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MAKING USE OF BRAGG'S DIFFRACTION FOR INVESTIGATIONS OF THE ACOUSTIC ACTIVITY OF BISMUTH-GERMANIUM OXIDE

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The essence of the acoustic activity of crystals has been discussed. A measuring system for the investigation of this activity has been presented. The measuring method makes use of Bragg's type diffraction of laser light in an acoustic wave. Measurements of the gyration constant G_{23} for bismuth-germanium oxide have been made.

1. Acoustic activity of crystals

Acoustic activity of crystals is the result of the existence of the dispersion phenomenon, i.e., the dependence of the velocity wave propagation, and thus the elastic constants on the wave wector k. Assuming that $c_{ijkl} = c_{ijkl}(k)$, we can write Hooke's law, with an accuracy of the second order terms, in the form:

$$\sigma_{ij} = c_{ijkl} s_{kl} + b_{ijklm} \frac{\partial s_{kl}}{\partial x_m}, \tag{1}$$

where σ_{ij} is the stress tensor, c_{ijkl} – the tensor of elastic constants, s_{kl} – deformation tensor, b_{ijklm} – tensor of acoustic activity. As this is the tensor of the odd rank, the phenomenon of the acoustic activity occurs only in the crystals with no centre of symmetry. It may be shown [1] that its numerical value is of the order $c_{ijkl} \cdot a$, where a is the lattice constant. In view of this, taking into account dependence (1), we infer that dispersion, and thus also the acoustic activity, may appear at frequencies of at least 10^8 Hz and higher.

The tensor of acoustic activity b_{ijklm} is usually written in the form:

$$b_{ijklm} = \delta_{ijq} G_{qklm}, \tag{2}$$

where G_{qklm} is the tensor of acoustic gyration, whereas δ_{ijq} is a unitary tensor defined as follows:

$$\delta_{ijq} = \begin{cases} 1 \text{ with } ijq = 123,\ 231,\ 312, \\ -1 \text{ with } ijq = 132,\ 213,\ 321, \\ 0 \text{ in remaining cases.} \end{cases}$$

If the dependence expressed by formula (2) is substituted in the equation of motion

$$\varrho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} \tag{3}$$

we obtain

$$\varrho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_k}{\partial x_l \partial x_j} + b_{ijklm} \frac{\partial^3 u_k}{\partial x_l \partial x_j \partial x_m}. \tag{4}$$

Assuming the solution from equation (4) in the form of a plane wave

$$u_i = u_{oi} e^{i(k_t x_t - \omega t)} \tag{5}$$

we obtain Christoffel equation

$$(c_{ijkl} k_l k_j + i b_{ijklm} k_l k_j k_m - \varrho \omega^2 \delta_{ik}) u_{ok} = 0 \tag{6a}$$

and respectively

$$|c_{ijkl} \varkappa_l \varkappa_j + i b_{ijklm} \varkappa_l \varkappa_j k_m - \varrho v^2 \delta_{ik}| = 0, \tag{6b}$$

where k_m is the component of the wave vector, $\varkappa_l$ – component of the unitary vector in the direction of wave propagation, v – the velocity of wave propagation, δ_{ik} – Kronecker delta.

After making use of the definition of the gyration tensor equation (6b) will be written in the form:

$$|\Gamma_{ik} + ik G_{ik} - \varrho v^2 \delta_{ik}| = 0, \tag{7}$$

where $\Gamma_{ik} = c_{ijkl} \varkappa_j \varkappa_l$, $G_{ik} = \delta_{ikp} G_{mpjl} \varkappa_m \varkappa_j \varkappa_l$.

