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ACOUSTIC RADIATION OF CIRCULAR PLATE INCLUDING THE ATTENUATION EFFECT AND INFLUENCE OF SURROUNDINGS

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This paper presents an analysis of the acoustic power of a thin circular plate, which includes internal dissipation in the plate's material and the influence of the acoustic wave radiated by the plate on it's vibrations. A fully fixed plate was accepted, which vibrates in a rigid and flat acoustic baffle under the influence of a definite external pressure, sinusoidal in terms of time, and radiates into a lossless and homogeneous liquid medium. The factor which forced vibrations was accepted as fixed. The acoustic power was calculated on the basis of the known distribution of vibration velocity into a series of eigenfunctions. The power radiated by the plate was expressed in the form of a strongly convergent series, so the frequency characteristics could be determined. Also the effect of the plate's internal attenuation was estimated, as well as the influence of the acoustic field radiated by the plate on the modification of its vibrations.

1. Introduction

At the present stage of development of experimental and technical acoustics, theoretical investigations should be aimed at the analysis of models representing more closely actual vibrating systems. This means that the analysis of acoustic power radiated by a circular plate should include the effect of attenuation inside the plates material and the influence of the acoustic wave, generated by the plate, on the form of its vibrations.

The problem of acoustic power radiated by a thin circular plate has been given particular attention in the present decade. In paper [3] the author has analysed acoustic power of single axially-symmetric vibration modes of a circular plate for high frequencies. Levine and Leppington [1] have considered this subject in greater detail and included the term defining the oscillating character in the expression for the real component of power. Paper [4] presents acoustic interaction of the field radiated by a circular plate. It also contains an analytical assesment of mutual resistance for high frequencies. The acoustic power of a plate, including the factor forcing vibrations, but excluding losses and the influence of associated vibrations of the air column, has been considered in paper [5].

From among classical problems of generation and propagation of acoustic waves this paper takes up the problem of acoustic power of a thin circular plate, including the attenuation effect and the influence of the acoustic wave radiated by the plate on its vibrations. The external pressure, displacement and velocity of plates vibrations were expressed with familiar distributions into serieses with respect to a complete system of eigenfunctions. As a result, acoustic power was given in the form of a series with 3 expansion coefficients forming a system of linear algebraic equations. In a wide frequency range expansion coefficients can be calculated with the application of the method of successive approximations, for example.

Results of numerical calculations of the acoustic power, including the effect of internal attenuation and the influence of the acoustic field of the radiating plate on the form of its vibrations, are also presented in graphical form.

2. Assumptions of analysis

A fixed thin circular plate, excited to vibrate by an external pressure $\mathrm{Re}[f(r)\exp(-i\omega t)]$ for $0 \leqslant r \leqslant a$, is placed in a liquid medium with rest density ϱ_0 in a flat and rigid acoustic baffle. The analysis is based on the equation of vibrations (see [2])

$$B_0 \nabla^4 \eta(r, t) + M \frac{\partial^2 \eta(r, t)}{\partial t^2} = f(r, t) - R \frac{\partial}{\partial t} [\nabla^4 \eta(r, t)] - 2p(r, t) \tag{1}$$

where: η is the transverse dislocation of a point on the plates surface, R – loss of plates material, M – plates mass per surface unit, B_0 – plates bending rigidity, $p(r, t)$ – acoustic pressure radiated by plate for $z = +0$. Because accepted processes are sinusoidal with respect to time, we have: $p(r) = i p_0 \omega \varphi(r)$, $v(r) = -i\omega\eta(r)$, and instead of Equation (1) we obtain the following equation of vibrations [1]

$$(\nabla^4 k_p^{-4} - 1) v(r) + 2\varepsilon_1 k_0 \varphi(r) = -\frac{i}{M\omega} f(r) \tag{2}$$

where: $\varphi(r)$ is the amplitude of the acoustic potential, $B = B_0(1 - i\omega R/B_0)$, $\varepsilon_1 = \varrho_0/(Mk_0)$, $k_0 = 2\pi/\lambda$, $k_p^4 = M\omega^2/\beta$.

