1/E_1$	K_{24}	$-C_1G_1/E_1$
K_{15}	0	K_{25}	0
K_{16}	0	K_{26}	0
K_{31}	C_3G_1/E_2	K_{41}	$-G_1/E_2$
K_{32}	$B_1C_2G_1/E_2$	K_{42}	$-B_1G_1/E_2$
K_{33}	$1-C_3G_3/E_2$	K_{43}	G_3/E_2
K_{34}	$C_1-C_3[1-C_1(C_2-G_3)]/E_2$	K_{44}	$[1-C_1(C_2-G_3)]/E_2$
K_{35}	C_3G_2/E_2	K_{45}	$-G_2/E_2$
K_{36}	$C_3D_1G_2/E_2$	K_{46}	$-D_1G_2/E_2$
K_{51}	0	K_{61}	0
K_{52}	0	K_{62}	0
K_{53}	D_3G_2/E_3	K_{63}	$-G_2/E_3$
K_{54}	$C_1D_3G_2/E_3$	K_{64}	$-C_1G_2/E_3$
K_{55}	$1-D_3G_2/E_3$	K_{65}	G_2/E_3
K_{56}	$D_1-D_3[1-D_1(D_2-G_2)]/E_3$	K_{66}	$[1-D_1(D_2-G_2)]/E_3$

$B_1 = Z_1M_{1,j}$ $B_2 = M_{1,j}/Z_1$ $B_3 = Z_1M_{1,j+1}$ $B_4 = M_{1,j+1}/Z_1$
$C_1 = Z_2M_{2,j}$ $C_2 = M_{2,j}/Z_2$ $C_3 = Z_2M_{2,j+1}$ $C_4 = M_{2,j+1}/Z_2$
$D_1 = Z_3M_{3,j}$ $D_2 = M_{3,j}/Z_3$ $D_3 = Z_3M_{3,j+1}$ $D_4 = M_{3,j+1}/Z_3$
$E_1 = 1-M_{1,j+1}^2;$ $E_2 = 1-M_{2,j+1}^2;$ $E_3 = 1-M_{3,j+1}^2;$
$G_1 = A_{1,j}(1-Mo_{1,j}^2)$ $G_2 = A_{2,j}(1-Mo_{2,j}^2)$ $G_3 = G_1+G_2$

can be formally noted as

$$\begin{bmatrix} P_{1,N} \\ U_{1,N} \\ P_{2,N} \\ U_{2,N} \\ P_{3,N} \\ U_{3,N} \end{bmatrix} = [T] \begin{bmatrix} P_{1,1} \\ U_{1,1} \\ P_{2,1} \\ U_{2,1} \\ P_{3,1} \\ U_{3,1} \end{bmatrix}. \tag{7}$$

Taking advantage of the properties of a catenary matrix, the transmission matrix of the whole filter $[T]$ can be determined from successive multiplications of the transmission matrices $[K_j]$ and $[L_j]$

$$T = \prod_{k=1}^{N} [K_k][L_k], \tag{8}$$

where

$$L_N \equiv [I] \tag{9}$$

Table 2. Formulae for elements of the transmission matrix L_j

L_{kl}	Calculation formula	L_{kl}	Calculation formula
L_{11}	$F_1(\cos\alpha_1 + iM_{1,j+1}\sin\alpha_1)$	L_{21}	$i(F_1\sin\alpha_1)/Z_1$
L_{12}	$iF_1Z_1(1-M^2_{1,j+1})\sin\alpha_1$	L_{22}	$F_1(\cos\alpha_1 - iM_{1,j+1}\sin\alpha_1)$
L_{13}	0	L_{23}	0
L_{14}	0	L_{24}	0
L_{15}	0	L_{25}	0
L_{16}	0	L_{26}	0
L_{31}	0	L_{41}	0
L_{32}	0	L_{42}	0
L_{33}	$F_2(\cos\alpha_2 + iM_{2,j+1}\sin\alpha_2)$	L_{43}	$i(F_2\sin\alpha_2)/Z_2$
L_{34}	$iF_2Z_2(1-M^2_{2,j+1})\sin\alpha_2$	L_{44}	$F_2(\cos\alpha_2 - iM_{2,j+1}\sin\alpha_2)$
L_{35}	0	L_{45}	0
L_{36}	0	L_{46}	0
L_{51}	0	L_{61}	0
L_{52}	0	L_{62}	0
L_{53}	0	L_{63}	0
L_{54}	0	L_{64}	0
L_{55}	$F_3(\cos\alpha_3 + iM_{3,j+1}\sin\alpha_3)$	L_{65}	$i(F_3\sin\alpha_3)/Z_3$
L_{56}	$iF_3Z_3(1-M_{3,j+1})\sin\alpha_3$	L_{66}	$F_3(\cos\alpha_3 - iM_{3,j+1}\sin\alpha_3)$