Let us now consider the effect of the acoustic activity of a crystal on the propagation of an acoustic wave. By the way of an example let us take a crystal of cubic structure in which an acoustic wave is propagated in one of the principal directions. We then have:

$$\Gamma_{11} = c_{11}, \tag{8a}$$

$$\Gamma_{12} = \Gamma_{13} = \Gamma_{23} = 0, \tag{8b}$$

$$\Gamma_{22} = \Gamma_{33} = c_{44}, \tag{8c}$$

$$G_{12} = G_{31} = 0, \tag{8d}$$

$$G_{23} = -G_{32} = G. \tag{8e}$$

In such case, the determinant of Christoffel equation assumes the form:

$$\begin{vmatrix} c_{11}-\varrho v^2 & 0 & 0 \\ 0 & c_{44}-\varrho v^2 & ikG \\ 0 & -ikG & c_{44}-\varrho v^2 \end{vmatrix} = 0. \tag{9}$$

Solving equation (9) we obtain the expressions for the velocity of propagation of longitudinal wave $v_L = (c_{11}/\varrho)^{1/2}$, and of two transverse waves

$$v_{T1} = \sqrt{\frac{c_{44}+kG}{\varrho}} \cong v_{T0}\left(1+\frac{1}{2}\frac{kG}{c_{44}}\right), \tag{10a}$$

$$v_{T2} = \sqrt{\frac{c_{44}-kG}{\varrho}} \cong v_{T0}\left(1-\frac{1}{2}\frac{kG}{c_{44}}\right), \tag{10b}$$

where $v_{T0} = \sqrt{\frac{c_{44}}{\varrho}}$.

Substituting the velocities determined in Christoffel equation we may determine the polarization of the wave of the velocity given. For the waves v_{T1} and v_{T2} we shall get respectively:

$$u_y = iu_z, \tag{11a}$$

$$u_y = -iu_z. \tag{11b}$$

This means that these waves have circular laevorotatory and dextrorotatory polarization.

If we put together two waves of circular laevorotatory and dextrorotatory polarization, we obtain a linearly polarized wave of the amplitude twice higher the amplitude of each of the components.

In the case when $v_{T1} = v_{T2}$, there is no phase difference between the waves, and the vector of polarization has a constant direction in space (Fig. 1a). However, if

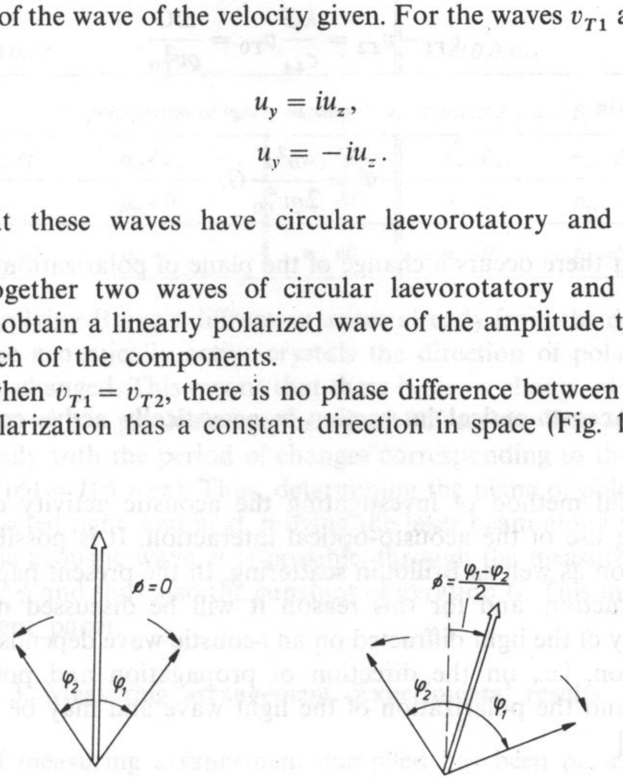


FIG. 1. Direction of wave polarization in the case when: a) $v_{T1} = v_{T2}$, b) $v_{T2} \neq v_{T1}$

dispersion occurs in the medium, that is $v_{T1} \neq v_{T2}$, a difference of phases will be created between the waves (Fig. 1b). After the waves have travelled path l, the phases will be

