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LOW FREQUENCY APPROXIMATION OF MUTUAL MODAL RADIATION EFFICIENCY OF A VIBRATING RECTANGULAR PLATE

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This paper presents some elementary formulations for the mutual modal radiation efficiency of a simply supported rectangular plate embedded into a rigid infinite baffle. The magnitude makes it possible to introduce the intermodal plate's interactions into the total radiation efficiency of the plate vibrating under the influence of an external surface force. The approximate formula has been expressed as a combinations of some trigonometric and special functions. The formula is convenient for some numerical computations of the modal and total radiation efficiency values of the plate.

Key words: modal and total radiation efficiency, sound radiation, sound pressure, sound power.

1. Introduction

The problem of modal radiation resistance of a rectangular plate was reported in the literature earlier. So far, some integral formulations for radiation self-resistance as well as for their low-frequency and high-frequency approximations were presented [1–5]. However, approximations presented by DAVIES [2] are useful only. The author expressed only a part of the corresponding integrand as its expansion series and left all the functions oscillating with a change in frequency unchanged. As a result he obtained much higher accuracy than the others – formulas weakly dependent on the modal numbers. Nonetheless, he included the zero expansion term, only, and his formulas show a big level of approximation error. Therefore, the formulas are useful for some rough numerical computations only. Paper [6] presents an approximation for the radiation self-resistance of a rectangular plate similar to that presented by Davies but including some more expansion terms. As a consequence, the frequency range, where the approximation is valid, became wider and the approximation error became much smaller.

For some computations of such vibroacoustic magnitudes as radiated sound power and acoustic radiation impedance it is necessary to know the modal radiation impedance as well as the intermodal mutual radiation impedance. So far, there were no approximation intermodal mutual radiation resistance formulas of a rectangular plate presented in the literature. Therefore, presenting such formulas is the main aim of this paper.

2. Fourier representation

A flat harmonically vibrating simply supported rectangular plate has been embedded into a flat rigid infinite baffle. Internal friction has been neglected. It has been assumed that transverse deflections of the plate are small as compared with the plate edge lengths a and b. A linear model of plate by Kirchhoff–Love has been used. The plate mode shape of mode mn is [7]

$$W_{mn}(x,y) = 2\sin m\pi\left(\frac{x}{a}+\frac{1}{2}\right)\sin n\pi\left(\frac{y}{b}+\frac{1}{2}\right), \tag{1}$$

where x, y – Cartesian coordinates of the plate point, $0x$ axis is parallel to the plate edges of length a and $0y$ axis is parallel to the plate edges of length b, m, $n = 1,\ 2,\ 3,\ \ldots$ – modal numbers. Solution (1) satisfies the equation of motion of the plate $(k_{mn}^{-4}\nabla^4 - 1)$ $W_{mn}(x,y) = 0$, where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $k_{mn}^2 = \pi^2[(m/a)^2 + (n/b)^2]$ – structural wavenumber of the plate raised to its second power. The integral for the time-averaged intermodal sound radiation power of modes mn and pq can be formulated as [8]

$$\mathit{\Pi}_{mn,pq} = \frac{1}{2}\int_S p_{mn}\, v_{pq}^*\,\mathrm{d}S, \tag{2}$$

where S – surface enclosing the plate, p_{mn} – modal amplitude of radiated acoustic sound pressure exerted by the plate via its mode mn on surface S, v_{pq}^* – conjugate value for vibration velocity of acoustic particle $v_{pq} = -i\omega_{pq}W_{pq}$ related to mode pq and normal to surface S given that the time dependence is $e^{-i\omega t}$, $\omega_{pq} = k_{pq}^2\sqrt{D_E/\rho h}$ – eigenfrequency, $D_E = Eh^3/[12(1-\nu^2)]$ – bending stiffness, ν – Poisson ratio, ρ, h – the plate density and thickness, respectively. It is possible to express the modal sound pressure amplitude as

