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# A NEW ATTEMPT AT THE ESTIMATION FOR THE NOISE LEVEL PROBABILITY DISTRIBUTION BASED ON SPECIFIC $L_x$ NOISE EVALUATION INDICES

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In this paper, a new attempt at the estimation for the noise level probability distribution is proposed based on the known values of several specific $L_x$ noise evaluation indices. Here, the effect of the restricted amplitude fluctuation range matched to the actual stochastic phenomenon on the estimation accuracy is reasonably taken into consideration. The effectiveness of the proposed method is experimentally confirmed by applying it to the actually measured road traffic noise data.

## 1. Introduction

It is well-known that the actual environmental stochastic noises exhibit various types of probability distributions, due to the diversified causes of the noise fluctuations. Therefore, their stochastic property shows very often the arbitrary non-Gaussian distribution forms. Of course, every noise evaluation index for these stochastic phenomena can be extracted along to its definition as one of the representative values, by grasping precisely the population probability distribution of the original stochastic phenomenon. From this point of view, many researchers already found various types of important relations among more than two noise evaluation indices in close connection with the above population probability distribution [1–4]. In the authors' previous studies [5, 6], a general theory for estimating the original noise level probability distribution as the above population one has been proposed in a generalized form by employing the known values of *Leq* and several specific $L_x$ levels. In this paper, in a similar way, a new attempt at the estimation for the original population distribution for the noise level fluctuation is considered based on several known specific values of only $L_x$ noise evaluation indices. Here, the measured levels fluctuate only in a restricted amplitude fluctuation range. Thus, in the theoretical consideration, the effect of this restricted amplitude fluctuation range on the estimation accuracy is theoretically taken into consideration in comparison with the ordinary method having no effect of the above amplitude fluctuation range (i.e., its amplitude fluctuation range is within $(-\infty, \infty)$).

Finally, the effectiveness of the proposed method has been experimentally confirmed by applying it to the actually measured road traffic noise data.

## 2. Theoretical consideration

### *2.1 Estimation method with no restriction of amplitude fluctuation range*

Let us consider the noise level fluctuation, $L$, with no restriction of an amplitude fluctuation range. In this case, a statistical Hermite series expansion type expression for the cumulative distribution function of the noise level fluctuating within $(-\infty, \infty)$ can be introduced at the starting point of the analysis, as follows [7]:

$$Q(L) = \int_{-\infty}^{L} N(\xi; \mu, \sigma^2)\, d\xi - \sum_{n=3}^{\infty} A_n \sigma N(L; \mu, \sigma^2) H_{n-1}\left(\frac{L-\mu}{\sigma}\right)$$

with

$$\mu \triangleq \langle L \rangle, \sigma^2 \triangleq \langle (L-\mu)^2 \rangle, A_n \triangleq \frac{1}{n!}\left\langle H_n\left(\frac{L-\mu}{\sigma}\right)\right\rangle$$

and

$$N(\xi; \mu, \sigma^2) \triangleq \frac{1}{\sqrt{2\pi}\sigma} \exp[(\xi-\mu)^2/2\sigma^2], \tag{1}$$

where $\langle * \rangle$ denotes an averaging operation with respect to $*$ and $H_n(.)$ denotes the $n$th order Hermite polynomial. Based on the definition of $L_x$ noise evaluation indices, the following relationship can be derived from Eq. (1):

$$1 - \frac{x}{100} = \int_{-\infty}^{L_x} N(\xi; \mu, \sigma^2)\, d\xi - \sum_{n=3}^{\infty} A_n \sigma N(L_x; \mu, \sigma^2) H_{n-1}\left(\frac{L_x-\mu}{\sigma}\right). \tag{2}$$

Thus, after employing several specific $L_x$ levels to the above relationship, one can construct the $N$-dimensional simultaneous equations for the coupling coefficient $A_n$ ($n = 3, 4, \ldots, N-2$). After solving these equations, one can evaluate the objective noise level probability distribution $Q(L)$ by substituting the mean value $\mu$, the variance $\sigma^2$ and the estimated expansion coefficient $A_n$ into Eq. (1).

