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METHODS OF MEASUREMENTS OF THE QUALITY FACTOR OF PIEZOELECTRIC RESONATORS WITH HIGH ELECTRICAL AND MECHANICAL LOSSES

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In the paper the analysis of methods of measurements of the piezoelectric resonators quality factors has been presented, as well as the possibility of their applications for resonators with high electrical and mechanical losses, e.g. ceramics and piezoelectric foils. The problem of the quality factor measurements for resonators excited by high electric field has been also discussed. To this end the authors propose to apply a ring-shaped piezoelectric transformer. This metod enables us to measure ceramic parameters in the quasi-linear range, to separate mechanical and electrical losses, and to determine the range of the linear work. The conducted analysis makes it possible to conclude the lack of a precise method of the quality factor and also the losses measurement for piezoelectric resonators with low quality factors, also in the case when nonlinear effects occur.

1. Introduction

Measurements of the mechanical and electrical quality factors of resonant vibrating elements give very important information about the piezoelectric material and resonator parameters. These measurements can be performed both in the linear and the quasilinear range of a resonator work. From measurements of the quality factor one can determine the electrical and mechanical losses of a piezoelectric material and the linearity of resonator's working range.

Methods of measurements of the mechanical or electrical quality factor of piezoelectric elements which are based on the resonator equivalent circuit are not very precise, because it is difficult to separate the mechanical part from the electrical part. In these methods one assumes that electrical losses are negligible at low voltage resonances and that the measured quality factor is connected with mechanical losses only. This assumption is not satisfied for measurements in high intensity electric fields [5], because electrical losses are proportional to the second power of the electric field intensity. In this case the measured factor of quality is connected with electrical losses as well as with mechanical losses of a vibrating resonator. Electrical losses are also considerable in resonators made of conducting piezoelectric materials [1].

The purpose of this work was to estimate the possibilities of application of the known methods of the quality factor measurements for investigations of piezoelectric resonators with high electrical and mechanical losses, also in the range of high electrical and mechanical fields. For the measurements in the case of high electric field excitation of a resonator, the authors propose to apply a ring-shaped piezoelectric transformer.

2. Measurements of the mechanical quality factor of resonators with moderate losses

The electromechanical equivalent circuit proposed by Van Dyke [16] is the basis for measurements of the mechanical and electrical quality factors. Using this circuit (Fig. 1) one can determine the input addmitance of a resonator, considering the mechanical losses.

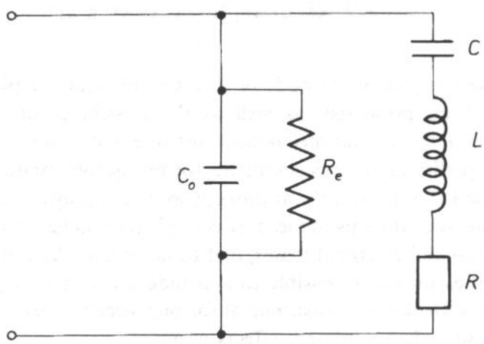

Fig. 1. Van Dyke's electromechanical equivalent circuit of a piezoelectric resonator.

$$Y=\frac{\omega^2RC^2}{(1-\omega^2CL)^2+\omega^2C^2R^2}-j\omega\left[C_0+\frac{C(1-\omega^2CL)}{(1-\omega^2CL)^2+\omega^2C^2R^2}\right]. \tag{2.1}$$

R — resistance corresponding to mechanical losses, L — motional inductance in the equivalent circuit, C — motional capacitance in the equivalent circuit, C_0 — shunt capacitance in the equivalent circuit. For the mechanical resonance $(1-\omega_s^2CL)=0$, and the equation (2.1) simplified to the form

$$Y=\frac{1}{R}-j\omega_sC_0, \tag{2.2}$$

$\omega_s=2\pi f_s$, f_s — series resonance frequency.
Taking into consideration the electrical losses one obtains

$$Y=\left(\frac{1}{R}+\frac{1}{R_e}\right)-j\omega_sC_0, \tag{2.3}$$

where R_e — resistance corresponding to electrical losses.

Both the electrical and mechanical losses depend on frequency, electrical field intensity and mechanical stress.