where: α_m – phase shift, [rad]; $\alpha_m = \frac{Klm,j}{1-M^2_{n,j+1}}$; m – number of tube ($m = 1, 2, 3$),
k – wave number [m^{-1}]; $k = \omega/c$; $= 2\pi f$,
ω – angular velocity (pulsation), [rad/s]; f – frequency [Hz],
$l_{m,j}$ – length of wave-guides between rows of perforations, [m],
Z_m – wave impedance, [$Pa\cdot s/m^3$] for tube m ($m = 1, 2, 3$),
$M_{m,j+1}$ – Mach number behind j–row of perforations (Figs. 1 and 2) in tube m,
F_m – auxiliary function $F_m = \cos(M_{m,j+1}\alpha_m - i\sin M_{m,j+1}\alpha_m) = e^{-i}M_{m,j+1}\alpha_m$.

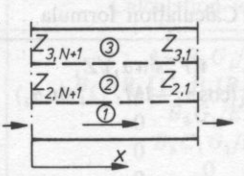
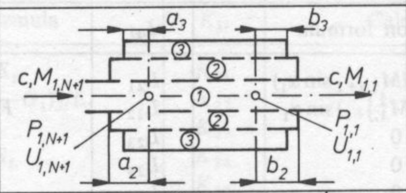
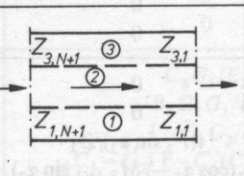
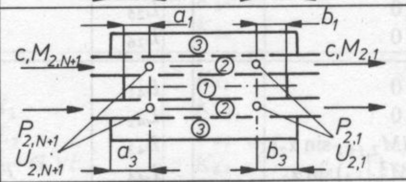
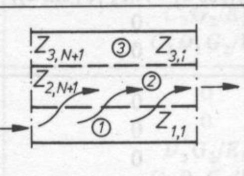
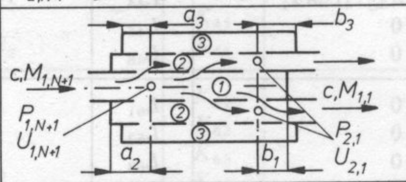
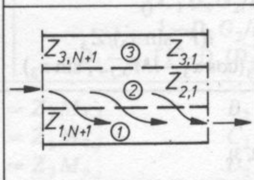
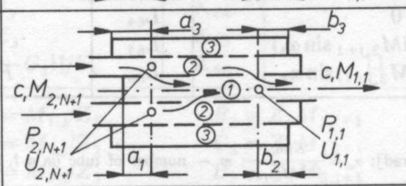
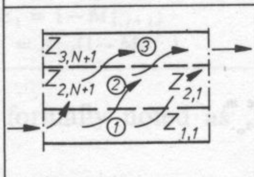
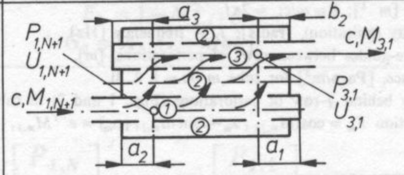
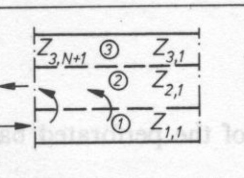
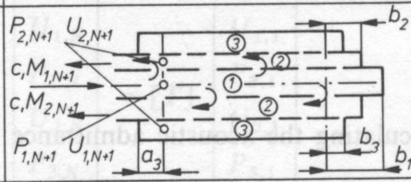
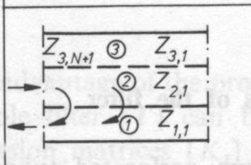
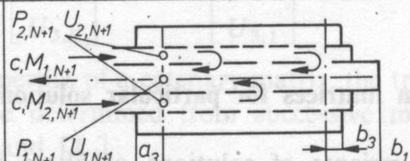
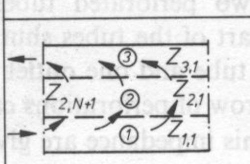
$[I]$ – identity matrix.