$$p_{mn}(x,y) = i\rho_0 kc\int_{S_0} v_{mn}(x_0,y_0)\, G(x,y,0\,|\,x_0,y_0,0)\,\mathrm{d}x_0\,\mathrm{d}y_0, \tag{3}$$

where Green's function in its Fourier representation for the Neumann boundary value problem for the Helmholtz equation for half-space $z \geq 0$ has been formulated as

$$G(x, y, z \,|\, x_0, y_0, 0) = \frac{i}{4\pi^2} \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \exp\left\{i[\xi(x - x_0) + \eta(y - y_0) + \zeta z]\right\} \frac{\mathrm{d}\xi \, \mathrm{d}\eta}{\zeta}, \tag{4}$$

where $\zeta^2 = k^2 - \xi^2 - \eta^2$. In the case of impedance approach, surface S is extended closely to plate's surface, i.e. for $z = 0$. Inserting Green's function (4) into Eq. (3) makes it possible to formulate the modal sound pressure amplitude as

$$\begin{aligned} p_{mn}(x, y) \;&=\; -\frac{i\rho_0 k^2 c\omega_{mn}}{4\pi^2} \\ &\times \int\limits_0^{2\pi} \int\limits_0^{\pi/2 - i\infty} U_{mn}(\vartheta, \varphi) \exp\big[ik \sin\vartheta (x \cos\varphi + y \sin\varphi)\big] \sin\vartheta \mathrm{d}\vartheta \, \mathrm{d}\varphi, \end{aligned} \tag{5}$$

where the following transformations have been applied $\xi = k \sin\vartheta \cos\varphi$, $\eta = k \sin\vartheta$ $\sin\varphi$, $\zeta = k \cos\vartheta$, $\mathrm{d}\xi \, \mathrm{d}\eta = k^2 \sin\vartheta \, \cos\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$, $\vartheta = \vartheta' + i\vartheta'' \in \mathbb{C}$, $0 \leq \varphi \leq 2\pi$ and the following denotations have been used: ρ_0 – rest density of the surrounding medium, c – sound velocity, $k = 2\pi/\lambda$ – acoustic wavenumber, and

$$\begin{aligned} U_{mn}(\vartheta, \varphi) \;&=\; \int\limits_{S_0} W_{mn}(x_0, y_0) \exp\big[-ik \sin\vartheta (x_0 \cos\varphi + y_0 \sin\varphi)\big] \, \mathrm{d}x_0 \, \mathrm{d}y_0 \\ &= \frac{2ab}{\pi^2 mn} \, \frac{\exp(i\alpha/2) - (-1)^m \exp(-i\alpha/2)}{1 - (\alpha/m\pi)^2} \\ &\quad \times \frac{\exp(i\beta/2) - (-1)^n \exp(-i\beta/2)}{1 - (\beta/n\pi)^2}, \end{aligned} \tag{6}$$

$\alpha = ka \sin\vartheta \cos\varphi$, $\beta = kb \sin\vartheta \sin\varphi$. The intermodal radiation sound power can be expressed in its Fourier representation for modenumber pairs mp and nq (each pair must contains modenumbers of the same parity)

$$\varPi_{mn,pq} = \frac{\rho_0 k^2 c\omega_{mn}\omega_{pq}}{8\pi^2} \int\limits_0^{2\pi} \int\limits_0^{\pi/2 - i\infty} U_{mn}(\vartheta, \varphi) \, U^*_{pq}(\vartheta, \varphi) \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi. \tag{7}$$

The intermodal reference radiation sound power can be defined as $\varPi^{(\infty)}_{mn,pq} = \sqrt{\varPi^{(\infty)}_{mn}} \cdot \sqrt{\varPi^{(\infty)}_{pq}}$ where $\varPi^{(\infty)}_{mn} = (\rho_0 c/2) \int\limits_S v_{mn} v^*_{mn} \, \mathrm{d}S$, $v_{mn} v^*_{mn} = \omega^2_{mn} W^2_{mn}(x, y)$ for some time harmonic processes. Therefore, $\varPi^{(\infty)}_{mn} = (\rho_0 c/2) \int\limits_{-a/2}^{a/2} \int\limits_{-b/2}^{b/2} \omega^2_{mn} W^2_{mn}(x, y) \, \mathrm{d}y \, \mathrm{d}x =$