Equation (2.1) can be graphically presented as an admittance circle with characteristic points Fig. 2 corresponding to frequencies of: mechanical resonance f_s, parallel resonance f_p, resonance f_r, antiresonance f_a, and frequencies of minimal and maximal absolute admittance f_n and f_m. For small losses the circle diameter is equal to the real part of admittance.

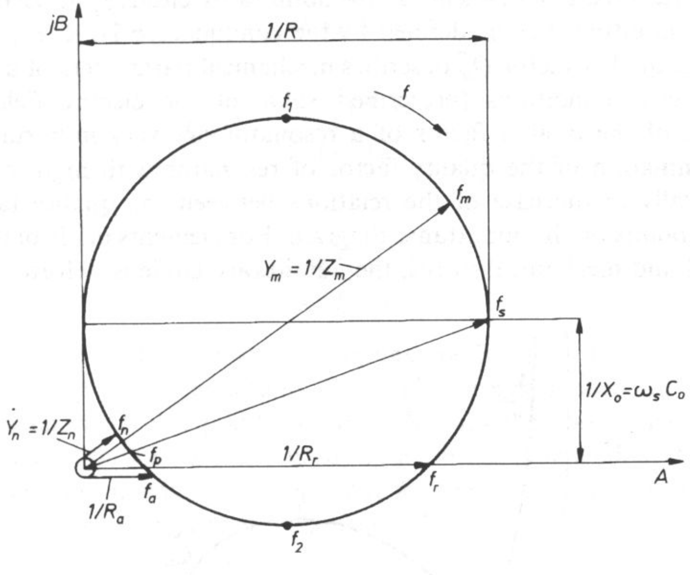

Fig. 2. Admittance circle of a piezoelectric resonator.

$$Y_{re}=\frac{1}{R}. \tag{2.4}$$

Van Dyke's equivalent circuit is very useful for measurements of the parameters of resonators with high quality factor. However, for the analysis of resonators working under variable conditions and, especially, in the nonlinear range, its application is limited. In this case Van Dyke's circuit can be used only as an indication for the determination of theoretical relations and the measurement of admittance as the function of frequency.

The mechanical quality factor is determined by means of lumped electric elements of the equivalent circuit:

$$Q_m^E=\frac{1}{\omega_s RC}, \tag{2.5}$$

and it can be determined using the admittance circle or the resonance curve

$$Q_m^E = \frac{f_s}{\Delta f}, \tag{2.6}$$

where Δf — difference of frequencies for the 3 dB — decrease of voltage.

$$Q_m^E = \frac{f_s}{f_2 - f_1}, \tag{2.7}$$

f_s, f_1, f_2 — characteristic frequencies on the admittance circle. f_1, f_2 correspond to the points on the admittance circle defined by the conductance $1/2R$ (Fig. 2).

Mechanical quality factor Q_m^E describes mechanical parameters of a resonator for prescribed electric conditions (prescribed value of the electric field). Therefore measurements of the quality factor of a resonator are very important. However, precise determination of the quality factor of resonator with high losses is complicated, especially in the case of the relations between the quality factor and the characteristic points on the admittance diagram. For elements made of materials with high electrical and mechanical losses, the admittance circle is deformed (cf. Fig. 3).

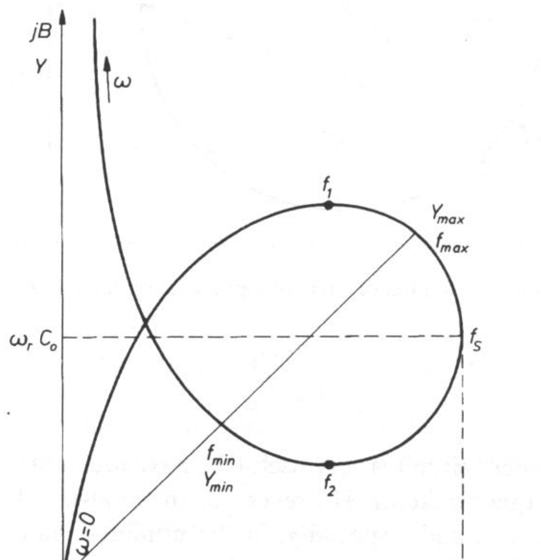

Fig. 3. Admittance diagram for a piezoelectric resonator with high losses.

