APPLICATION OF A VIBRATIONAL IMPEDANCE HEAD TO THE MEASUREMENT OF THE COMPLEX ELASTIC MODULUS BY THE DRIVING POINT IMPEDANCE METHOD

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A rheological model of an impedance head is considered and the expression for its mechanical impedance Z_H has been derived. The numerical results of the modulus $|Z_H|$ and the experimental data were compared. Also, the phase angle dependence of the measuring head Φ_{num} for a given rheological model and the experimental curve Φ_{exp} of the Brüel-Kjaer type 8001 impedance head are compared. The presented model of the measuring head impedance was used to determine the impedance for the following mechanical system: beam–measuring head–element joining the head and the sample. The experimental results of the complex modulus of elasticity against frequency for samples of a few polymers: plexiglass, PVC and Razotex B are presented as a verification of the model.

1. Introduction

Dynamical viscoelastic properties of a material are characterized by a complex modulus of elasticity defined as [1–11]

$$E_f^* = E_f(1+j\eta) = E_f + jE_f'$$

where E_f is a dynamical Young modulus and η is a loss factor. The real component of the modulus (dynamical Young modulus) describes the elastic properties of a material and the imaginary part its damping properties. A knowledge of the dynamical characteristics (E_f and η) of sound- and vibro-insulating materials is necessary for the proper choice of materials in technical noise and vibration control. Examination of damping mechanisms in solid polymers, for instance, requires the applications of measurement methods for E^* in the vide frequency range. The methods should also provide the measurement of η in a large of its variation. In the literature there exists a gap of the complex modulus of elasticity measurements in solid polymers within the range from a few kHz to the lower limit of the ultrasonic frequencies. On the other hand, just in that range strong relaxation processes in solid

polymers are observed due to the macromolecular structure of a polymer. For this reason, a good and reliable method of the complex modulus of elasticity with the possibility of evaluating exactly the experimental errors seemed to be necessary and useful.

The point impedance of a beam method permits to determine the complex modulus of elasticity in the relatively wide range of frequency, namely between 20–15000 Hz and to obtain η values in the range of 10^{-4}–1.0. Theoretical considerations on the application of the method based on the Bernoulli-Euler beam theory were performed by J. C. SNOWDOWN [3]. His results have been applied in the experimental method for measurements of visco elastic beams using the impedance head [1, 2]. In the paper [2] the influence of the inaccuracy in determining the midpoint position of a beam sample on the measurement of the complex elastic modulus E^* when using the driving sinusoidal force in the point impedance method was discussed in details. Now, the possibilities and limits of the method will be considered. In particular, two problems are important to determine the complex elasticity modulus of the beam, namely, the influence of the mechanical impedance and the phase angle of the measuring head. In the paper the numerical results for $|Z_u|$ and Φ_n as a function of the nondimensional parameter $(na) \sim \sqrt{\omega}$ for large values of the loss factor are presented.

In the theoretical considerations an approximative rheological model of the impedance head was applied and experimentally verified. The impedance head model was implemented in further considerations to determine the mechanical impedance of the total system: the beam sample, the impedance head including the element joining the head and the sample. Consequently, the formulas for the modulus and the phase angle of the impedance of the system were derived. In the experimental part the measuring setup is described and the results are presented together with the error evaluation.

2. Theory and numerical results

The mechanical point impedance of the system: beam sample-impedance head Z_u is the sum of the beam point impedance Z_0 [1, 3] and the head impedance Z_H at the driving point (Fig. 1):

$$Z_u = Z_0 + Z_H \tag{1}$$

The measurement method for small values of the loss factor of viscoelastic beams driving by a sinusoidal force to excite bending vibrational modes was described
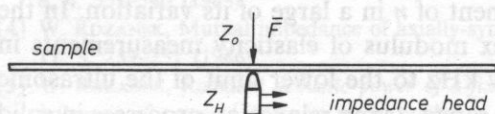

FIG. 1. The system: sample — impedance head

in the papers [1, 2]. The influence of the reactant mass of the support situated above the force transducer of the impedance head on the determination of E^* was discussed. In the case of samples of larger values of the mechanical impedance modulus $|Z_0|$ for the (antyresonance frequencies or for samples having masses much larger than the mass situated above the force transducer of the head), the influence of the reactant mass can be neglected. In the case when the ratio of those masses is significant, the component Z_H becomes comparable to the component Z_0 and neglecting Z_0 leads to great errors in determining the complex modulus of elasticity. The error is the greatest when the measurement takes place for the resonance frequency. For the measurement at the antyresonance frequencies, the error is about 10% when $\left|\dfrac{Z_H}{Z_u}\right| < 0.1$.

2.1. The mechanical impedance of the measuring head

The subject of the theoretical analysis as well as the examinations was the Bruel and Kjaer type 8001 impedance head. In the head the piezoelectric element is joined with a movable support of the mass of 10^{-3} kg. The support is connected elastically with the head casting with a silicon rubber ring. Its influence is not only the kind of the mass; also, its elasticity and viscosity play an important role. When masses of samples are several times greater than the mass of the movable support, the influence of the impedance of the tightening ring on the accuracy of determination of the loss factor η, as will be seen, is essential, particularly when it is measured for resonance frequencies. The producer of the head does not say anything about the influence of that ring on the phase characteristics and as far as the authors know there is nothing to find on the matter in the literature.