$$\varphi_1 = \frac{\omega}{v_{T1}} l, \tag{12a}$$

$$\varphi_2 = \frac{\omega}{v_{T2}} l, \tag{12b}$$

whereas the difference of phases $\Delta\varphi$ and the rotation of the polarization plane Φ are respectively

$$\Delta\varphi = \varphi_1 - \varphi_2 = \omega l \left(\frac{1}{v_{T1}} - \frac{1}{v_{T2}} \right), \tag{13a}$$

$$\Phi = \frac{\varphi_1 - \varphi_2}{2} = \frac{\omega l}{2} \frac{v_{T1} - v_{T2}}{v_{T1} v_{T2}} \cong \frac{\omega l}{2} \frac{\Delta v}{v_{T0}^2}. \tag{13b}$$

Considering that

$$v_{T1} - v_{T2} = \frac{kG}{c_{44}} v_{T0} = \frac{\omega G}{\varrho v_{T0}^2}, \tag{14}$$

we finally obtain

$$\Phi = \frac{\omega^2 l}{2 \varrho v_{T0}^4} G. \tag{15}$$

This means that there occurs a change of the plane of polarization of the transverse acoustic wave.

2. Acousto-optical interaction in acoustically active crystals

A very useful method of investigating the acoustic activity of crystals is the method making use of the acousto-optical interaction. It is possible to apply here Bragg's difraction as well as Brillouin scattering. In the present paper use was made of Bragg's diffraction, and for this reason it will be discussed more fully.

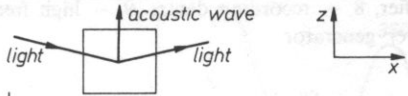
The intensity of the light diffracted on an acoustic wave depends on the geometry of the diffraction, i.e., on the direction of propagation and polarization of the acoustic wave and the polarization of the light wave and may be expressed by the dependence [2]

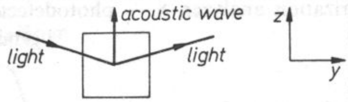
$$\frac{I}{I_0} \sim (p_{ijkl} \alpha_i \beta_j \varkappa_k \gamma_l)^2, \tag{16}$$

where p_{ijkl} are the components of the tensor of photoelastic constants, α_i, β_j – constants of the unitary vector determining the polarization of incident and diffracted light, $\varkappa_k$, γ_l – unitary vectors in the direction of propagation and polarization of an acoustic wave. This means that in the particular crystal, and thus in the determined form of the tensor p_{ijkl}, diffraction is possible only for a certain polarization of the diffracted and incident light, and for a certain polarization and direction of propagation of an acoustic wave. A change of the plane of polarization of an acoustic wave, which is what takes place in acoustically active crystals, results in the change of the intensity of the diffracted light. In particular, this intensity may be equal to zero.

In table 1 are given, as an example, some possible geometries of diffraction at a particular direction of propagation of an acoustic wave for a cubic system.

Table 1.
diffraction of light in acoustic waves in crystals of cubic system





polarization of acoustic wave						
	$\varkappa=(0,0,1)$			$\varkappa=(0,0,1)$		
	polarization of light: incident / diffracted (α_i/β_j)					
	α_y/β_y	α_z/β_z	α_z/β_y	α_x/β_x	α_z/β_z	α_z/β_x
γ_x	$p_{25}=0$	$p_{35}=0$	$p_{45}=0$	$p_{15}=0$	$p_{35}=0$	$p_{55}\neq 0$
γ_y	$p_{24}=0$	$p_{34}=0$	$p_{44}\neq 0$	$p_{14}=0$	$p_{34}=0$	$p_{54}=0$

The idea of applying Bragg's diffraction arises already from the data above. It is known that in the acoustically active crystals the direction of polarization of the transverse wave is changed. This means that there is also a change of the intensity of the diffracted light with a particular geometry of diffraction. This intensity will be changed periodically with the period of changes corresponding to the change of the phase $\phi = \pi$, or $I(\phi) = I(\phi + n\pi)$. Thus, determining the plane of polarization of the incident and diffracted light, and next, moving the laser beam along the direction of propagation of the acoustic wave, it is possible, through the measurement of $I(l)$ or $I(\phi)$, to determine ϕ and thus also the constant of gyration G. This method has been used in the present paper.