In this case, the specific formulae for estimating $A_n$ are explicitly expressed as follows:

(1) Case for $A_3$ with use of one reference level $L_{x1}$:

$$A_3 = \frac{\int_{-\infty}^{L_{x1}} N(\xi; \mu, \sigma^2)\, d\xi + \frac{x_1}{100} - 1}{\sigma N(L_{x1}; \mu, \sigma^2) H_2\left(\frac{L_{x1}-\mu}{\sigma}\right)}. \tag{3}$$

(2) Case for $A_3$ and $A_4$ with use of two reference levels $L_{x1}$ and $L_{x2}$:

$$A_3 = \frac{\left[\int_{-\infty}^{L_{x2}} N(\xi;\mu,\sigma^2)\,d\xi + \frac{x_2}{100} - 1\right] F_2(L_{x1};\mu,\sigma^2) - \left[\int_{-\infty}^{L_{x1}} N(\xi;\mu,\sigma^2)\,d\xi + \frac{x_1}{100} - 1\right] F_2(L_{x2};\mu,\sigma^2)}{\sigma F_1(L_{x2};\mu,\sigma^2)\,F_2(L_{x1};\mu,\sigma^2) - \sigma F_1(L_{x1};\mu,\sigma^2)\,F_2(L_{x2};\mu,\sigma^2)}, \tag{4}$$

$$A_4 = \frac{\left[\int_{-\infty}^{L_{x1}} N(\xi;\mu,\sigma^2)\,d\xi + \frac{x_1}{100} - 1\right] F_1(L_{x2};\mu,\sigma^2) - \left[\int_{-\infty}^{L_{x2}} N(\xi;\mu,\sigma^2)\,d\xi + \frac{x_2}{100} - 1\right] F_1(L_{x1};\mu,\sigma^2)}{\sigma F_1(L_{x2};\mu,\sigma^2)\,F_2(L_{x1};\mu,\sigma^2) - \sigma F_1(L_{x1};\mu,\sigma^2)\,F_2(L_{x2};\mu,\sigma^2)}$$

with

$$F_1(\xi;\mu,\sigma^2) \triangleq N(\xi;\mu,\sigma^2)\,H_2\!\left(\frac{\xi-\mu}{\sigma}\right), \tag{5}$$

$$F_2(\xi;\mu,\sigma^2) \triangleq N(\xi;\mu,\sigma^2)\,H_3\!\left(\frac{\xi-\mu}{\sigma}\right).$$

### 2.2. *Estimation method with restriction of amplitude fluctuation range*

Let us consider the noise level fluctuation, $L$, with restriction of an amplitude fluctuation range matched to the actually measured amplitude range of a stochastic phenomenon and a dynamic range of a measurement device. In this case, one can employ a statistical Jacobi series expansion type expression as the cumulative distribution function of the noise level fluctuation, as follows [8]:

$$Q(L) = \int_a^L \frac{1}{B(\gamma,\alpha-\gamma+1)(b-a)}\left(\frac{\xi-a}{b-a}\right)^{\gamma-1}\left(1-\frac{\xi-a}{b-a}\right)^{\alpha-\gamma} d\xi$$
$$+ \sum_{n=3}^{\infty} \frac{B_n}{B(\gamma,\alpha-\gamma+1)}\cdot\frac{1}{\gamma}\left(\frac{L-a}{b-a}\right)^{\gamma}\left(1-\frac{L-a}{b-a}\right)^{\alpha-\gamma+1} G_{n-1}\!\left(\alpha+2,\gamma+1;\frac{L-a}{b-a}\right)$$

with

$$\alpha \triangleq \frac{(\mu-a)(b-\mu)}{\sigma^2} - 2,\quad \gamma \triangleq \frac{(\alpha+1)(\mu-a)}{b-a}$$

and

$$B_n \triangleq \frac{\Gamma(\gamma)\,\Gamma(\alpha-\gamma+1)}{\Gamma(\alpha+1)}\cdot\frac{(\alpha+2n)\,\Gamma(\alpha+n)\,\Gamma(\gamma+n)}{n!\,\Gamma(n+\alpha-\gamma+1)\,[\Gamma(\gamma)]^2}\left\langle G_n\!\left(\alpha,\gamma;\frac{L-a}{b-a}\right)\right\rangle, \tag{6}$$

where $B(.)$, $\Gamma(.)$ and $G_n(.)$ denote respectively Beta function, Gamma function and the $n$th order Jacobi's polynomial. It should be noticed that the above formula is a generalized one including the well-known statistical Hermite series expansion type expression (taking a well-known standard Gaussian distribution as the first expansion term) in a special case when $\gamma \to \infty\,(a = b \to \infty)$ (see Ref. [8]). According to the definition of $L_x$ noise evaluation indices, the following relationship can be directly derived from Eq. (6).