The approach which takes into account the additional influence of the impedance of the ring in the total impedance of the head gives reasons for the necessity to enlarge the measuring range of the complex modulus of elasticity in the frequency range, particularly in higher frequencies. In experiments very thin samples (about 10^{-3} m) and narrow (10^{-2} m) are used. The limits of lateral dimensions of a sample result from the assumptions of the Bernoulli-Euler theory for bending waves of a beam [1]. The small mass of samples becomes comparable with the mass of the element above the force transducer and the tightening ring of the head. Therefore, the mechanical impedance of the head must be taken into account when measuring the mechanical impedance of the sample-beam.

The mechanical impedance of the head can be described by the system of rheological elements a spring, a mass and a damper connected by the way presented in Fig. 2.

The ring tightening the impedance head is fixed to the edge of a movable mass M_p situated above the piezoelectric element and can be treated as a dynamic absorber. The mechanical impedance of the ring (dynamic absorber) Z_R can be
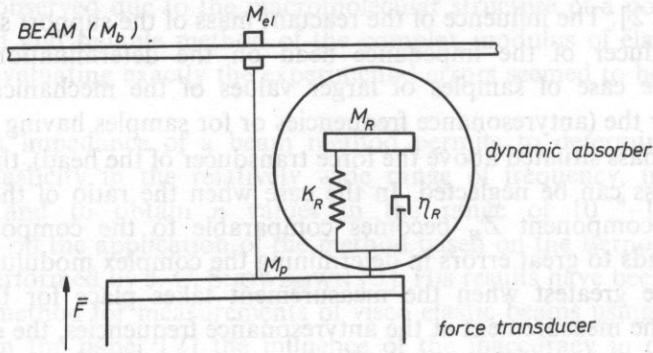
FIG. 2. Mechanical rheological model of the impedance head type 8001, Bruel-Kjaer

written as

$$Z_R = \left[\frac{1}{j\omega M_R} + \frac{1}{\frac{K_R}{j\omega} + \eta_R} \right]^{-1} \tag{2}$$

where M_R is the mass of the silicon ring, K_R is its stiffness, η_R its loss factor.

So, one can write in the first approximation the expression for the complete mechanical impedance of the head Z_H as follows:

$$Z_H = j\omega M_p + Z_R = \frac{(M_p + M_R)K_R - \omega^2 M_R M_p + j\omega \eta_R (M_p + M_R)}{\eta_R + j\omega(M_R - K_R/\omega^2)} = \frac{R_N + jI_N}{R_D + jI_D} \tag{3}$$

where M_p is the mass above the piezoelectric element. The eventual mechanical coupling between the piezoelement and the casting was not taken into account.

The modulus of the mechanical impedance of the head is equal to

$$|Z_H| = \left[\frac{R_N^2 + I_N^2}{R_D^2 + I_D^2} \right]^{1/2} \tag{4}$$

where

$$R_N = (M_p + M_R)K_R - M_R M_p \omega^2$$

$$I_N = \omega \eta_R (M_p + M_R)$$

$$R_p = \eta_R$$

$$I_p = \omega \left(M_R - \frac{K_R}{\omega^2} \right)$$

For a quantitative evaluation of the influence of the impedance of the ring against the frequency, one can take the value of 1000 Hz up to which the influence is small and above that it becomes more and more significant.

Dividing the numerator and the denominator of the expression by K_R^2 and substituting $k_R \approx 10^7$ N/m and $\eta \approx 40$ kg/s for $f < 1000$ Hz, one obtains for the modulus of the reactant mass

$$Z_H \approx (M_P + M_R)\omega \tag{5}$$

It is seen from Eq. (5) and Fig. 2 that for frequencies less than 1000 Hz the modulus of the mechanical impedance of the head is equal to the modulus of reactance of the system: movable support − tightening ring. By measuring the modulus of the reactance head Z_H in the low frequency, range one can determine the dynamical mass of the ring M_R, namely,

$$|Z_H| = 2\pi f (M_R + M_P) = \frac{U_{RMS}}{C V_{RMS}}, \tag{6}$$

where U_{RMS} is the root mean square voltage on the force transducer. V_{RMS} − the root mean square vibrational velocity, and C is the sensitivity of the transducer.

The dynamical mass M_R was experimentally determined as equal to 1.4 x 10^{-3} kg.

The stiffness k_R may be calculated from the expression of the antyresonance frequency of the head which can be obtained from Eq. (4) by putting the first derivatives equal to zero, namely $|Z'| = 0$. The antyresonance frequency of the head was measured (Fig. 3a) as $f_a = 1.4$ x 10^4 Hz. Having Z_H and f_a as well as the values M_R, M_p and η, one can use the formula (4) to calculate $k_R = 2.38$ x 10^7 N/m.

For further experimental vrification of the assumed theoretical rheological model of the head the numerical calculations of the modulus of the mechanical impedance of the head were performed.