$(ab/2)\,\rho_0 c\omega_{mn}^2$ and finally $\Pi_{mn,pq}^{(\infty)} = (ab/2)\rho_0 c\omega_{mn}\omega_{pq}$. Using Eqs. (6) and (7) makes it possible to express the normalized intermodal radiation impedance of mode pair mn and pq in its Fourier representation as

$$\zeta_{mn,pq} \equiv \frac{\Pi_{mn,pq}}{\Pi_{mn,pq}^{(\infty)}} = \frac{4k^2ab}{\pi^6 mnpq} \int\limits_0^{2\pi} \int\limits_0^{\pi/2-i\infty} \frac{1-(-1)^m\cos\alpha}{[1-(\alpha/\pi m)^2][1-(\beta/\pi n)^2]} \times \frac{1-(-1)^n\cos\beta}{[1-(\alpha/\pi p)^2][1-(\beta/\pi q)^2]} \sin\vartheta \,\mathrm{d}\vartheta\,\mathrm{d}\varphi, \tag{8}$$

where the pairs mp and nq must contain modenumbers of the same parity as in Eq. (7). For all the other modenumber combinations $\zeta_{mn,pq} = 0$. Equation (8) represents a complex magnitude which integrated along ϑ variable within its limits of $(0, \pi/2)$ gives the normalized intermodal radiation resistance and integrated within the limits of $(\pi/2, \pi/2 - i\infty)$ gives the normalized intermodal radiation reactance.

3. Low frequency approximation

It is necessary to compute the approximate value of integral from Eq. (8) applying the method analogous as presented in [2] and [6]. For this purpose, the denominators of the corresponding integrands have been expressed as their expansion series expanded around points $\alpha_m \equiv \alpha/m\pi = 0$, $\alpha_p \equiv \alpha/p\pi = 0$, $\beta_n \equiv \beta/n\pi = 0$, $\beta_q \equiv \beta/q\pi = 0$ giving

$$\left[(1-\alpha_m^2)(1-\alpha_p^2)(1-\beta_n^2)(1-\beta_q^2)\right]^{-1} \simeq \varepsilon_0 - \varepsilon_1 + \varepsilon_2 + O(\alpha_m^6 + \alpha_p^6 + \beta_n^6 + \beta_q^6), \tag{9}$$

where function $O(\cdot)$ denotes the approximation error order and

$$\begin{aligned} \varepsilon_0 &= 1, \qquad \varepsilon_1 = \alpha_m^2 + \alpha_p^2 + \beta_n^2 + \beta_q^2, \\ \varepsilon_2 &= \alpha_m^2\alpha_p^2 + \alpha_m^2\beta_n^2 + \alpha_p^2\beta_n^2 + \alpha_m^2\beta_q^2 + \alpha_p^2\beta_q^2 + \beta_n^2\beta_q^2 + \alpha_m^4 + \alpha_p^4 + \beta_n^4 + \beta_q^4. \end{aligned} \tag{10}$$

Further, the expansion series from Eq. (9) has been inserted into integral (8), and integrated term by term giving the low frequency approximation for the radiation resistance covering the three initial expansion terms

$$\operatorname{Re}\zeta_{mn,pq} = \frac{4k^2ab}{\pi^6 mnpq}\left[\sum_{r=0}^{N-1}(-1)^r I_r + O(\varepsilon^{2N})\right], \tag{11}$$