3. Measurements of resonators with low quality factor

Measurements of the mechanical quality factor in the range of 10–100 is a difficult problem. Recently new piezoelectric materials are used, e.g. polymer foils (mainly PVDF) and composites with low mechanical and electrical quality factors (far below 100). The commonly applied equations involving characteristic frequencies of the equivalent circuit for low losses case appear to be insufficient for the accurate determination of the quality factor. The equations in which frequencies $f_{\min}$, $f_{\max}$ are used (often denoted also by f_n, f_m), corresponding to the minimal and maximal absolute admittance corresponding f_1, f_2 and to the conductance $1/2R$, have been introduced [4, 9]

$$Q_m^E = \frac{f_{\min}^2 + f_{\max}^2}{f_{\min}^2 - f_{\max}^2}, \tag{3.1}$$

and

$$(Q_m^E)^2 = \frac{1}{4}\left[\left(\frac{f_1+f_2}{f_2-f_1}\right)^2 - 1\right], \tag{3.2}$$

or approximately

$$Q_m^E \cong \frac{1}{2}\left(\frac{f_1+f_2}{f_2-f_1}\right). \tag{3.3}$$

In these equations the electromechanical coupling coefficient is included not explicitly. For higher quality factors equations (3.1)–(3.3) are equivalent to the equation (2.7), since their characteristic points coincide. Besides the above mentioned equations, the following equations are also used in the case of independent determination of the electromechanical coupling coefficient k from the ratios of the measured fundamental and overtone resonant frequencies [10]:

$$Q_m^E = \frac{1}{k^2}\sqrt{\frac{Y_m}{Y_n}}, \tag{3.4}$$

$$Q_m^E = \frac{1}{k^2}\frac{\left(\frac{Y_m}{Y_n} - 1\right)}{\sqrt{\frac{Y_m}{Y_n}}}, \tag{3.5}$$

and [5]:

$$Q_m^E = \frac{1-k^2}{k^2}\left(\frac{Y_m}{Y_n}\frac{f_{\min}}{f_{\max}} - 1\right)\sqrt{\frac{Y_n}{Y_m}} \tag{3.6}$$

Y_m, Y_n — maximal and minimal value of the absolute admittance determined by means of the admittance diagram or the absolute admittance dependence on the frequency (Fig. 4).

In Table 1 the calculated values of the mechanical quality factor Q_m^E are presented. The values of Q_m^E were calculated for the electromechanical coupling coefficient $k=0.3$ and for different ratios of admittances, using different formulae. It can be seen

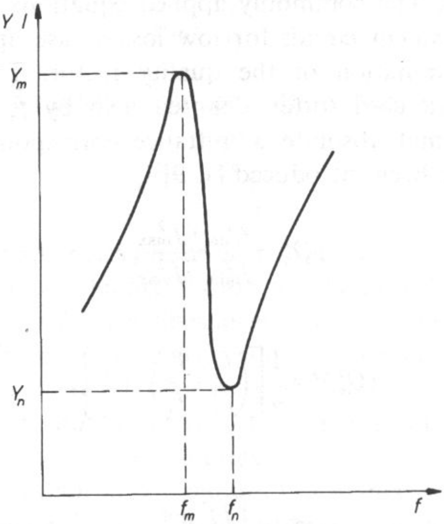

Fig. 4. Absolute admittance of a piezoelectric resonator as a function of frequency.

Table 1. Mechanical quality factors calculated by different formulae

$\sqrt{\frac{Y_m}{Y_n}}$	1.2	1.5	2	3	4	5	Formula
	13.3	16.7	22.2	33.3	44,4	55.5	(3.4)
Q_m^E	4.1	9.3	16.7	29.6	41.7	53.3	(3.5)
	5	10.2	17.4	30	42.4	54.1	(3.6)

in Table 1 that the differences are large for the small values of $(Y_m/Y_n)^{1/2}$ (it is also connected with high losses).