The values assumed above for the viscoelastic elements of the head, namely, k_R, M_R, η and M_p were introduced into Eq. (4). Figure 3 represents the results of the experimental measurement (a) in comparison with the numerical ones (b) of the
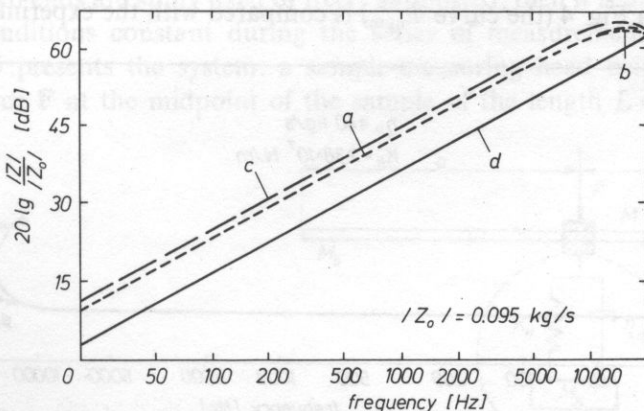

FIG. 3. Mechanical impedance modulus level of the impedance head against frequency

modulus level of the mechanical impedance of the head, as $20 \log \left| \dfrac{Z_u}{Z_0} \right|$, for different excitation velocities: 0.5; 2.0; 5.0 mm/s.

The experimental curve (a) and the theoretical one (b) are overlapped in the low frequency range. A small difference, about 0.5 dB, occurs at the range of the antyresonance frequency; however, both curves have the same behaviour. It results from Fig. 3 that the impedance of the tightening ring (the curves (a) and (b)) contributes significantly to the total impedance of the head when compared with the curve (d) calculated assuming the ring impedance is neglected.

The curve (c) in Fig. 3 represents the total mechanical impedance of the system: the head — the element which joins the head and the sample.

It results from the theoretical analysis and the experimental examinations that the contribution from the impedance of the measuring head should be taken into account when one determines the visco-elastic parameters from the experimental data of the modulus of the sample impedance.

2.2. Phase angle of the measuring head

The influence of the tightening ring appears also as an additional phase shift between the vibrational velocity and the exciting force. The phase angle of the impedance head can be expressed as the ratio of the imaginary part to the real part of the head impedance Z_H.

After corresponding transformation of the formula (3), and according to the definition, one obtains the following expression for the phase angle of the impedance of the head:

$$\Phi = \text{arctg} \frac{I_N R_D - R_N I_D}{R_N R_D + I_N I_D}. \tag{7}$$

The calculated phase angle for the assumed rheological model of the impedance head presented in Fig. 4 (the curve Φ_{num}) is compared with the experimental one (the
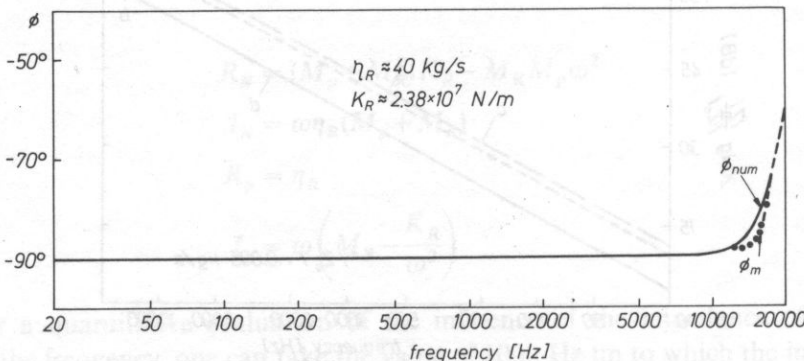

FIG. 4. Phase angle of the measuring impedance head

curve Φ_{pom}). Both curves Φ_{num} and Φ_{pom} overlap each other up to 14 kHz within the experimental error of the phase measured with a phasemeter of the accuracy $\pm 1°$. Above the range of 14 kHz, the divergence of the curves is observed due to the resonance range of the B and K vibrator (type 4809) used and to the resonance of the impedance head.

This is worth mentioning, too, that in the consideration of the impedance head model from which the formulas for mechanical impedance and for the phase angle were derived, the coupling between the casting and the accelerometer as well as the transducer of the force has been neglected. It is also possible that the coupling is responsible for the divergence of both curves in Fig. 4 above 14 kHz.

In materials of great values of η the modulus of the point impedance in the vicinity of resonance and antiresonance frequencies of the beam is a function of diffused extrema. The function $|Z_u(f)|$ in the increase of the frequency has a small dynamic of variations of its value for the consecutive resonance and antiresonance frequencies. So, to determine the loss factor on the basis of experimental data for materials of great internal losses, namely $\eta > 0.1$, it is necessary to measure not only the modulus of the impedance but also the phase Φ between the vibrational velocity and the force in the driving point.

2.3. The point impedance of the system: beam sample — measuring impedance head

Analyzing the application of the point impedance method to the complex modulus of elasticity measurements of solid materials, a beam shape sample was considered, two ends of which were free and driven at its midpoint with the harmonic force to obtain bending vibrations. The choice of such boundary conditions was dictated by the fact that their practical realization is relatively simple. It is easier and more reproducible to fix a sample in those conditions comparing, for instance with cases when two ends are stiffly fixed or freely supported; then it is difficult to keep the boundary conditions constant during the series of measurements.

Figure 5 presents the system: a sample-measuring head being driven by the harmonic force $\mathbf{F}$ at the midpoint of the sample of the length $L = 2a$. Mechanical
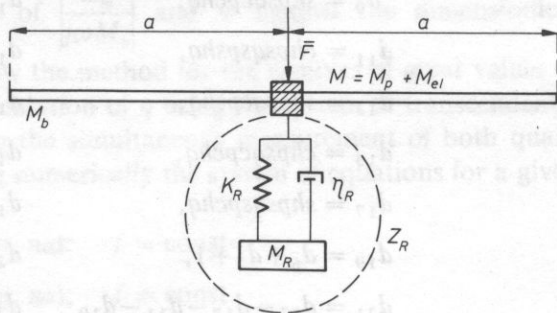

FIG. 5. The system: sample — head driven with the harmonic force to bending vibrations

point impedance of the system: beam-head is the sum of the point impedance of the beam Z_0 and the impedance of the measuring head Z_H:

$$Z_u = Z_0 + Z_H = Z_0 + Z_R + j\omega M \qquad (8)$$

where Z_R is the mechanical impedance of the tightening ring, $M = M_p + M_{el} + M_b$, M_p is the mass of the force transducer and M_{el} the mass of the element joining the head with the sample, M_b the mass of the sample.