The method of calculating the acoustic admittance of the perforated baffle is presented in Section 4.

4. Transmission matrices for particular solutions of the filter

The fundamental variants of solutions of filters two perforated tubes are presented in Table 3. Models under consideration have part of the tubes shut with rigid acoustically impermeable baffles with only one inlet tube and one outlet tube. The impedance of segments from the baffle to the nearest row of perforations can be determined in closed channels. Formulae for calculating this impedance are given in Table 3.

Table 3. Fundamental variants of acoustic have propagation in a three-tube model

direction of sound propagation		diagram of model	example of solution	notation and remarks
longitudinal				1a $Z_{2,1} = -iZ_2\,ctg\,(kb_2)$ $Z_{3,1} = -iZ_3\,ctg\,(kb_3)$ $Z_{2,N+1} = iZ_2\,ctg\,(ka_2)$ $Z_{3,N+1} = iZ_3\,ctg\,(ka_3)$
				1b $Z_{1,1} = -iZ_1\,ctg\,(kb_1)$ $Z_{3,1} = -iZ_3\,ctg\,(kb_3)$ $Z_{1,N+1} = iZ_1\,ctg\,(ka_1)$ $Z_{3,N+1} = iZ_3\,ctg\,(ka_3)$
	transverse			2a $Z_{1,1} = -iZ_1\,ctg\,(kb_1)$ $Z_{3,1} = -iZ_3\,ctg\,(kb_3)$ $Z_{2,N+1} = iZ_2\,ctg\,(ka_2)$ $Z_{3,N+1} = iZ_3\,ctg\,(ka_3)$
				2b $Z_{2,1} = -iZ_2\,ctg\,(kb_2)$ $Z_{3,1} = -iZ_3\,ctg\,(kb_3)$ $Z_{1,N+1} = iZ_1\,ctg\,(ka_1)$ $Z_{3,N+1} = iZ_3\,ctg\,(ka_3)$
				2c $Z_{1,1} = -iZ_1\,ctg\,(kb_1)$ $Z_{2,1} = -iZ_2\,ctg\,(kb_2)$ $Z_{2,N+1} = iZ_2\,ctg\,(ka_2)$ $Z_{3,N+1} = iZ_3\,ctg\,(ka_3)$
	reversible			3a $Z_{1,1} = -iZ_1\,ctg\,(kb_1)$ $Z_{2,1} = -iZ_2\,ctg\,(kb_2)$ $Z_{3,1} = -iZ_3\,ctg\,(kb_3)$ $Z_{3,N+1} = iZ_3\,ctg\,(ka_3)$
				3b as for 3a
				3c $Z_{1,1} = -iZ_1\,ctg\,(kb_1)$ $Z_{2,1} = -iZ_2\,ctg\,(kb_2)$ $Z_{3,1} = -iZ_3\,ctg\,(kb_3)$ $Z_{2,N+1} = iZ_2\,ctg\,(ka_2)$

As in the case of a muffler with one perforated tube [3], the relationship between acoustic parameters of an input and output acoustic wave may be interesting for mentioned above systems. The relationship can be noted in the form of a four-element transmission matrix [T]. The flow of the medium, determined by Mach numbers, has the same direction as sound propagation in models under consideration. If the flow of the medium is oppositely directed to the direction of sound propagation, then negative values for Mach numbers have to be accepted in calculations.

The method of deriving formulae for elements of the transmission matrix [T′] has been presented in detail in paper [3]. While in Tables 4 and 5 we have formulae for individual cases presented in Table 3.

Table 4. Formulae for elements of the transmission matrix $[T']$ for a case of longitudinal and transverse sound propagation

Notation and description of specific solution		Values of indices				
		i	m	j	k	n
1a	Propagation along a single perforated baffle	1	1	3	2	2
1b	Propagation along a double perforated baffle	2	2	3	1	1
2a	Transverse propagation to the adjacent tube with double perforated baffle	1	2	3	2	1
2b	Transverse propagation to the adjacent tube with single perforated baffle	2	1	3	1	2
2c	Transverse propagation to the opposite tube	1	3	2	3	1