$$1-\frac{x}{100}=\int_a^{L_x}\frac{1}{B(\gamma,\alpha-\gamma+1)(b-a)}\left(\frac{\xi-a}{b-a}\right)^{\gamma-1}\left(1-\frac{\xi-a}{b-a}\right)^{\alpha-\gamma}d\xi$$

$$+\sum_{n=3}^{\infty}\frac{B_n}{B(\gamma,\alpha-\gamma+1)}\cdot\frac{1}{\gamma}\left(\frac{L_x-a}{b-a}\right)^{\gamma}\left(1-\frac{L_x-\mu}{b-a}\right)^{\alpha-\gamma+1}G_{n-1}\left(\alpha+2,\gamma+1;\frac{L_x-a}{b-a}\right). \quad (7)$$

Therefore, after employing several specific $L_x$ levels to the above relationship, one can obtain the $N$-dimensional simultaneous equations for the expansion coefficient $B_n$ ($n=3, 4, \ldots, N-2$). The objective noise level probability distribution can be obtained by substituting the estimated $B_n$ and the known values of $\mu$, $\sigma^2$, $a$ and $b$ into Eq. (6).

More explicitly, the explicit formulae for estimating the expansion coefficient $B_n$ can be obtained as follows:

(1) Case for $B_3$ with use of one reference level $L_{x1}$:

$$B_3=\frac{1-\frac{x_1}{100}-Q_0(L_{x1};\alpha,\gamma,a,b)}{\frac{1}{B(\gamma,\alpha-\gamma+1)}\cdot\frac{1}{\gamma}\left(\frac{L_{x1}-a}{b-a}\right)^{\gamma}\left(1-\frac{L_{x1}-a}{b-a}\right)^{\alpha-\gamma+1}G_2\left(\alpha+2,\gamma+1;\frac{L_{x1}-a}{b-a}\right)},$$

$$Q_0(\xi;\alpha,\gamma,a,b)\triangleq\int_a^{\xi}\frac{1}{B(\gamma,\alpha-\gamma+1)(b-a)}\left(\frac{\xi-a}{b-a}\right)^{\gamma-1}\left(1-\frac{\xi-a}{b-a}\right)^{\alpha-\gamma}d\xi. \quad (8)$$

(2) Case for $B_3$ and $B_4$ with use of two reference levels $L_{x1}$ and $L_{x2}$:

$$B_3=\frac{\left[1-\frac{x_1}{100}-Q_0(L_{x1};\alpha,\gamma,a,b)\right]J_2(L_{x2};\alpha,\gamma,a,b)-\left[1-\frac{x_2}{100}-Q_0(L_{x2};\alpha,\gamma,a,b)\right]J_2(L_{x1};\alpha,\gamma,a,b)}{J_1(L_{x1};\alpha,\gamma,a,b)J_2(L_{x2};\alpha,\gamma,a,b)-J_1(L_{x2};\alpha,\gamma,a,b)J_2(L_{x1};\alpha,\gamma,a,b)},$$

$$B_4=\frac{\left[1-\frac{x_2}{100}-Q_0(L_{x2};\alpha,\gamma,a,b)\right]J_1(L_{x1};\alpha,\gamma,a,b)-\left[1-\frac{x_1}{100}-Q_0(L_{x1};\alpha,\gamma,a,b)\right]J_1(L_{x2};\alpha,\gamma,a,b)}{J_1(L_{x1};\alpha,\gamma,a,b)J_2(L_{x2};\alpha,\gamma,a,b)-J_1(L_{x2};\alpha,\gamma,a,b)J_2(L_{x1};\alpha,\gamma,a,b)}, \quad (9)$$