The velocity of the plates vibrations is presented in the form of a series

$$v(r) = \sum_n c_n v_n(r) \tag{3}$$

where

$$v_n(r) = \mathrm{v}_{0n} \left[J_0(\gamma_n r/a) - \frac{J_0(\gamma_n)}{I_0(\gamma_n)} I_0(\gamma_n r/a) \right]$$

for $v_{0n} = [a]\, J_0(\gamma_n)]^{-1}$ presenting the complete, orthonormal system of eigenfunctions of a homogeneous equation $(\nabla^4 k_n^{-4} - 1)\, v_n = 0$, while $\gamma_n = k_n a$ is a solution to the frequency equation $J_0(\gamma_n) I_1(\gamma_n) + J_1(\gamma_n) I_0(\gamma_n)$, $n = 1, 2, \ldots$ (see [2, 3]).

Expansions coefficients c_n are determined from a system of algebraic equations [1]

$$c_n\left(\frac{k_n^4}{k_p^4} - 1\right) - 2\varepsilon_1 \sum_m i\zeta_{mn} c_m = f_n \tag{5}$$

where

$$f_n = -\frac{i}{M\omega} \int_0^a f(r) v_n(r) r dr \tag{6}$$

while [1, 4]

$$\zeta_{mn} = 4\gamma_m^3 \gamma_n^3 \int_0^\infty \frac{x}{\gamma}\left(\frac{a_m J_0(x) - x/\gamma_m J_1(x)}{x^4 - \gamma_m^4}\right)\left(\frac{a_n J_0(x) - x/\gamma_n J_1(x)}{x^4 - \gamma_n^4}\right) dx \tag{7}$$

is an expression for the normalized mutual impedance of axially-symmetrical modes of the same circular plate in a case of free vibrations. Quantity γ is defined as follows:

$$\gamma = [1 - (x/k_0 a)^2]^{1/2} \quad \text{for } 0 \leqslant x \leqslant k_0 a \text{ and } \gamma = i\,[(x/k_0 a^2) - 1]^{1/2}$$

for $k_0 a \leqslant x < \infty$ while $a_n = J_1(\gamma_n)/J_0(\gamma_n)$.

3. Calculation of acoustic power

The acoustic power of the vibrating system under consideration is calculated according to definition

$$N = 1/2 \int_\sigma p(r) v^*(r) d\sigma \tag{8}$$

where [3]

$$p(r) = \varrho_0 c k_0^2 \int_0^{\pi/2 - i\infty} J_0(k_0 r \sin\vartheta) \Big[\int_0^a v(r_0) J_0(k_0 r_0 \sin\vartheta) r_0 dr_0\Big] \sin\vartheta d\vartheta \tag{9}$$

quantity c is the velocity of wave propagation in a medium with density ϱ_0 and $v(r_0)$ is calculated from formula (3).

The characteristic function of a circular plate for a spherical distribution of velocity of vibrations (3) is defined as follows [4]:

$$W(\vartheta) = \Sigma c_n W_n(\vartheta) \tag{10}$$

where

$$W_n(\vartheta) = \int\limits_0^a v_n(r_0) J_0(k_0 r_0 \sin\vartheta) r_0 dr_0 \tag{10'}$$

is the plates partial characteristic function, i.e. when the plate is excited to vibrate with free vibrations $v_n(r_0)$ for an axially-symmetric mode of vibrations $(0, n)$. After integrating (10′) we obtain

$$W_n(\vartheta) = 2a\gamma_n^3 \frac{a_n J_0(x) - x/\gamma_n J_1(x)}{\gamma_n^4 - x^4} \tag{10''}$$

where $x = k_0 a \sin\vartheta$.

The acoustic power (8) is equal to

$$N = \pi \varrho_0 c k_0^2 \sum_n \sum_s c_n c_s^* \int\limits_0^{\pi/2 - i\infty} W_n(\vartheta) W_s^*(\vartheta) \sin\vartheta d\vartheta \tag{11}$$

or finally, after expressions (10″) and (7) are included, we reach

$$N = \varrho_0 c\pi \sum_n \sum_s c_n c_s^* \zeta_{ns}. \tag{11'}$$

This expression is converge convenient for numerical calculations, because the series are very strongly convergent with respect to values of normalized impedances ζ_{ns}. A detailed analysis and methods of calculating quantity ζ_{ns} in specific cases are presented in papers [1, 3] and [4].