In the case of piezoelectric foil materials with high losses, Q_m^E can be determined from the slope of the straight line X_a/R_a (the ratio of the imaginary part of impedance to the real part of impedance near the resonance) [2]. The quality factor is calculated from the measurements of the impedance of the series circuit being a function of frequency near the mechanical resonance of a resonator. The dependence of X_a/R_a on the frequency is linear and the quality factor corresponds to the inclination of the straight line to the frequency axis,

$$\frac{X_a}{R_a} = -2Q_m^E \frac{\omega - \omega_0}{\omega_0}. \tag{3.7}$$

Here $2Q_m^E$ corresponds to the slope of the straight line $\frac{X_a}{R_a}(\omega)$, $\omega_0 = 2\pi f_0 = (\pi/t)(c^D/\rho)^{1/2}$, f_0 — thickness resonant frequency, t — resonator thickness, c^D — stiffened elastic constant, ρ — density.

As a result, one obtains the formula for the mechanical quality factor

$$Q_m^E = \frac{1}{2}\frac{X_a}{R_a}\frac{\omega_0}{\omega-\omega_0}. \tag{3.8}$$

This formula may be used also for the parallel circuit

$$Q_m^E = \frac{1}{2}\frac{B_a}{G_a}\frac{\omega_0}{\omega-\omega_0}; \tag{3.9}$$

B_a/G_a is the ratio of the imaginary part of admittance to the real part of admittance near the resonance.
In this case one calculates the ratio B_a/G_a from the admittance diagram by moving its centre along the conductance line and subtracting the value of the conductance connected with the electrical losses.

$$Q_m^E = \frac{1}{2}\frac{\omega_0 C_0 - \omega C}{\left(\frac{1}{R}-\frac{1}{R_e}\right)}\frac{\omega_0}{\omega-\omega_0}; \tag{3.10}$$

Notations C, C_0, R, R_e — cf. Fig. 1.
For the determination of the angle of inclination of the straight line B_a/G_a to the frequency axis it is sufficient to find the coordinates of one point in addition to the point ω_0, in the vicinity of the resonance. The results obtained from the calculations based on the formula (3.10) are very close to the results obtained by the formulae (3.4) and (3.6). The formulae (3.4) and (3.10) have been applied for the calculations of the quality factor of a barium-modified lead meta-niobate piezoelectric ceramic with small quality factor. Fot this purpose we have used the admittance diagrams obtained during investigations of this ceramic [11]. Figure 5 presents an example of such a diagram. The results of the calculations are presented in Table 2. In the same way we have also calculated the data from [2] for the piezoelectric foil PVF_2, $f_0=20.65$ MHz, $Q_B=14.6$, $Q_f=17.2$, $\frac{Q_f-Q_B}{Q_f}=15\%$, where Q_B, Q_f are the quality factors calculated by means of the formula (3.10) and (3.4) respectively.

Table 2. Quality factors of plates of $Pb_{0.9}Ba_{0.1}Nb_2O_6$ ceramic determined from measurements and calculations using the formula (3.10) — Q_B and (3.4) — Q_f

f_0 [kHz]	2.25	3.3	2.8	3.05	2.32	4.7	5.3
Q_B	13.0	15.8	18.2	25.7	21.1	55.4	23.6
Q_f	13.4	16.5	18.6	30.5	23.2	71.0	25.2
$\frac{Q_f-Q_B}{Q_f}$ [%]	3.0	4.2	2.2	15.7	9.0	22.0	6.3