The expression for the modulus of the normalized point impedance of the total system is as follows:

$$\left|\frac{Z_u}{j\omega M_b}\right| = \left\{\frac{X_N^2 + Y_N^2}{X_D^2 + Y_D^2}\right\}^{1/2} \qquad (9)$$

and for the phase angle

$$\Phi_u = \text{arctg}\,\frac{Y_N X_D - X_N Y_D}{X_N X_D + Y_N Y_D} \qquad (10)$$

where

$$X_D = d_{18}d_{24} - d_{19}d_{23},$$
$$Y_D = d_{18}d_{23} + d_{19}d_{24},$$
$$X_N = d_{18}d_{26} + d_{19}d_{25} + d_4 d_{20} + d_5 d_{21},$$
$$Y_N = -d_{18}d_{25} + d_{19}d_{26} + d_4 d_{22} + d_5 d_{20}$$

and

$$d_1 = (M + M_R)k_R, \qquad\qquad d_2 = \omega^2 M M_R,$$
$$d_3 = \omega \eta_R (M + M_R), \qquad\qquad d_4 = \omega M_l \eta_R,$$
$$d_5 = \omega^2 M_b\left(M_R - \frac{k_R}{\omega^2}\right), \qquad d_6 = chpcqcpchq,$$
$$d_7 = shpsqspshq, \qquad\qquad d_8 = chpcqspshq,$$
$$d_9 = shpsqcpchq, \qquad\qquad d_{10} = shpcqcpchq,$$
$$d_{11} = chpsqspshq, \qquad\qquad d_{12} = chpcqspchq,$$
$$d_{13} = shpsqcpshq, \qquad\qquad d_{14} = shpcqspshq,$$
$$d_{15} = chpsqcpchq, \qquad\qquad d_{16} = chpcqcpshq,$$
$$d_{17} = shpsqspchq, \qquad\qquad d_{18} = d_8 - d_9,$$
$$d_{19} = d_6 + d_7 + 1, \qquad\qquad d_{20} = d_{14} - d_{15} - d_{16} - d_{17},$$
$$d_{21} = d_{13} - d_{12} - d_{11} - d_{10}, \qquad d_{22} = d_9 + d_{11} + d_{15} - d_{13},$$

$$d_{23} = pd_5 + qd_4, \qquad\qquad d_{24} = pd_4 - qd_5,$$
$$d_{25} = pd_1 - pd_2 - qd_3, \qquad d_{26} = qd_1 - qd_2 + pd_3.$$

The parameters p and q are determined as

$$p = na\left(\frac{E_0}{E_f}\right)^{1/4}\left[\frac{1}{2\sqrt{D}} + \frac{(1+D)^{1/2}}{2\sqrt{2D}}\right]^{1/2},$$

$$q = -na\left(\frac{E_0}{E_f}\right)^{1/4}\left[\frac{1}{2\sqrt{D}} - \frac{(1+D)^{1/2}}{2\sqrt{2D}}\right]^{1/2},$$

where

$$D = (1+\eta^2)^{1/2}.$$

The dynamical Young modulus E_f is given by the formula

$$E_f = \frac{\omega^2 \varrho a^4 A}{(na)^4 I} \tag{11}$$

where ϱ is the density of the material, A is the lateral cross section of the beam sample, I is the momentum of of the lateral cross section, (na) is the dimensionless parameter proportional to $\sqrt{\omega}$, $n = \sqrt{\dfrac{\omega^2 \varrho A}{E_f I}}$ is the wave number and a is the half of the length of the beam.

The Young modulus E_0 is the reference value equal to the dynamical Young modulus for a given reference frequency f_0. Usually, the first antiresonant frequency of the beam is chosen as f_0.

Using the formulas (9)–(11) numerical calculation of the modulus of the point impedance and of the phase angle of the system: beam-head being driven at the midpoint were performed for great values of the loss factor (the formula (10)). For the calculations the following values were taken: $\gamma = \dfrac{M}{M_b} = 0.1$ and $\eta = 0.1$ (Fig. 6a), $\eta = 1.0$ (Fig. 6b) and $\eta = 5.0$ (Fig. 6c).

In the numerical analysis the viscosity η_R and the stiffness k_R of the tightening ring of the head was neglected.