It is convenient to use the convenient to use the notion of relative acoustic power N/N_0 in numerical calculations, where N_0 denotes the plates active power for $k_0 \to \infty$. If $k_0 \to \infty$, then $p(r) = \varrho_0 c v(r)$, hence on the basis of formula (8) we have

$$N_0 = \pi \varrho_0 c \int\limits_0^a v^2(r) r dr, \tag{12}$$

and in a case of velocity of vibrations (3)

$$N_0 = \pi \varrho_0 c \sum_n c_n^2. \tag{12'}$$

Also a representation for acoustic power different from expression (11′) is possible. Both sides of the system of Equations (5) are multiplied by c_s^* and summed up with respect to index s, and expression (11′) is used, then we finally obtain the following formula for the acoustic power of a circular plate

$$N = \frac{i\varrho_0 c\pi}{2\varepsilon_1} \sum_n \left[c_n^* f_n - c_n^2 \left(\frac{k_n^4}{k_p^4} - 1 \right) \right]. \tag{13}$$

In comparison to expression (11′) this formula is less convenient for numerical calculations, because the series occuring here is slowly convergent in general.

4. Numerical example

The acoustic power of a circular plate was numerically calculated under the assumption that the amplitude distribution of the external pressure exciting the plate to vibrate has the following (from see [5]):

$$f(r) = \begin{bmatrix} f_0 & \text{for } 0 < r < a_0 \\ 0 & \text{for } a_0 < r < a \end{bmatrix} \tag{14}$$

where $f_0 = \text{const}$. In this case according to definition (6) the n Fourier coefficient f_n is equal to

$$f_n = -i\frac{af_0\varepsilon_0}{M\omega}\frac{J_1(\varepsilon_0\gamma_n)}{\gamma_n J_0(\gamma_n)}\left[1+\frac{J_1(\gamma_n)}{J_1(\varepsilon_0\gamma_n)}\frac{I_1(\varepsilon_0\gamma_n)}{I_1(\gamma_n)}\right] \tag{15}$$

where $\varepsilon_0 = a_0/a$.

If the entire surface of the plate is excited to vibrate by a factor not equal to zero, then for $a_0 = a$ we have

$$f_n = -i\frac{2af_0}{M\omega}\frac{a_n}{\gamma_n}. \tag{15'}$$

The acoustic power was calculated from formula (11′) and Fourier coefficients c_n were calculated from the system of equations (5). For a wide frequency range coefficients c_n can be calculated with approximate methods, e.g. method of successive approximations. In the case under consideration it was sufficient to limit calculations to the (2) approximation, and to the first few terms of expansion in formula (5) $(n, s \leqslant 4)$.

The notion of relative apparent power was applied in numerical calculations and graphical illustrations, i.e. apparent power $N' = [N_x^2+N_y^2]^{1/2}$ is related to quantity $N'_0 = \pi a^2 \varrho_0 c(f_0/M\omega_1)^2$, where $N = N_x - iN_y$. The influence of the interaction of the acoustic field radiated by the plate on the form of its vibration is characterized with the constant $\varepsilon'_1 = 2\varrho_0 c/(M\omega_1)$ while $\varepsilon_1 = 0.5\varepsilon'_1\omega_1/\omega$ $\omega_1 = \sqrt{B_0/M}(\gamma_1/\mathrm{a})^2$. The quantity $\varepsilon'_2 = \omega_1 R/B_0$, was accepted as the measure of internal attenuation in the plates material. Also parameter $b = k_0 a/(ka)^2$ $= (h/2a)\cdot\sqrt{E/[3\varrho c^2(1-v^2)]}$, was introduced. It characterizes the quotient of the plates thickness h to its diameter $2a$, E is the Young's modulus ϱ — density of plate's material, v — Poisson's ratio. This parameter has a limited top value, because of the analysis of the problem for thin plates and adequate values of material constants.

The numerical example includes several different values of the a_0/a parameter. This makes it possible to estimate the effect of the external pressure, which forces vibrations, on the value of radiated power. Also such a_0/a values were chosen which correspond with the plates volumetric displacements equal to zero. These values are equal to 0.5995, 0.7373, 0.4064, respectively for the second and third resonance frequency.

5. Conclusions

Expressions for acoustic power radiated by a thin circular plate were derived within the performed theoretical analysis.

They included:

1) external pressure forcing the plates vibrations,

2) effects of internal dissipation,

3) the influence of the acoustic wave, radiated by the plate on the form of its vibrations.