Formulae for elements of transmission matrix $[T']$

$$T'_{11} = T_{(2i-1)(2m-1)} + a_{(2i-1)}(AH-CF) + b_{(2i-1)}(CE-AG)/M$$
$$T'_{12} = T_{(2i-1)(2m)} + a_{(2i-1)}(BH-DF) + b_{(2i-1)}(DE-BG)/M$$
$$T'_{21} = T_{(2i)(2m-1)} + a_{(2i)}(AH-CF) + b_{(2i)}(CE-AG)/M$$
$$T'_{22} = T_{(2i)(2m)} + a_{(2i)}(BH-DF) + b_{(2i)}(DE-BG)/M$$

where: T_{xy} – transmission matrix between the $N+1$ and 1 row of perforations, with dimensions (6×6)

$a_x = T_{x(2j-1)}Z_{j,1} + T_{x(2j)}$	$b_x = T_{x(2n-1)}Z_{n,1} + T_{x(2n)}$
$A = T_{(2k-1)(2m-1)} - Z_{k,N+1}\, T_{(2k)(2m-1)}$	$B = T_{(2k-1)(2m)} - Z_{k,N+1}\, T_{(2k)(2m)}$
$C = T_{(2j-1)(2m-1)} - Z_{j,N+1}\, T_{(2j)(2m-1)}$	$D = T_{(2j-1)(2m)} - Z_{j,N+1}\, T_{(2j)(2m)}$
$E = Z_{k,N+1}\, a_{(2k)} - a_{(2k-1)}$	$F = Z_{k,N+1}\, b_{(2k)} - b_{(2k-1)}$
$G = Z_{j,N+1}\, a_{(2j)} - a_{(2j-1)}$	$H = Z_{j,N+1}\, b_{(2j)} - b_{(2j-1)}$
$M = EH - FG$	$M \neq 0$

i – index of inlet tube; m index of outlet tube;
j – tube closed at both ends; $x = 1, 2, \ldots 6$;
$k = 1, 2, 3$ where $k \neq i$ and $k \neq j$; $n = 1, 2, 3$ where $n \neq j$ and $n \neq m$.

Table 5. Formulae for elements of the transmission matrix [T′] for a case of reversible sound propagation

	Notation and description of specific solution	Values of indices		
		i	m	j
3a	Propagation reversible to adjacent tube with double perforated baffle	1	2	3
3b	Propagation reversible to adjacent tube with single perforated baffle	2	1	3
3a	Propagation reversible to opposite tube	1	3	2

Formulae for elements of transmission matrix T'

$$T'_{11} = B_{(2i-1)}\,E_{(2m)} - A_{(2i-1)}\,F_{(2m)} \qquad T'_{12} = A_{(2i-1)}\,F_{(2m-1)} - B_{(2i-1)}\,E_{(2m-1)}$$
$$T'_{21} = B_{(2i)}\,E_{(2m)} - A_{(2i)}\,F_{(2m)} \qquad T'_{22} = A_{(2i)}\,F_{(2m-1)} - B_{(2i)}\,F_{(2m-1)}$$

where

$$A_x = a_x + C_x(a_{(2j-1)} - Z_{j,N+1}\,a_{(2j)})/M \qquad x = 1, 2, \ldots, 6$$
$$B_x = b_x + C_x(b_{(2j-1)} - Z_{j,N+1}\,b_{(2j)})/M$$
$$M = Z_{j,N+1}\,C_{(2j)} - C_{(2j-1)} \qquad M \neq 0$$
$$a_x = T_{x1}\,Z_{1,1} + T_{x2} \qquad b_x = T_{x3}\,Z_{2,1} + T_{x4}$$
$$C_x = T_{x5}\,Z_{3,1} + T_{x6}$$
$$E_x = A_x/W \qquad F_x = B_x/W$$
$$W = A_{(2m)}\,B_{(2m-1)} - A_{(2m-1)}\,B_{(2m)}$$

T_{xy} – transmission matrix between the $N+1$ and 1 row of perforations, with dimensions (6×6), i – index of inlet tube; m – index of outlet tube, $j = 1, 2, 3$ where $j \neq i$ oraz $j \neq m$

5. Admittance of a perforated surface

It is most convenient to determine the admittance of a perforated baffle on the basis of impedance measurements, as it was done by SULLIVAN [6] or MUNJAL [4]. The conversion formula for the j–row of perforations is as follows

$$A_{i,j} = \frac{1}{Z_{i,j}} = \frac{\sigma_{i,j}}{Zo_{i,j}}\,\frac{\theta_{i,j} - i\chi_{i,j}}{\theta_{i,j} + \chi_{i,j}} \tag{10}$$

where: $A_{i,j}$ – acoustic admittance of perforated baffles, [m³/Pa·s], $Z_{i,j}$ – acoustic impedance of perforated baffles walls of perforated tubes $i = 1, 2$, [Pa·s/m³], $\sigma_{i,j}$ – factor for the i–tube and j–row of perforations, $\theta_{i,j}$ – specific acoustic resistance of perforations, $\chi_{i,j}$ – specific acoustic reactance of perforations.