3. Measuring arrangement. Experimental results

A diagram of measuring arrangement compiled has been presented in Fig. 2.

The source of light is a laser He-Ne of 50 mW. Suitable rotators, polarizers and analyzers provide suitable polarization of the incidence and diffraction light. The

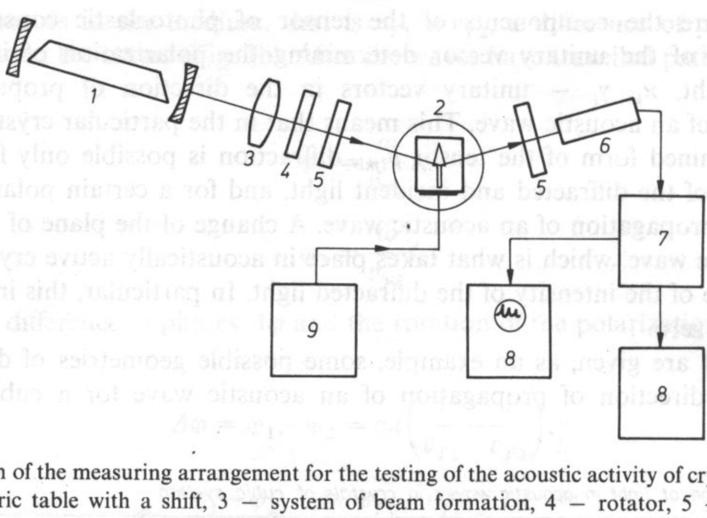


FIG. 2. Diagram of the measuring arrangement for the testing of the acoustic activity of crystals. 1 – laser, 2 – goniometric table with a shift, 3 – system of beam formation, 4 – rotator, 5 – polarizer and polarization analyzer, 8 – photodetector, 7 – amplifier, 8 – recording device, 9 – high frequency generator and power generator

samples tested are placed on a geometric table which may be moved in a horizontal plane by means of a motor, in the direction of propagation of an acoustic wave with the speed of about 1 cm/min. The diffracted light is registered by means of a photoelectric multiplier, amplifier, osciloscope and recorder. Piezoelectric transducers are activated by means of high frequency generators G3–20 and G4–37A.

Measurements were made for the crystals of bismuth-germanium oxide $Bi_{12}GeO_{20}$ (BGO). The crystals cut along principal directions were in the shape of rectangular prisms of the dimensions $8 \times 8 \times 60$ mm, $LiNbO_3$ transducers of the X cut were glued on the butting faces of the crystal so that the direction of polarization of the acoustic wave at the transducer was in the diffraction plane (Fig. 3). The sample

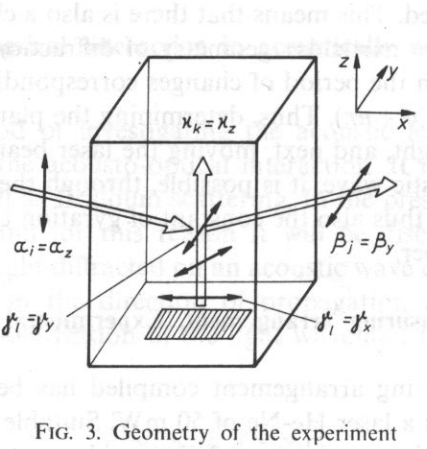


FIG. 3. Geometry of the experiment

was placed between the intersecting polarizer and analizer, i.e., diffraction was studied while changing the plane of polarization. The initial conditions are thus as follows $\alpha_i = \alpha_z$, $\beta_j = \beta_y$, $\varkappa_k = \varkappa_z$, $\gamma_l = \gamma_x$. The photoelastic constant corresponding to these conditions is $p_{3231} = p_{45}$. For the crystal BGO (cubic structure) $p_{45} = 0$ [3]. This means that the initial intensity of the diffracted light $I(z = 0) = 0$. When moving the crystal along the direction of propagation of an acoustic wave (with an immobile laser beam) there is a diffraction of light in the transverse wave of the polarization γ_y which appears as a result of the rotation of the polarization plane. Responsible for this diffraction is the photoelastic constant $p_{3232} = p_{44}$ which is, for the BGO crystals, different from zero.