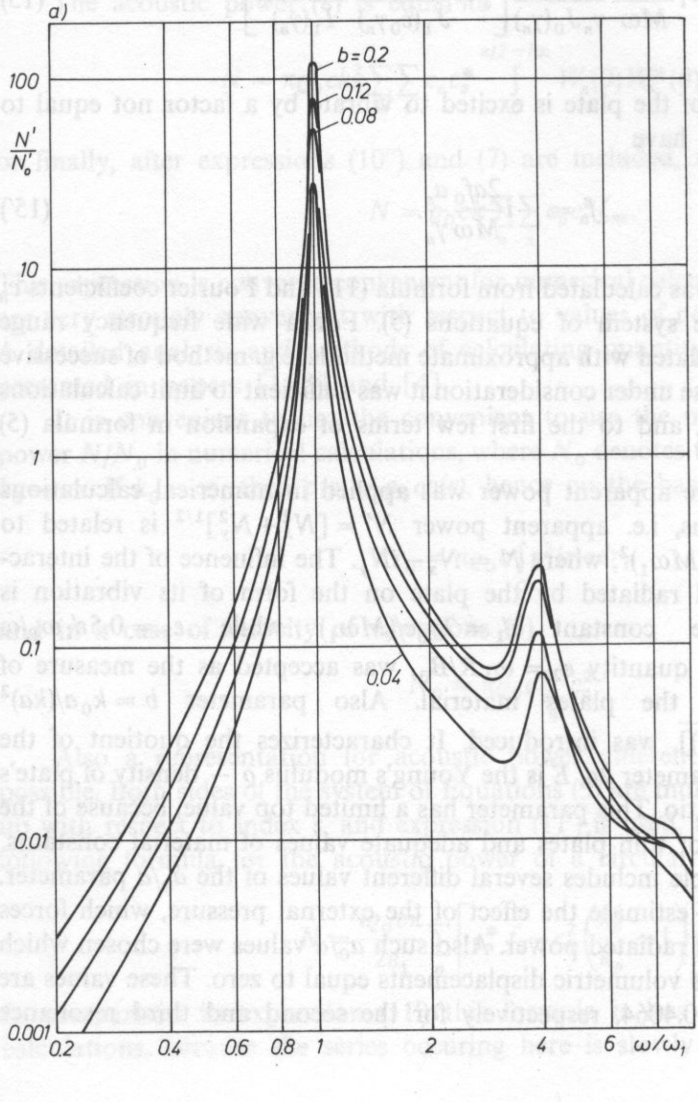
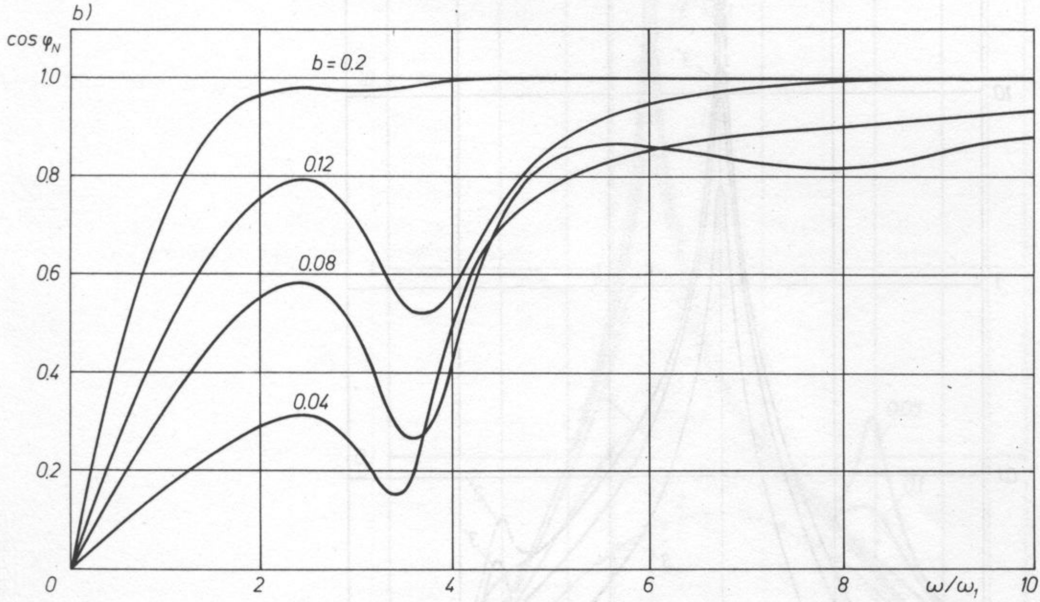


FIG. 1. Relative frequency characteristics of apparent power (a) and power factor (b) of circular plate. For $a_0/a = 1$, $\varepsilon_1' = 0$, $\varepsilon_2' = 0.05$



[Fig. 12]

The value of radiated power is significantly depend upon the distribution of the external pressure, which forces the plates vibrations. The analysed example proves that an increase of the central surface excited to vibrate is accompanied by an increase of the value of radiated power (see Figs. 2 a, b).