The behaviour of the curves of $\left|\dfrac{Z_m}{j\omega M_b}\right|$ and Φ against the dimensionless parameter $(na) \sim \sqrt{\omega}$ points out how the method for the samples of great values of η should be chosen. The correct calculation of η using the system of transcendental equations is possible by performing the simultaneous measurement of both quantities, namely $|Z_u|$ and Φ and solving numerically the system of equations for a given frequency:

$$\eta = h(\eta, na); \quad f = \text{const}$$
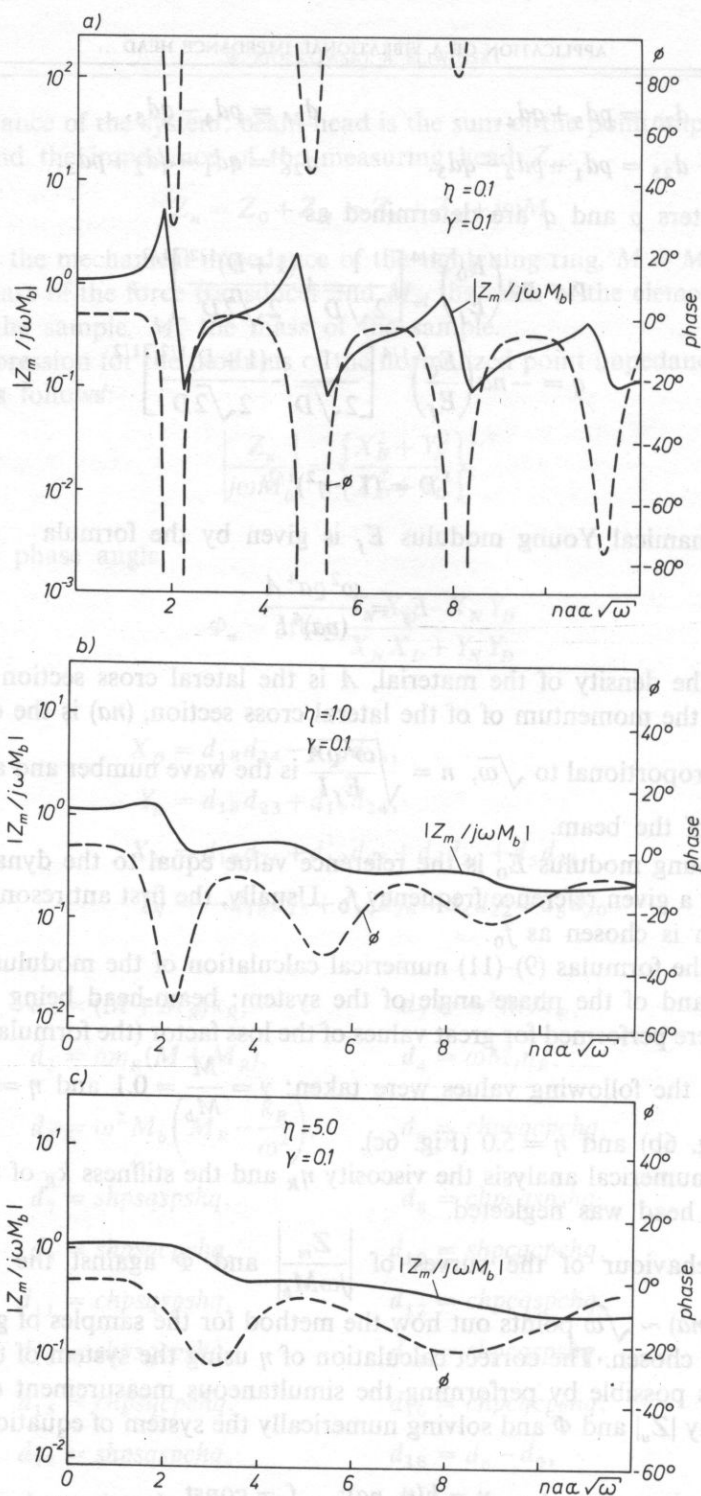$$\Phi = g(\eta, na); \quad f = \text{const}.$$

FIG. 6. Dependance of the impedance modulus and the phase angle against the frequency: a) $\eta = 0.1$; b) $\eta = 1.0$; c) $\eta = 5.0$

3. Experimental results

The experimental results presented in this paper are based on the first approximation, i.e., assuming that the mechanical impedance of the measuring head is equal to the reactance of the mass of component elements of the impedance head, namely, to the reactance of the mass situated above the force transducer and the reactance of the mass of the tightening ring (Eqs. (9)–(10)).

Solid polymer methyl-polymethacrilate (plexiglass) was chosen for measurements because the substance was already examined by many authors who applied various measuring methods to determine the complex modulus of elasticity in dependance on frequency and temperature [4–11].

The complex modulus of elasticity was numerically calculated for three following mass ratio:

1) $\gamma = \dfrac{M_{el} + M_p + M_R}{M_b}$

2) $\gamma = \dfrac{M_{el} + M_p}{M_b}$ (the contribution of the ring is neglected)

3) $\gamma = 0$ (the total contribution of the impedance head is neglected, i.e., $|Z_H| = 0$.