with

$$J_1(\xi;\alpha,\gamma,a,b)\triangleq\frac{1}{B(\gamma,\alpha-\gamma+1)}\cdot\frac{1}{\gamma}\left(\frac{\xi-a}{b-a}\right)^{\gamma}\left(1-\frac{\xi-a}{b-a}\right)^{\alpha-\gamma+1}G_2\left(\alpha+2,\gamma+1;\frac{\xi-a}{b-a}\right),$$

$$J_2(\xi;\alpha,\gamma,a,b)\triangleq\frac{1}{B(\gamma,\alpha-\gamma+1)}\cdot\frac{1}{\gamma}\left(\frac{\xi-a}{b-a}\right)^{\gamma}\left(1-\frac{\xi-a}{b-a}\right)^{\alpha-\gamma+1}G_3\left(\alpha+2,\gamma+1;\frac{\xi-a}{b-a}\right). \quad (10)$$

## 3. Experimental consideration

In order to confirm the effectiveness of the proposed method, it has been applied to two kinds of road traffic noise data measured actually in Hiroshima Prefecture. That is, first, the road traffic noise data have been measured in an urban area with a fairly large traffic volume. From the notational viewpoint, the above case is defined here as "Case *A*". Next, the other road traffic noise data have been measured in a rural area with a small traffic volume. At this time, this case is defined as "Case *B*". Figure 1 shows the actual situation in Case *A* of measuring the road traffic noise in an urban area. Similarly, Fig. 2 also shows the actual situation in Case *B* of measuring the road traffic noise in a rural area.

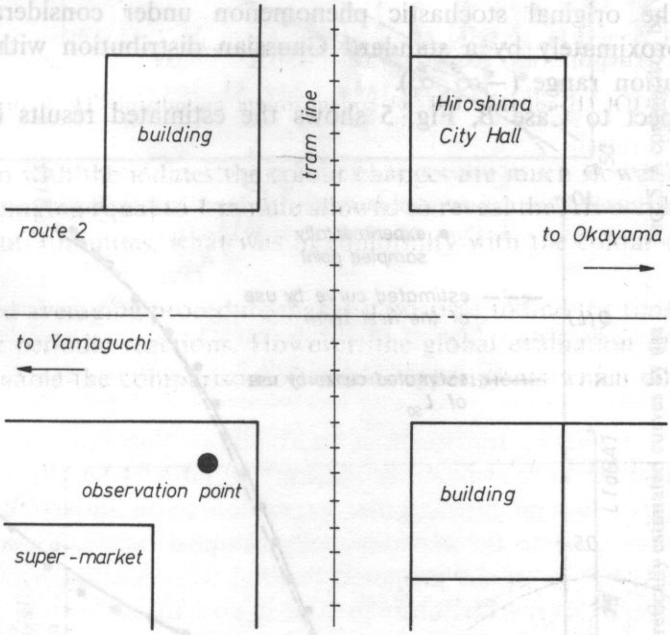


FIG. 1. Actual situation of measuring the road traffic noise in an urban area (Case *A*)

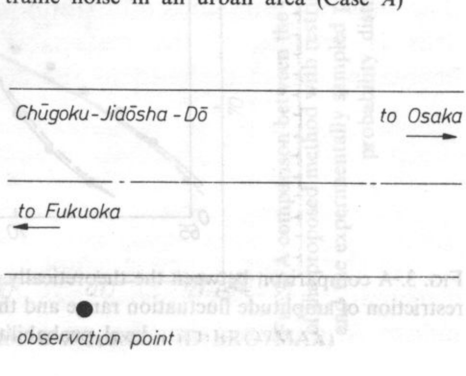


FIG. 2. Actual situation of measuring the road traffic noise in a rural area (Case *B*)

Figure 3 shows a comparison between the theoretically estimated curves by use of the proposed method with no restriction of an amplitude fluctuation range and the experimentally sampled points for the cumulative noise level probability distribution with respect to Case *A*. As shown in this figure, it is obvious that these data exhibit approximately a standard Gaussian distribution corresponding to the first expansion term of Eq.(1). A comparison between the theoretically estimated curves by use of the proposed method with consideration of the restricted fluctuation range and the experimentally sampled points is shown in Fig. 4. From these results, the estimated results due to both methods are in good agreement with the experimental results, because both methods are generally applicable to the case when the objective stochastic phenomenon can be expressed by not only a non-Gaussian distribution but also a standard Gaussian one. In this case, the effect of a constant restriction of an amplitude fluctuation range on the estimation accuracy is not so clear, since the original stochastic phenomenon under consideration can be expressed approximately by a standard Gaussian distribution with an idealized infinite fluctuation range $(-\infty, \infty)$.