where $\varepsilon^{2N} = (k/\pi)^{2N}[a^{2N}(m^{-2N} + p^{-2N}) + b^{2N}(n^{-2N} + q^{-2N})]$, $N \in \{1,\ 2,\ 3\}$. Assuming in Eq. (11) that $N = 1$ results in formulas presented earlier by Davies for $m = p$, $n = q$ of the lowest numerical accuracy [2]. Assuming N = 2 or N = 3 results in increasing the numerical accuracy. Moreover, formulas identical as presented in [6] have been obtained for $m = p$, $n = q$. The elementary values of the integrals have been computed in the same way as presented in [6] which gives

$$\frac{1}{2\pi} I_0 = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi/2} \varepsilon_0 [1 - (-1)^m \cos\alpha][1 - (-1)^n \cos\beta] \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$$
$$= 1 - (-1)^m \mathrm{sinc} ka - (-1)^n \mathrm{sinc} kb + (-1)^{m+n} \mathrm{sinc}\gamma; \tag{12a}$$

$$\frac{\pi}{2} I_1 = \frac{\pi}{2} \int_0^{2\pi} \int_0^{\pi/2} \varepsilon_1 \big[1 - (-1)^m \cos\alpha\big] \big[1 - (-1)^n \cos\beta\big] \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$$
$$= \frac{1}{3} \left[(ka)^2 \left(\frac{1}{m^2} + \frac{1}{p^2} \right) + (kb)^2 \left(\frac{1}{n^2} + \frac{1}{q^2} \right) \right]$$
$$- (-1)^m \left\{ \left(\frac{1}{m^2} + \frac{1}{p^2} \right) \big[(ka)^2 \mathrm{sinc} ka - 2\mathrm{cs} ka \big] + \frac{b^2}{a^2} \left(\frac{1}{n^2} + \frac{1}{q^2} \right) \mathrm{cs} ka \right\}$$
$$- (-1)^n \left\{ \left(\frac{1}{n^2} + \frac{1}{q^2} \right) \big[(kb)^2 \sin \mathrm{c} kb - 2\mathrm{cs} kb \big] + \frac{a^2}{b^2} \left(\frac{1}{m^2} + \frac{1}{p^2} \right) \mathrm{cs} kb \right\}$$
$$+ (-1)^{m+n} \left\{ \frac{a^2}{a^2 + b^2} \left(\frac{1}{m^2} + \frac{1}{p^2} \right) \left[(ka)^2 \mathrm{sinc}\gamma + \left(1 - \frac{3a^2}{a^2 + b^2} \right) \mathrm{cs}\gamma \right] \right.$$
$$\left. + \frac{b^2}{a^2 + b^2} \left(\frac{1}{n^2} + \frac{1}{q^2} \right) \left[(kb)^2 \mathrm{sinc}\gamma + \left(1 - \frac{3b^2}{a^2 + b^2} \right) \mathrm{cs}\gamma \right] \right\}; \tag{12b}$$

$$\frac{\pi^3}{2} I_2 = \frac{\pi^3}{2} \int_0^{2\pi} \int_0^{\pi/2} \varepsilon_2 \big[1 - (-1)^m \cos\alpha\big] \big[1 - (-1)^n \cos\beta\big] \sin\vartheta \, \mathrm{d}\vartheta \, \mathrm{d}\varphi$$
$$= \frac{(ka)^4}{5} \left(\frac{1}{m^4} + \frac{1}{m^2 p^2} + \frac{1}{p^4} \right) + \frac{(k^2 ab)^2}{15} \left(\frac{1}{m^2} + \frac{1}{p^2} \right) \left(\frac{1}{n^2} + \frac{1}{q^2} \right)$$
$$+ \frac{(kb)^4}{5} \left(\frac{1}{n^4} + \frac{1}{n^2 q^2} + \frac{1}{q^4} \right)$$
$$- (-1)^m (ka)^2 \left\{ \left(\frac{1}{m^4} + \frac{1}{m^2 p^2} + \frac{1}{p^4} \right) \left[\left(1 - \frac{8}{(ka)^2} \right) (ka)^2 \mathrm{sinc} ka \right. \right.$$
$$\left. -4 \left(1 - \frac{6}{(ka)^2} \right) \mathrm{cs} ka \right]$$
$$+ \frac{b^2}{a^2} \left(\frac{1}{m^2} + \frac{1}{p^2} \right) \left(\frac{1}{n^2} + \frac{1}{q^2} \right) \left[4\mathrm{sinc} ka + \left(1 - \frac{12}{(ka)^2} \right) \mathrm{cs} ka \right]$$
$$\left. - 3 \frac{b^4}{a^4} \left(\frac{1}{n^4} + \frac{1}{n^2 q^2} + \frac{1}{q^4} \right) \left(\mathrm{sinc} ka - \frac{3\mathrm{cs} ka}{(ka)^2} \right) \right\}$$
$$- (-1)^n (kb)^2 \left\{ \left(\frac{1}{n^4} + \frac{1}{n^2 q^2} + \frac{1}{q^4} \right) \left[\left(1 - \frac{8}{(kb)^2} \right) (kb)^2 \mathrm{sinc} kb \right. \right.$$