It was stated that the frequency characteristics of radiated power (Figs. 1 a, b) depends on the plates thickness, its diameter and material constants. An increase of the value of acoustic power accompanies and increase of the quotient $h/2a$ in the range between zero and the second resonance frequency. Frequency bands with radiation efficiency decreasing for a definite value of parameter b are distinctly outlined.

A clear influence of internal attenuation on the radiated acoustic field occurs for bands around resonance frequencies (see Figs. 2 and 3). The influence of attenuation decays for volumetric displacement of the vibrating plate equal to zero (curve 1 in Figs. 2a, b).

Attenuation resulting from the interaction of the acoustic wave radiated by the plate has a minor effect. A narrow band near the first resonance frequency is an exception. Damping of the fluid medium, surrounding the plate, influence the value of the radiated energy of the acoustic field in this band. (see Fig. 4).

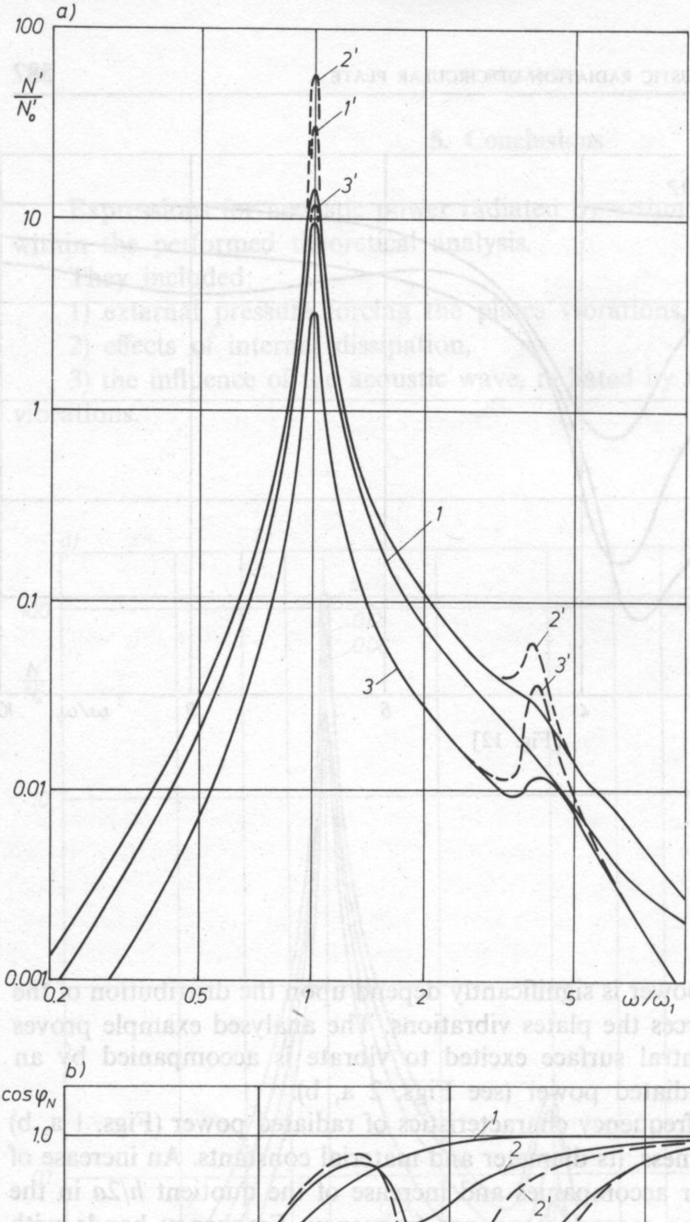
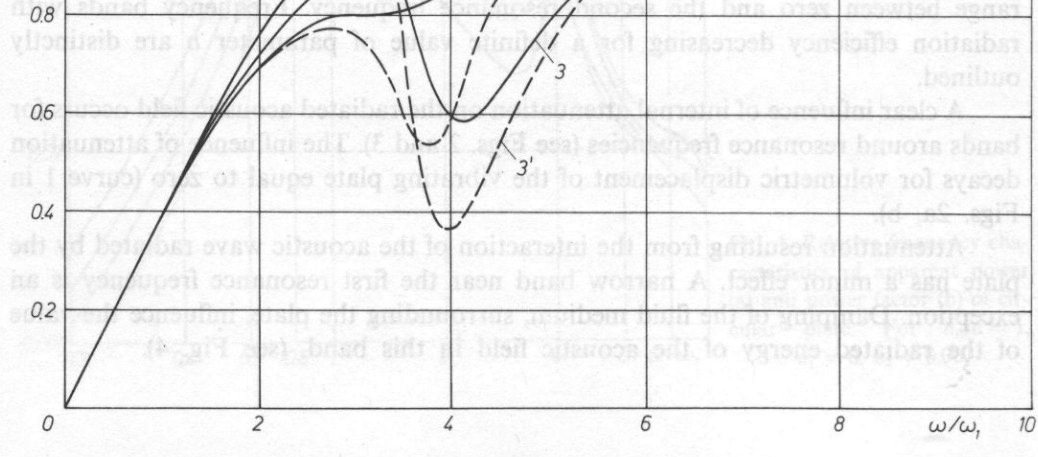