When empirical data is not available, then approximate formulae described in previous papers by the author [1, 3] can be applied

$$\theta_{i,j} = \frac{4\sqrt{\pi f \nu}}{C}\left[\frac{h}{d}+(1-\sigma_{i,j})\right]+2.57\,\frac{Mo_{i,j}}{\sigma_{i,j}}$$
$$\chi_{i,j} = \frac{2\pi}{C}\, f\, h_{\mathrm{ef}_{i,j}} \tag{11}$$

where: f – frequency of acoustic vibrations, [Hz], ν – kinematic viscosity of the medium, [m^2/s]; it is recommended to increase this coefficient by 114% [7] in order to include the effect of losses due to heat exchange between concentrated and rarefied places in the medium, b – graduation of holes of perforation, [m], d – diameter of holes of the perforation, [m], h – thickness of perforated baffle tube, [m], $h_{\mathrm{ef}_{i,j}}$ – effective thickness of j–row of holes of the perforation in the i–tube (including the share of the mass of the medium adjoining the hole), [m]

$$h_{\mathrm{ef}} = h+0.85\left(1-\frac{d}{2b}\right)d. \tag{12}$$

Expressions presented above (11) are valid in the range of laminar flow of the medium through the holes of the perforation, i.e. when the following condition is fulfilled

$$R_e = \frac{\overline{V}_0 \cdot d}{\nu} < 10^3 \tag{13}$$

where: $\overline{V}_0$ – average flow velocity in holes of perforation, [m/s], or in a case of low flow velocities through holes of the perforation ($\overline{V}_0 < 0.5$ m/s) for a limited frequency range

$$f < 0.06c/d \tag{14}$$

6. Generalization of the model

The previously considered physical model assumed that individual rows of perforations in both tubes overlap perfectly. However, this model is a specific and rather rare case among actual technological solutions. In general we have to do with mixed connections, where beside the connection described by the model, also a connection between two out of three considered tubes can occur in individual rows of perforations.

The mathematical model of a transmission matrix K_j for these cases can be described with the same formulae, which have been presented in Table 1, when additional substitutions are applied:

– for a connection between tube 1 and 2 only

$$M_{2,j} = 0 \quad \text{and} \quad A_{2,j} = 0 \tag{15}$$

Table 6. Formulae for elements of the transmission matrix K_j for a case of a connection of tube 1 and 2, only, through openings of the perforation

K_{Kl}	Calculation formula	K_{Kl}	Calculation formula
K_{11}	$1-B_3G_1/E_1$	K_{21}	G_1/E_1
K_{12}	$B_1-B_3[1-B_1B_2-G_1)]/E_1$	K_{22}	$[1-B_1(B_2-G_1)]/E_1$
K_{13}	B_3G_1/E_1	K_{23}	$-G_1/E_1$
K_{14}	$B_3C_1G_1/E_1$	K_{24}	$-C_1G_1/E_1$
K_{15}	0	K_{25}	0
K_{16}	0	K_{26}	0
K_{31}	C_3G_1/E_2	K_{41}	$-G_1/E_2$
K_{32}	$B_1C_3G_1/E_2$	K_{42}	$-B_1G_1/E_2$
K_{33}	$1-C_3G_1/E_2$	K_{43}	G_1/E_2
K_{34}	$C_1-C_3[1-C_1(C_2-G_1)/E_2$	K_{44}	$[1-C_1(C_2-G_1)/E_2$
K_{35}	0	K_{45}	0
K_{36}	0	K_{46}	0
K_{51}	0	K_{61}	0
K_{52}	0	K_{62}	0
K_{53}	0	K_{63}	0
K_{54}	0	K_{64}	0
K_{55}	1	K_{65}	0
K_{56}	0	K_{66}	1

Constants B_i, C_i, D_i, E_i, G_i $i = 1, 2, \ldots$ – calculated according to formulae as in Table 1, when substitutions are introduced
$A_{2,j} = 0$ and $Mo_{2,j} = 0 \rightarrow G_3 = G_1$

– for a connection between tube 2 and 3, only

$$M_{1,j} = 0 \quad \text{and} \quad A_{2,j} = 0 \tag{16}$$

when these substitutions are applies, formulae are converted into the form presented in Tables 6 and 7.