$$- 4\left(1-\frac{6}{(kb)^2}\right)\text{cs}kb\Bigg] + \frac{a^2}{b^2}\left(\frac{1}{m^2}+\frac{1}{p^2}\right)\left(\frac{1}{n^2}+\frac{1}{q^2}\right)\left[4\text{sinc}kb + \left(1-\frac{12}{(kb)^2}\right)\text{cs}kb\right]$$

$$- 3\frac{a^4}{b^4}\left(\frac{1}{m^4}+\frac{1}{m^2p^2}+\frac{1}{p^4}\right)\left(\text{sinc}kb - \frac{3\text{cs}kb}{(kb)^2}\right)\Bigg\}$$

$$+ \frac{(-1)^{m+n}}{4}\Bigg((ka)^4\left(\frac{1}{m^4}+\frac{1}{m^2p^2}+\frac{1}{p^4}\right)\Bigg\{\left(1+\frac{13}{\gamma^2}\right)\text{sinc}\gamma + \left(2-\frac{39}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2}$$

$$- 2\varepsilon^2\left[\left(1-\frac{5}{\gamma^2}\right)\text{sinc}\gamma - \left(4-\frac{15}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2}\right]$$

$$+ \varepsilon^4\left[\left(1-\frac{35}{\gamma^2}\right)\text{sinc}\gamma - 5\left(2-\frac{21}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2}\right]\Bigg\}$$

$$+ (k^2ab)^2\left(\frac{1}{m^2}+\frac{1}{p^4}\right)\left(\frac{1}{n^2}+\frac{1}{q^4}\right)\Bigg\{\left(1-\frac{19}{\gamma^2}\right)\text{sinc}\gamma$$

$$-3\left(2-\frac{19}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2} - \varepsilon^4\left[\left(1-\frac{35}{\gamma^2}\right)\text{sinc}\gamma - 5\left(2-\frac{21}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2}\right]\Bigg\}$$

$$+ (kb)^4\left(\frac{1}{n^4}+\frac{1}{n^2q^2}+\frac{1}{q^4}\right)\Bigg\{\left(1+\frac{13}{\gamma^2}\right)\text{sinc}\gamma + \left(2-\frac{39}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2}$$

$$+ 2\varepsilon^2\left[\left(1-\frac{5}{\gamma^2}\right)\text{sinc}\gamma - \left(4-\frac{15}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2}\right]$$

$$+\varepsilon^4\left[\left(1-\frac{35}{\gamma^2}\right)\text{sinc}\gamma - 5\left(2-\frac{21}{\gamma^2}\right)\frac{\text{cs}\gamma}{\gamma^2}\right]\Bigg\}\Bigg), \tag{12c}$$

where $\gamma^2 = k^2(a^2+b^2)$, $\text{sinc}\, u = \sin u/u$, $\text{cs}u = \text{sinc}u - \cos u$, $\varepsilon^2 = (b^2-a^2)/(b^2+a^2)$. Equations (11), (12) are the generalization of all the earlier presented formulas. They are valid for computations of the modal radiation resistance as well as for computations of the intermodal radiation resistance, and they assure the highest numerical accuracy known so far. Moreover, assuming $m = p$ and $n = q$ results in the modal radiation resistance formula known from [6] just from the intermodal radiation resistance from Eq. (11).