The measuring setup is presented in Fig. 7. The samples of plexi glass had three different dimensions (obtained by the consecutive shortening by cutting out (305, 285, 265 x 10.76 x 1.99) x 10^{-3} m^3. All measurements were performed at room temperature.
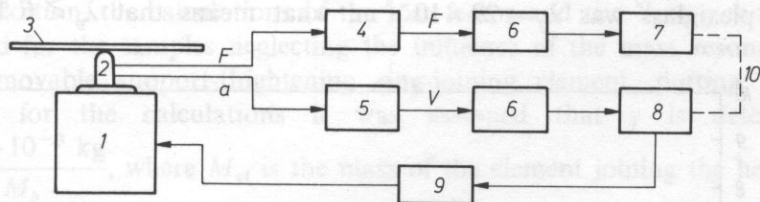

FIG. 7. Measuring arrangement. 1. vibration exciter Bruel-Kjaer type 4809; 2. impedance head B-K type 8001; 3. beam being tested; 4. preamplifier; 5. preamplifier and integrator; 6. frequency analyzer B-K type 2120; 7. level recorder B-K type 2307; 8. since generator B-K type 1024; 9. power amplifier B-K type 2706; 10. mechanical linkage

The measurements were carried out for three different values of the vibrational velocity applied at the driving point of the sample: $V_{RMS} = (2, 5, 15)$ x 10^{-3} m/s. In the whole frequency range the values of the vibrational velocity were kept constant. The measured values of F_{RMS}, V_{RMS} at given f were used to calculate the modulus of impedance for the system: beam-head mass from the formula $|Z_u(f)| = \dfrac{F_{RMS}}{V_{RMS}}$, at the resonance and antiresonance conditions.

The resulting values of $|Z_u(f)|$ were introduced to the formulas (10) or (11), respectively, and the loss factor and the dynamic Young modulus were calculated for the given resonance frequencies of the whole system: beam-head mass. Finding η from the transcendental equation (9), the dependence of the loss factor on the dimensionless parameter $\eta(na)$ was exploited at the vicinity of the resonance and antiresonance frequencies for the bending vibrations of the total system. The loss factor approaches its maximum value for $(na)_{ext}$ corresponding to the resonance frequency or antiresonance frequency, respectively. At the vicinity of the extremal value na_{ext} the loss factor decreases. Hence the calculation of the loss factor consists in seaeching the maximal value $\eta_{max} = \eta(na)_{rez}$ (or $\eta (na)_{antirez}$) over a small range of the dimensional parameter na within a narrow vicinity of the resonance or antiresonance frequency (see [1]).

In the case of the determination of η and E_f out of the discrete resonance and antiresonance values one must measure not only the modulus of the point impedance but also the phase angle between the force and the velocity.

The measurement results of η and E_f for the driving velocities $V_{RMS} = (2$ and $15) \times 10^{-3}$ m/s have been overlapped (in the limit of the experimental error) with those ones for $V_{RMS} = 5 \times 10^{-3}$ m/s, therefore only the last ones are presented in this paper.

Anyway, the results for three different values of velocities have shown the linear range of vibrations what was the condition for the validity of the Bernoulli-Euler theory. The maximal vibrational velocity (driving takes place at the loop of wave) at the resonance may be evaluated from the relation $V_{RMS} \ll \lambda_g f/10$.

The wavelength for the highest frequency measured 8000 Hz (the 9-th resonance mode) in plexiglass was $\lambda_g = 29 \times 10^{-3}$ m, what means that $\lambda_g \gtrsim 6$ h and the
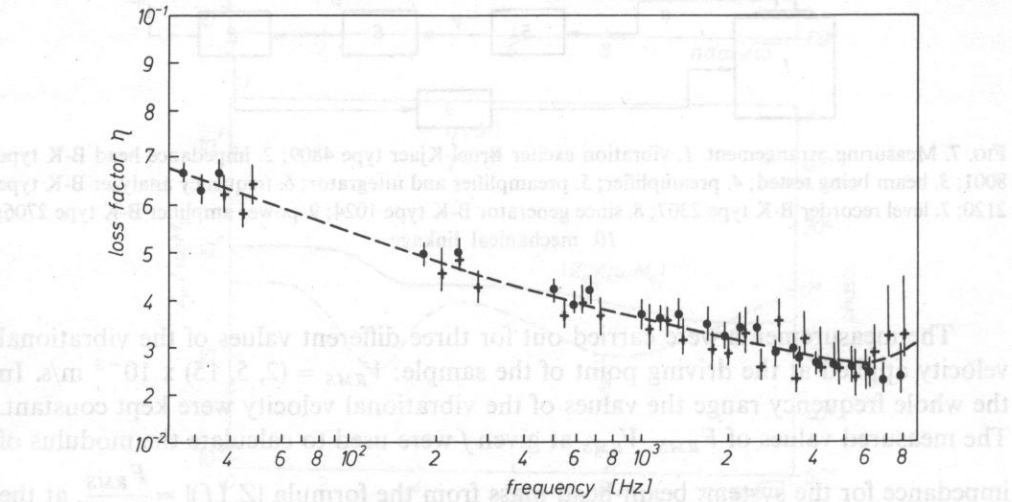

FIG. 8. Dependance of the loss factor η against the frequency for samples of plexiglass •, + — values of the loss factor measured at antiresonances and resonances of the sample, respectively

assumption of the Bernoulli-Euler theory is satisfied. Such evaluation is not possible for the antiresonance frequencies because then driving takes place at the mode of the sample.

Figure 8 represents the dependance of the loss factor in the plexiglass sample against the frequency. The range of experimental errors is shown in the figure, too.