Contents of the tables indicate that relations in force between parameters of neighbouring and connected in a given j–row of perforations two tubes are identical with parameters obtained for a model, in which only propagation in two tubes connected by one perforated baffle was considered [3]. While the multiplication by an identity matrix is performed in the case of the third tube (not connected with the other two tubes in the given j–row). This means that acoustic parameters in this particle remain unchanged and that this is in accordance with the accepted physical model.

Table 7. Formulae for elements of the transmission matrix K_j for a case of a connection of tube 2 and 3, only, through openings of the perforation

K_{Kl}	Calculation formula	K_{Kl}	Calculation formula
K_{11}	1	K_{21}	0
K_{12}	0	K_{22}	1
K_{13}	0	K_{23}	0
K_{14}	0	K_{24}	0
K_{15}	0	K_{25}	0
K_{16}	0	K_{26}	0
K_{31}	0	K_{41}	0
K_{32}	0	K_{42}	0
K_{33}	$1-C_3G$	K_{43}	G_2/E_2
K_{34}	$C_1-C_3[1-C_1(C_2-G_2)]/E_2$	K_{44}	$[1-C_1(C_2-G_2)]/E_2$
K_{35}	C_3G_2/E_2	K_{45}	$-G_2/E_2$
K_{36}	$C_3D_1G_2/E_2$	K_{46}	$-D_1G_2/E_2$
K_{51}	0	K_{61}	0
K_{52}	0	K_{62}	0
K_{53}	D_3G_2/E_3	K_{63}	$-G_2/E_2$
K_{54}	$C_1D_3G_2/E_3$	K_{64}	$-C_2G_2/E_2$
K_{55}	$1-D_3G_2/E_3$	K_{65}	G_2/E_3
K_{56}	$D_1-D_3[1-D_1(D_2-G_2)]/E_3$	K_{66}	$[1-D_1(D_2-G_2)]/E_3$

Constants B_i, C_i, D_i, E_i, G_i $i = 1, 2, \ldots$ — calculated according to formulae as in Table 1, when substitutions are introduced

$$A_{1,j} = 0 \text{ and } Mo_{1,j} = 0 \rightarrow G_3 = G_2$$

7. Conclusions

This paper presents an analytical model of a filter with two perforated tubes. A segmentation principle for reproducible segments of tubes, analogical to that accepted in a previously presented model [3] for one perforated tube was accepted here. In [3] properties of individual segments were described with a transmission matrix with the application of a discrete parameter to describe properties of acoustic particles related with openings of the perforation, and a model with distributed parameters to describe the propagation of an acoustic wave along sections of tubes between individual rows of perforations.

The presented above description of acoustic properties of discussed filters is very convenient for further computer processing, because transmission matrices related to the filters individual elements are catemary matrices for the filter configuration under consideration. Hence the transmission matrix for the whole filter can be easily achieved by multiplying the transmission matrix for succesive components. Values of generally applied attenuation measures transmission loss TL and insertion loss IL can be equally easily determined on the basis of the transmission matrix, if only the impedance of the sound source and its outlet from the tube of the muffler or installation is determined [3].

Simple substitutions make it possible to use the presented mathematical model for such cases, where there is no connection between tubes on both perforated surfaces in a single row of perforations at the same time. This simplifies the analytical programme.

The author has included a large number of possible particular solutions of filters in the mathematical model. They include fundamental combinations of closures of parts of tubes, applied in Table 3. Yet, it was possible to formulate a generalized mathematical model for elements of the transmission matrix $[T']$ with the use of not many calculation formulae. This also influenced the simplicity of the programme and calculation time.

The shortcoming of the presented model is that its application is limited to: small dimensions of holes of the perforation discrete parameter model, low flow velocities (laminar flow range) and bottom frequency range (conservation of propagation conditions of a plane wave and a harmonic acoustic wave).

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