With respect to Case *B*, Fig. 5 shows the estimated results by use of the

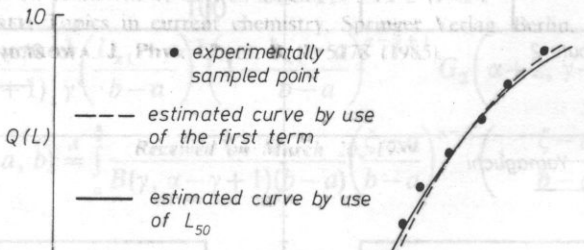


FIG. 3. A comparison between the theoretically estimated curves by use of the proposed method with no restriction of amplitude fluctuation range and the experimentally sampled points for the cumulative noise level probability distribution (Case *A*)

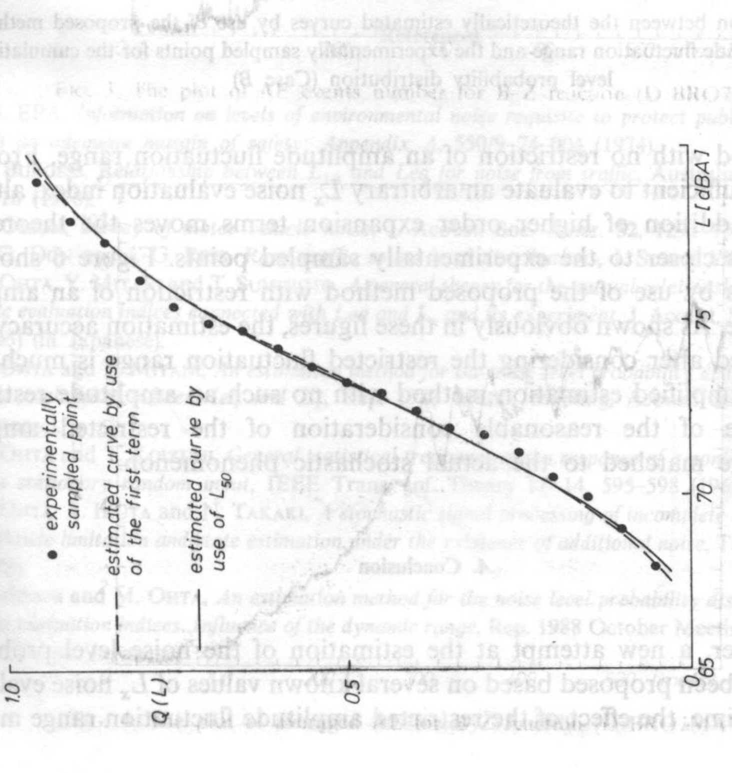


FIG. 4. A comparison between the theoretically estimated curves by use of the proposed method with restriction of amplitude fluctuation range and the experimentally sampled points for the cumulative noise level probability distribution (Case A)