In Fig. 9 the dependance of the dynamical Young modulus is shown against the frequency. Using the formula (11) and taking into account the geometrical dimensions of the sample the antiresonance or resonance frequency and calculated (from the transcendental equation $\eta = h(\eta, na)$) dimensionless parameter na were determined. In some other methods [1, 9, 10] the values of na are taken such as for the pure elastic samples, what leads to the wrong calculations of E_f.
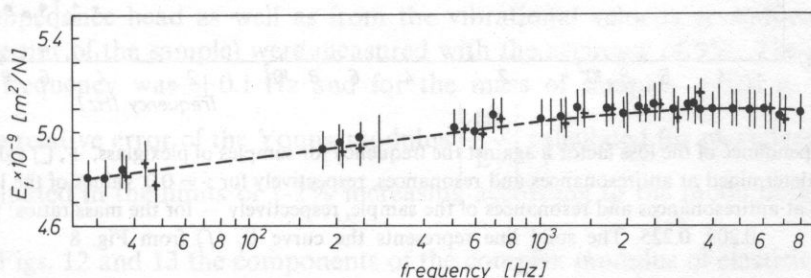

FIG. 9. Dependance of the Young modulus E_f against the frequency for samples of plexiglass, •, + values of the Young modulus measured at antiresonances and resonances of the sample, respectively

In addition, the calculations of the loss factor and the Young modulus were performed for the samples neglecting the influence of the mass resonance of the system: movable support-thightening ring-joining element, putting $\gamma = 0.0$. In addition, for the calculations it was assumed that γ is determined as $\gamma = \dfrac{M_{el} + 10^{-3} \text{ kg}}{M_b}$, where M_{el} is the mass of the element joining the head and the sample and 10^{-3} kg is the mass of the support above the force transducer in the impedance head of the Bruel and Kjaer type 8001.

Figure 10 illustrating the dependance of the loss factor against the frequency for the plexiglass samples shows the divergence of η values (increasing for the resonance frequencies and decreasing for the antiresonance frequencies) when the frequency increases, for the mass ratios $\gamma = 0.0$; 0.195; 0.209; 0.225.

The agreement of η values measured in the corresponding resonance and antiresonance modes of the sample gives an evidence of correct determination of the loss factor.

The neglecting of the reactant mass of the system: moveable support-tightening ring-joining element (i.e. $\gamma = 0.0$) in calculations of the Young modulus results decreasing of E_f value determined at resonance and antiresonance frequencies of the sample (see Fig. 11).
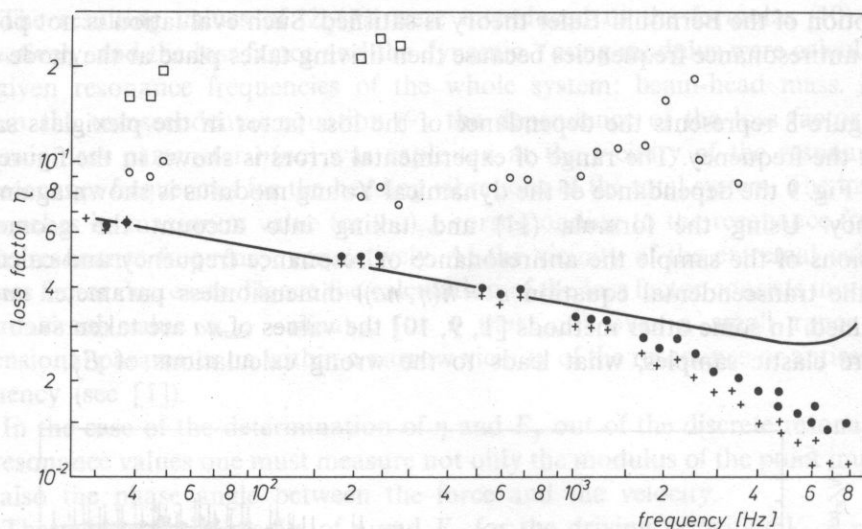
FIG. 10. Dependance of the loss factor η against the frequency for samples of plexiglass, $+$, $\square$ values of the loss factor determined at antiresonances and resonances, respectively for $\gamma = 0.0$, values of the loss factor determined at antiresonances and resonances of the sample, respectively $-$ for the mass ratios: $\gamma = 0.195$, 0.209, 0.225. The solid line represents the curve $E_f\,(f)$ from Fig. 8
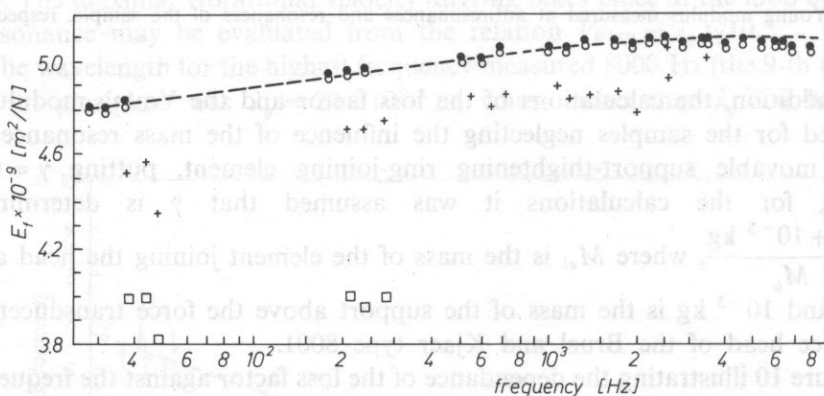

FIG. 11. Dependance of the Young modulus E_f against the frequency for plexiglass samples. $+$, $\square$ values determined at antiresonances and resonances of the sample, respectively, for $\gamma = 0.0$, $\bullet$, $\circ$ $-$ at resonances and resonances, respectively, for $\gamma = 0.195$; 0.209; 0.225. The dotted line illustrates the dependance of $E_f(f)$ from Fig. 9

Loss factor η values calculated for corresponding resonances and antiresonances are charged by an additional error due to the inaccuracy in determining the midpoint for driving the sample [2]. The influence of that error on the determination of η in plexiglass samples was neglected for the first 9 resonance and antiresonance

modes of the sample because the error could be minimized by a special selection of samples before the measurements [2].