4. Numerical analysis

All the numerical results have been prepared for a sample steel rectangular plate of sizes $a = 0.5$ [m], $b = 1.0$ [m] and $h = 1$ [mm]. Certainly, the formulas presented can be used for any simply supported rectangular plates given that they are thin as compared with their remaining sizes.

The normalized intermodal radiation resistance has been presented in Fig. 1a for some sample mode pairs. A good agreement of the low frequency approximation and the integral formulation is shown for $0 \le k/k_{mn,pq} < 0.2$ where $k_{mn,pq}^2 = k_{mn}^2 + k_{pq}^2$. The theoretical approximation error value Err has been computed from Eq. (11) and it is equal to ε^{2N} whereas the estimated approximation error value has been computed from

$$\text{Err} = |\sigma_{\text{Int}} - \sigma_{\text{Approx}}|, \tag{13}$$

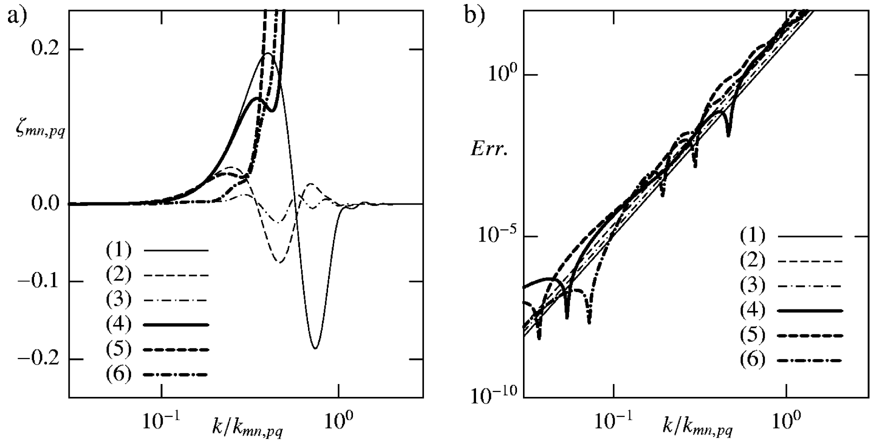

Fig. 1. a) Radiation efficiency $\zeta_{mn,pq}$. Key: (1) integral for modes (1,2) and (1,4); (2) integral for modes (3,4) and (1,2); (3) integral for modes (3,3) and (7,7); (4) approximation for modes (1,2) and (1,4); (5) approximation for modes (3,4) and (1,2); (6) approximation for modes (3,3) and (7,7). b) Approximation error Err. Key: (1) theory for modes (1,2) and (1,4); (2) theory for modes (3,4) and (1,2); (3) theory for modes (3,3) and (7,7); (4) estimation for modes (1,2) and (1,4); (5) estimation for modes (3,4) and (1,2); (6) estimation modes (3,3) and (7,7).

where values $\sigma_{\rm Int}$ and $\sigma_{\rm Approx}$ have been computed from Eqs. (8) and (11), respectively. Figure 1b shows that the estimated error value does not considerably exceed the theoretical error value within the whole low frequency range. Moreover, the error assumes values smaller than 10^{-2} for the relative frequencies $k/k_{mn,pq} < 0.2$ which confirms that the formulas presented herein gives a good approximation for the intermodal radiation resistance.

5. Concluding remarks

The low frequency formulas for the intermodal radiation resistance have been presented in the form useful for some numerical computations. The formulas gives considerably higher numerical accuracy than those presented earlier by Davies in [2]. Moreover, they are the generalization of those formulas and the enhanced formulas presented in [6] since they can be used for the modal radiation resistance as well as for the intermodal radiation resistance.

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