FIG. 2. Relative frequency characteristics of apparent power (a) and power factor (b) of circular plate. For $b = 0.1$; $\varepsilon'_1 = 0$ 1 – $a_0/a = 0.5995$; 2 – $a_0/a = 0.7373$; 3 – $a_0/a = 0.4064$ – – – – – – $\varepsilon_2 = 0.05$ ——— $\varepsilon_2 = 0.1$



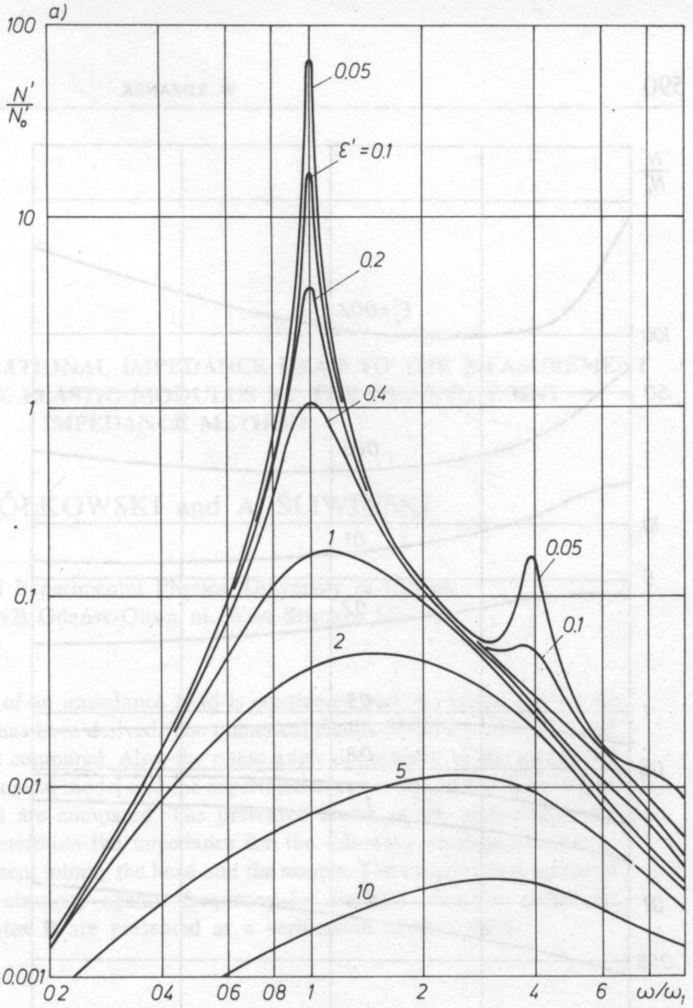
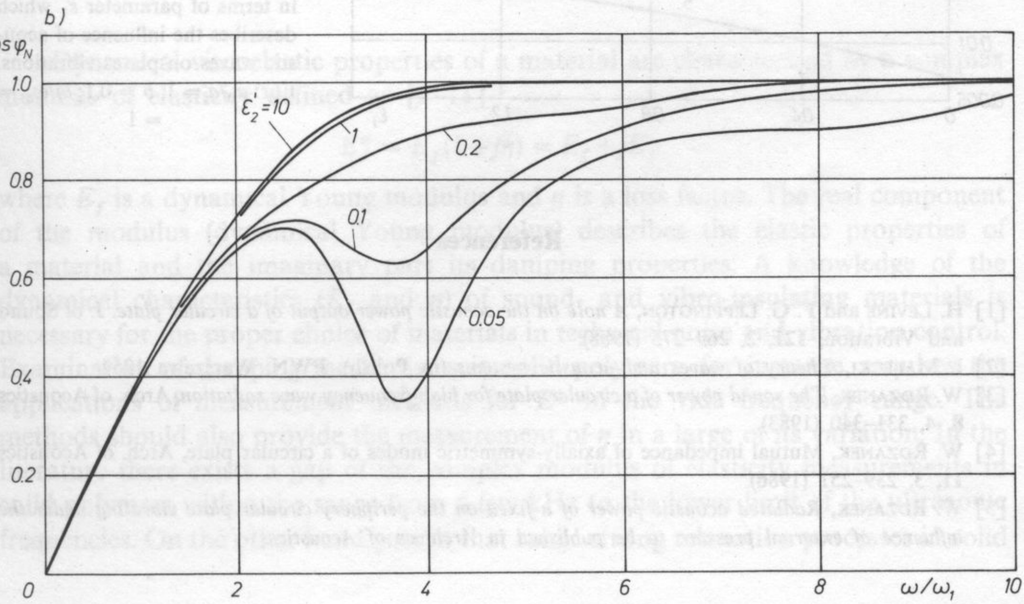