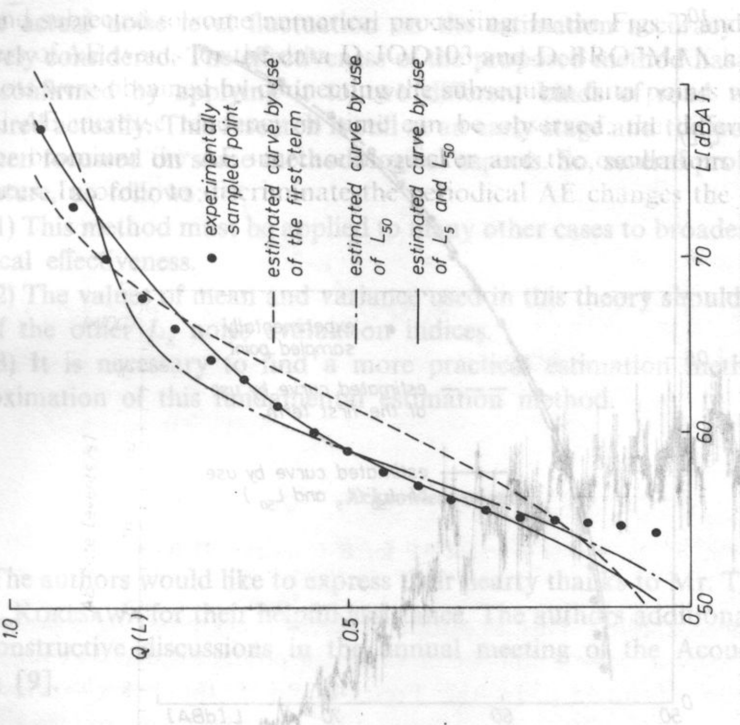


FIG. 5. A comparison between the theoretically estimated curves by use of the proposed method with no restriction of amplitude fluctuation range and the experimentally sampled points for the cumulative noise level probability distribution (Case B)

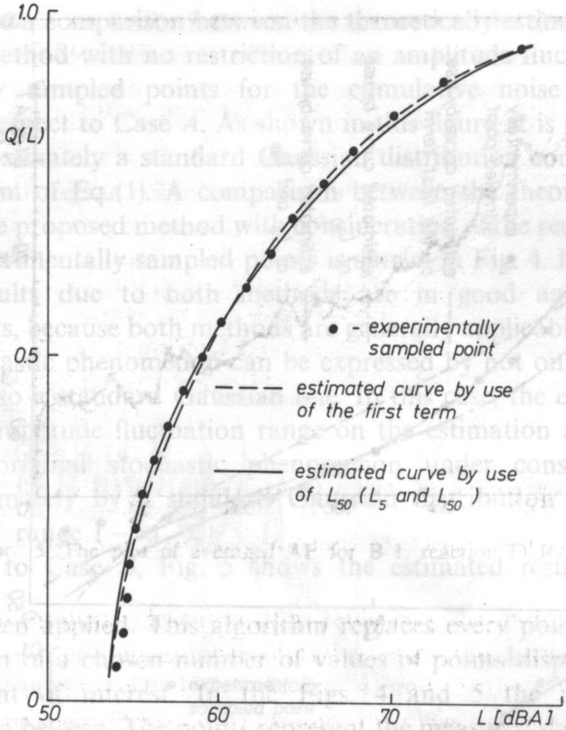


FIG. 6. A comparison between the theoretically estimated curves by use of the proposed method with restriction of amplitude fluctuation range and the experimentally sampled points for the cumulative noise level probability distribution (Case *B*)

proposed method with no restriction of an amplitude fluctuation range. From this figure, it is not sufficient to evaluate an arbitrary $L_x$ noise evaluation index, although the successive addition of higher order expansion terms moves the theoretically estimated curves closer to the experimentally sampled points. Figure 6 shows the estimated results by use of the proposed method with restriction of an amplitude fluctuation range. As shown obviously in these figures, the estimation accuracy of the method proposed after considering the restricted fluctuation range is much better than that of a simplified estimation method with no such an amplitude restriction. This is because of the reasonable consideration of the restricted amplitude fluctuation range matched to the actual stochastic phenomenon.

## 4. Conclusion

In this paper, a new attempt at the estimation of the noise level probability distribution has been proposed based on several known values of $L_x$ noise evaluation indices. At this time, the effect of the restricted amplitude fluctuation range matched to the actual noise level fluctuation on the estimation accuracy has been quantitatively considered. The effectiveness of the proposed method has been experimentally confirmed by applying it to two different kinds of road traffic noise data measured actually. This research is still at an early stage and the work reported here has been focussed on some methodological aspects. So, several problems are left for the future, as follows:

(1) This method must be applied to many other cases to broaden and confirm its practical effectiveness.

(2) The values of mean and variance used in this theory should be estimated by use of the other $L_x$ noise evaluation indices.

(3) It is necessary to find a more practical estimation method through the approximation of this fundamental estimation method.

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