The relative error of the loss factor

$$\left|\frac{\Delta\eta}{\eta}\right| = \left|\frac{\eta_+ - \eta_-}{\eta}\right|$$

for the most unadvantageous 9-th resonance mode is about 40% and decreases when the number of the mode decreases approaching about 10% for the 1-st mode. For the 9-th antiresonance mode the error is about 30% being, however, less than 10% for the first 6 modes.

For measuring the electric signals proportional to F_{RMS} and V_{RMS}, the Bruel-Kjaer analyzer 2120 was used. The values of voltages from the force transducer of the impedance head as well as from the vibrational velocity transducer (at the driving point of the sample) were measured with the accuracy of 5%. The accuracy for the frequency was ± 0.1 Hz and for the mass of samples ± 0.01 g.

The relative error of the Young modulus $\left|\dfrac{\Delta E_f}{E_f}\right|$ calculated for plexiglass samples was evaluated in the limits of 1-2% increasing according to the mode number from 1 to 9.

In Figs. 12 and 13 the components of the complex modulus of elasticity against the frequency are presented for softening PCW (polivinilchloride) and Razotex
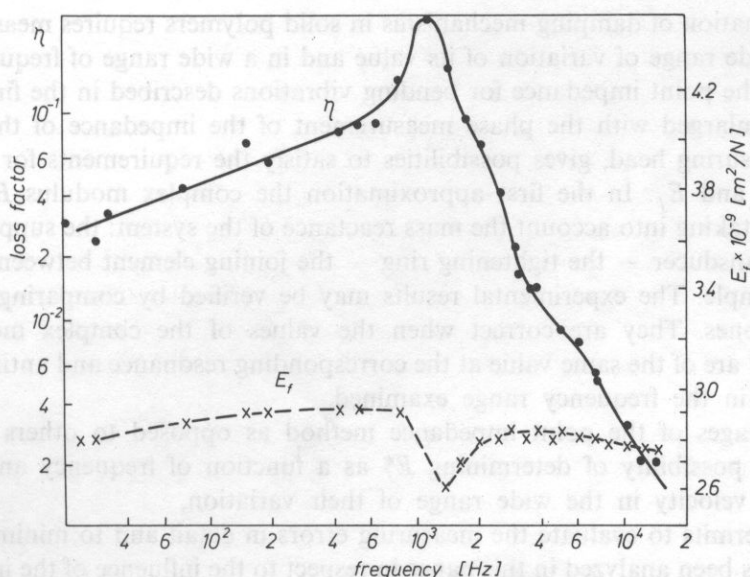

FIG. 12. Components of complex modulus of elasticity against the frequency for PCW

B (fenole epoxy), respectively. For PCW a strong damping for bending waves appears at the vicinity of 1000 Hz. For the maximum the inner structure of the polymer related to the relaxation mechanism is responsible. For the Razotex no relaxation process was observed within the frequency range examined.
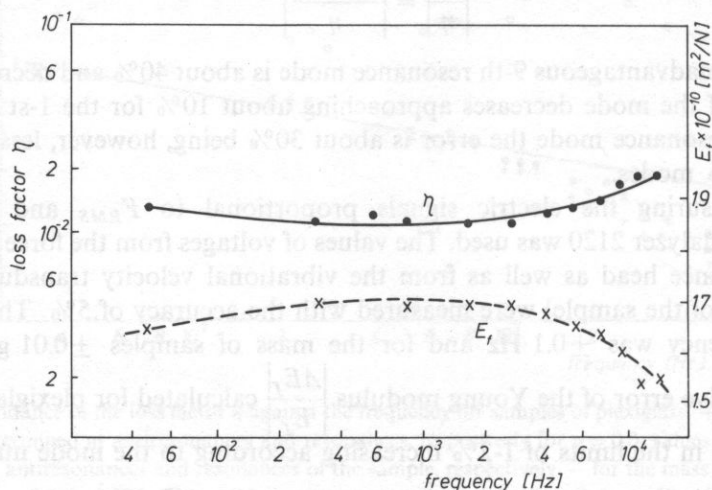

FIG. 13. Values of E_f and η against frequency for Rezotex B

4. Conclusions

Examination of damping mechanisms in solid polymers requires measurements of η in a wide range of variation of its value and in a wide range of frequency. The method of the point impedance for bending vibrations described in the first part of the paper enlarged with the phase measurement of the impedance of the system: sample-measuring head, gives possibilities to satisfy the requirements for measurements of η and E_f. In the first approximation the complex modulus E_f^* can be determined taking into account the mass reactance of the system: the support above the force transducer — the tightening ring — the joining element between the head and the sample. The experimental results may be verified by comparing with the numerical ones. They are correct when the values of the complex modulus of elasticity E_f^* are of the same value at the corresponding resonance and antiresonance frequencies in the frequency range examined.

Advantages of the point impedance method as opposed to others

1. The possibility of determining E_f^* as a function of frequency and driving vibrational velocity in the wide range of their variation,

2. It permits to evaluate the measuring errors in detail and to minimize them, too, as it has been analyzed in this paper in respect to the influence of the impedance of the measuring head on the determination of E_f^* for some viscoelastic materials.

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