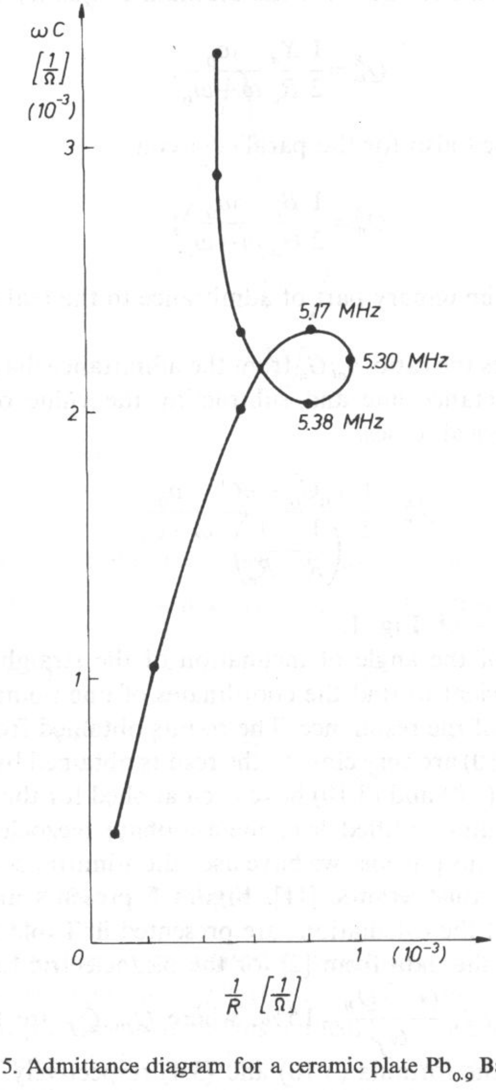

Fig. 5. Admittance diagram for a ceramic plate $Pb_{0.9}\,Ba_{0.1}\,Nb_2O_6$.

It can be seen in Table 2 that the quality factor values calculated from the formula (3.10) are by 10% or less lower than the quality factors calculated from the formula (3.4) for $Q_m^E < 20$. Application of these quality factor values for the calculations of the electromechanical coupling coefficient (formulae (3.4) – (3.6)) leads to a considerable disagreement with the results obtained from the ratio of overtone frequencies [10, 11]. In this case one obtains considerably higher values of the coefficient k than the values following from the method described in [10]. These discrepancies result probably from the fact that the electric losses are not taken into consideration in the formulae (3.4) – (3.6). In all calculations the equivalent circuit and the admittance circle are idealized, and the electric losses are not considered. The conventional formulae applied in the calculations are not sufficient in this case.

4. Measurements of the quality factor of a resonator in high electric fields

The formulae mentioned above cannot be used to calculate the quality factor for higher excitation voltages, when electrical losses increase and resonance curves deform. Their deformation is caused by nonlinear effects [3, 14]. When the amplitude of vibration increases more, changes of the resonance frequency and discontinuities of resonance curves appear — Fig. 6. In this case it is not possible to find the characteristic points on the admittance diagram. Discontinuities make it impossible to determine the quality factor. The resonance curve narrows apparently. It should be the proof of the increase of the quality factor. In fact, the quality factor decreases very

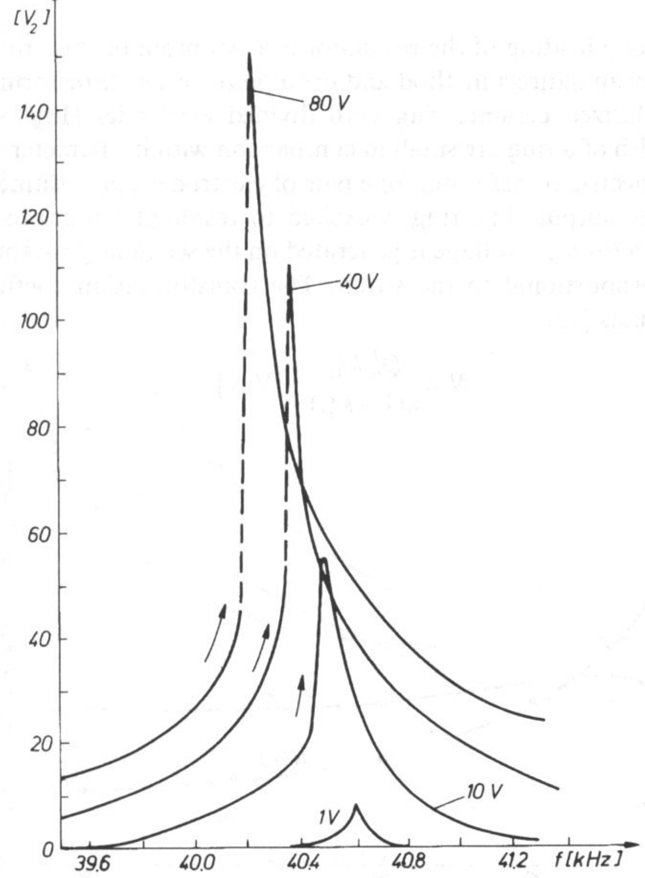

Fig. 6. Resonance curves of the PZT ring ($\phi_{ext}=30$ mm, $\phi_{int}=23$ mm, $d=3.6$ mm) for excitation voltages V_1. Transformer circuit, the secondary circuit is loaded by high resistance. V_1 values are written at the curves.