FIG. 3. Relative frequency characteristics of apparent power (*a*) and power factor (*b*) of circular plate. For: $a_0/a = 1$; $b = 0.1$; $\varepsilon'_1 = 0$



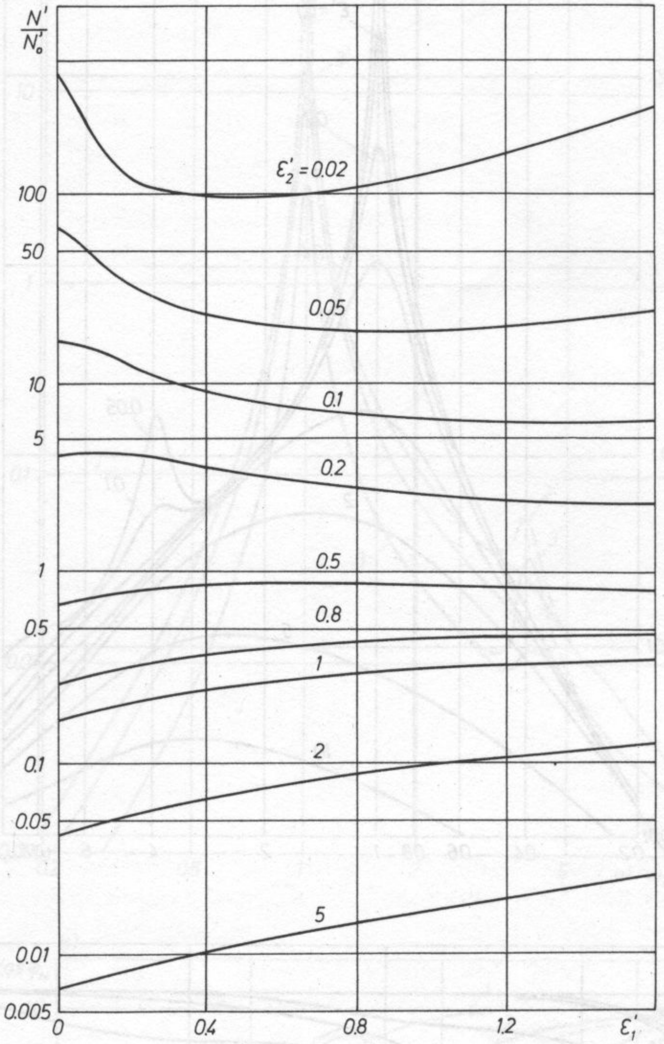


FIG 4. Relative apparent power in terms of parameter ε'_1 which describes the influence of acoustic waves on plates vibrations. For: $a_0/a = 1$, $b = 0.1$; $\omega/\omega_1 = 1$

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