The measurements were made for two frequencies: $f_1 = 360$ MHz and $f_2 = 450$ MHz. The measurement results are given in Fig. 4.

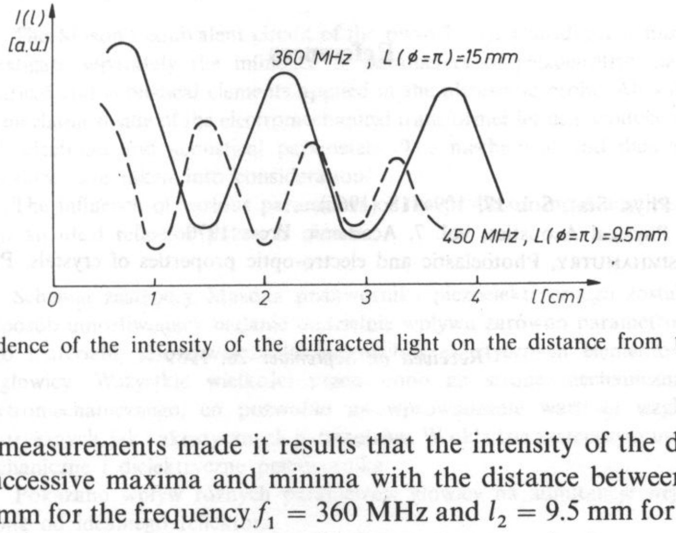


FIG. 4. Dependence of the intensity of the diffracted light on the distance from the transducer

From the measurements made it results that the intensity of the diffracted light reaches the successive maxima and minima with the distance between the maxima being $l_1 = 15$ mm for the frequency $f_1 = 360$ MHz and $l_2 = 9.5$ mm for the frequency $f_2 = 450$ MHz. Assuming that to these l values corresponds $\phi = \pi$, we may determine, on the basis of formula (15), the constant G which is:

$$G = \frac{\varrho v_{T0}^4}{2\pi l f^2}. \tag{20}$$

Substituting in formula (20) the experimental values obtained, as well as the material constants: $\varrho = 9.252 \cdot 10^3 \text{kg} \cdot \text{m}^{-3}$, $v_{T0} = 1662 \text{ m} \cdot \text{s}^{-1}$, we obtain the numerical value of the constant $G = 5.8 \text{ N m}^{-1}$. Taking into account the accuracy of the determination L which is mainly dependent on the diameter of the laser beam and is about 0,5 mm, we may write that the constant of acoustic gyration $G = G_{23}$ for the crystals BGO is $(5.8 \pm 0.8) \text{ Nm}^{-1}$.

4. Summary

The measurements made have shown that Bragg's diffraction of laser light in an acoustic wave is very useful for the investigating of the acoustic activity of crystals.

The investigation of the acoustic activity, or in general, of the polarization effects with the propagation of acoustic waves makes it possible to obtain significant information in the field of the acoustics of solids. In particular, it is possible to analyze dispersion and influence of acoustic waves, acoustic anisotropy, non-linear effects, accuracy of orientation of crystals.

It seems that of particular importance is the investigation of forced activity and of forced polarization effects since in this way it is possible to obtain information about the stresses and deformations in crystals which is of partial significance. Further research will be made in this direction.

References

[1] G. Kluge, Phys. Stat. Sol. **17**, 109–118 (1966).
[2] V. Mason, Physical Acoustics, vol. 7, Academic Press 1970.
[3] T. S. Narasimhamutry, Photoelastic and electro-optic properties of crystals, Plenum Press 1981.

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