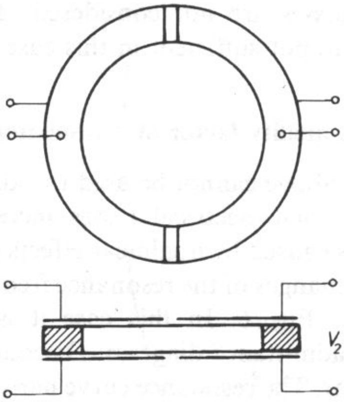

Fig. 7. Ring-shaped piezoelectric transformer

considerably. Strong heating of the resonator is a symptom of this effect [15]. In this case one can apply an indirect method and use a piezoelectric transformer in the form of an axially polarized ceramic ring with divided electrodes [13] — Fig. 7. The thickness and width of a ring are small in comparison with its diameter. Such a ring is a resonant piezoelectric transformer, one pair of electrodes constituting its input, the second pair — its output. The ring is excited to resonant vibrations by the input electrodes and as a result, a voltage is generated on the secondary (output) electrodes, its value being proportional to the strain. The transformation coefficient of such a transformer equals [12]

$$N=\frac{Q_m^E k_{31}^2}{\pi(1-k_{31}^2)} \quad [\mathrm{V/V}]$$

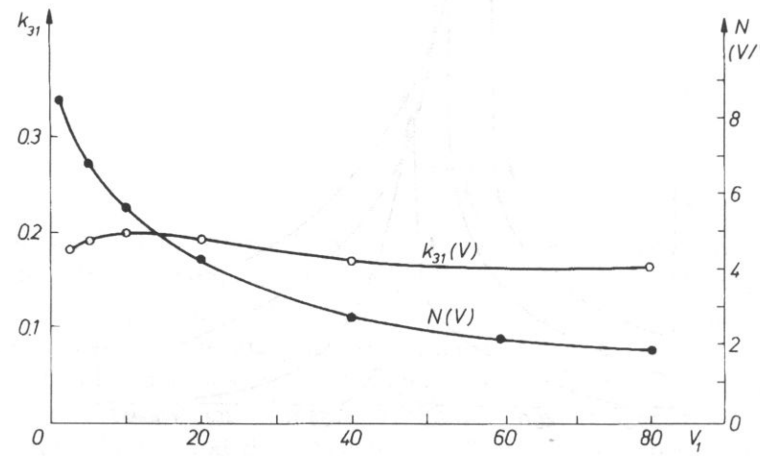

Fig. 8. Dependence of the electromechanical coupling coefficient k_{31} and the transformation coefficient N on the voltage exciting the ring transformer (PZT, $\phi_{ext}=30$ mm, $\phi_{int}=23$ mm, $d=3.6$ mm).

This formula can be used to calculate the quality factor for the measurement conditions. Then with a ring-shaped transformer it is possible to determine Q_m^E without the necessity of finding the characteristic points on the deformed admittance diagrams or resonance curves, it is very difficult or even impossible in certain cases.

The measurement of the transformation coefficient N is simple it is enough to measure the input and output voltages of the transformer. On the contrary, the measurement of the electromechanical coupling coefficient may be complicated. Experimental investigations of the transformer show that the electromechanical coupling coefficient does not change considerably with changes of the voltage exciting the transformer — Fig. 8. However, it is difficult to determine its value above the threshold of the discontinuity. The quality factor can be calculated in the quasilinear range by the determination of the k_{31} coefficient from the measurements of the

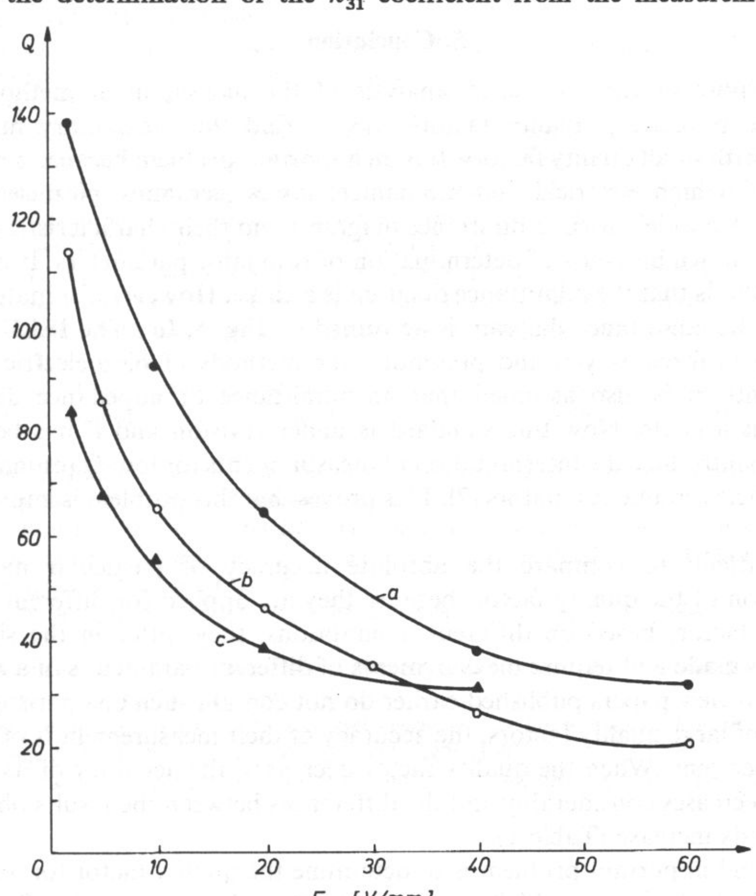

Fig. 9. Quality factor as a function of the electric field for the PZT ring with thickness 0.95 mm a) secondary electrodes shorted, b) secondary electrodes open, c) primary and secondary electrodes connected.

resonance and antiresonance [11] of a ring submerged in a viscous liquid. In this case discontinuities of resonance curves are eliminated. The real part of radiation impedance does not influence the resonance frequencies.

Figure 9 presents the quality factor as a function of the electric field for the $Pb/Zr_{0.45}\,Ti_{0.55}/O_3$ ring with the thickness 0.95 mm. The measurements have been conducted for three cases: a) the secondary electrodes shorted, b) the secondary electrodes open, c) the primary and secondary electrodes connected. The largest quality factor has been obtained for the shorted secondary electrodes, in this case the electrical losses in the secondary circuit are eliminated ($E=0$). For the connected electrodes and for the open secondary electrodes, the measured quality factor depends on the electrical and mechanical losses. The electrical losses can be calculated from the differences between the quality factors for the case b) and the case a) [13].

5. Conclusion

The purpose of the conducted analysis of the measurement methods of the piezoelectric resonators quality factors was to find the satisfactory method for resonators with small quality factors. It is an important problem because piezoelectric materials with high electrical and mechanical losses (ceramics, piezoelectric foils, composites) are widely used. Admittance diagrams and their characteristic points are used in the known methods of determination of resonator parameters. It is assumed in these methods that the admittance diagram is a circle. However, for materials with high losses the admittance diagram is deformed — Fig. 5. In "The IEEE Standard 177", being in force as yet, and presenting the methods of piezoelectric vibrators measurements, it is also assumed that an admittance or impedance diagram of a vibrator is a circle. Now this standard is under revision and it has been found necessary to introduce the interpretation of measurements for low-Q resonators, such as piezoelectric ceramic resonators [7]. This proves that this problem is important and live.

It is difficult to compare the absolute accuracy of particular methods of determination of the quality factor, because they are applied for different ranges of the quality factor, based on different assumptions, they differ in the simplifying assumptions made and require measurements of different parameters of a resonator. Also, some review papers published earlier do not contain such comparison, e.g. [8]. In the case of large quality factors, the accuracy of their measurement is of the order of several per cent. When the quality factor decreases, the accuracy of its measurement also decreases considerably and the differences between the results obtained by other methods increase (Table 1).

The second important problem is to determine the quality factor for a resonator excited by the high electric field. The measurement method proposed in Sect. 4 using a piezoelectric transformer in the form of a ceramic ring enables us to determine the ceramic parameters in the quasilinear range. It is possible to separate the electrical and mechanical losses and to determine the range of the linear work.

Finally, the authors conclude the lack of an accurate method of the quality factor measurement (and, consequently, of the losses) for piezoelectric resonators with low quality factors, and in the case when nonlinear effects occur.

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