ARCHIVES OF ACOUSTICS
31, 4 (Supplement), 137–142 (2006)

HOLOGRAPHIC IMAGING OF CYLINDRICAL ULTRASONIC WAVES

I. GRULKOWSKI, D. JANKOWSKI, P. KWIEK

University of Gdańsk
Institute of Experimental Physics
Division of Acousto-Optics and Laser Physics
Wita Stwosza 57, 80-952 Gdańsk-Oliwa, Poland
e-mail: fizpk@univ.gda.pl

(*received June 15, 2006; accepted September 30, 2006*)

This report is devoted to the visualisation of standing cylindrical ultrasonic wave, generated by a radially vibrating piezoelectric shell, by means of time-average holographic interferometry. The theoretical basis of this technique was introduced, and the expression of light intensity distribution during holographic reconstruction of cylindrical waves was presented. Finally, the theory was verified by experimental findings. Quite a good agreement between theoretical predictions and experimental results was found.

Key words: acousto-optics, optical holographic interferometry, cylindrical ultrasonic waves.

1. Introduction

It was not until laser had been invented that holography was recognised as a powerful technique in pure and applied science. The advances in laser technology resulted in a large number of applications of optical holographic interferometry, including among others the imaging of ultrasonic fields [1, 2]. However, most of the theoretical and experimental papers were confined to the case of plane ultrasound [3–5].

The interaction of light with ultrasonic waves of cylindrical symmetry has recently received considerable attention [6–9]. The aim of this report is to visualise the standing cylindrical ultrasonic wave, generated by a vibrating shell, by means of time-average holographic interferometry. The theoretical backgrounds of this technique were introduced, and the expression of light intensity distribution during holographic reconstruction of cylindrical waves was experimentally verified.

2. Ultrasonic field generated by a vibrating circular cylinder

It is a widely-known fact that the simplest radial mode of shell vibration is represented by the Bessel function of the first kind of zero order [7, 10]. This axisymmetric mode can be easily obtained when a piezoelectric circular shell (transducer) vibrates radially. Accordingly, the acoustic pressure Δp generated inside this shell is described by:

$$\Delta p(r,t) = p_1 J_0(Kr)\cos(\Omega t), \tag{1}$$

where p_1 stands for the amplitude of acoustic pressure variations, r is radial coordinate, K and Ω are the wave number and the circular frequency of ultrasound, respectively.

Assuming that acoustic pressure variations cause proportional modulation of the refraction index of a medium, which the cylinder is filled with (i.e. neglecting all the nonlinear phenomena), the expression for the refractive index distribution inside vibrating cylinder is given by the following form (for detailed derivation we refer the reader to our previous paper [7]):

$$n(r,t) = n_0 + n_1 J_0(Kr)\cos(\Omega t), \tag{2}$$

where n_0 is the refractive index of the undisturbed medium, n_1 is the amplitude of refractive index variations. It should be emphasised that the distribution in Eq. (2) should be regarded as a standing wave. From the mathematical point of view this fact manifests in the separation of the spatial and temporal coordinates.

3. Time-average optical holographic interferometry

In order to describe the processes of hologram recording and reconstruction theoretically, we have to make the following assumptions:

- the light wave of the object beam is affected only in its phase (the acousto-optic cell acts as a phase grating; Raman–Nath regime);
- the exposure time of a hologram is much longer than the period of ultrasound (time-average holography) [5];
- the linear part of the relation between amplitude transmittance of the recording material and exposure is used (linearity of the holographic recording process);
- technique of holographic addition is used (i.e. first exposure – without ultrasound, second exposure – in the presence of ultrasound).

Using the theoretical considerations given in the previous reports (e.g. [3]) and taking into account the distribution represented by Eq. (2), the light intensity transmitted by the hologram during its reconstruction is expressed as follows:

$$I(r) \propto \left[1 + J_0\left(v_{\max} J_0(Kr)\right)\right]^2 . \tag{3}$$

Additionally, we introduced here the Raman-Nath parameter $v_{\max} = kn_1L$, where k denotes the wave number of the light and L stands for the depth of ultrasonic field (the length of piezoelectric shell). Note that by acousto-optic convention – in contrast to the parameters of light wave (k and ω) – the parameters of ultrasound (K and Ω) are written in capitals.

We have not presented here all the mathematical steps leading to this final equation for the sake of brevity. It is easily to see from Eq. (3) that time-average holography enables us to visualise the spatial distribution of the refractive index, which is a result of standing wave propagation inside vibrating shell.

4. Experiment

4.1. Experimental set-up

In order to verify the presented theory we designed a set-up, which – as shown in Fig. 1 – consisted of two subsystems: the holographic subsystem and the calibration subsystem (the latter is depicted by dashed line in Fig. 1). The light emitted by an argon ion laser (ILA 120-1; Carl Zeiss Jena; $\lambda = 488$ nm) was divided into two beams by a beamsplitter BS. The light beams in both object and reference arms were extended and spatially filtered using expanders EXP1 and EXP2, respectively. Additionally, the object having been holographically imaged was a cylindrical ceramic transducer TR, which was excited by a function generator GEN (PM5193; Philips) via power amplifier AMP (325LA; ENI). The piezoelectric shell was closed by glasses of high optical quality and filled with water, so we generated the standing ultrasonic wave of cylindrical symmetry. The cylinder axis and the axis of the light beam coincided. While the acousto-optic cell was working, the refractive index in the object optical beam was modulated. Having been directed into the holographic plate H (Agfa) by two mirrors M1 and M2, the reference beam interfered with the object beam, which passed through the shell. As we placed a diffuser (ground glass plate) D immediately behind the transducer, all the information about light interaction with ultrasound was integrated along the propagation direction of light and projected onto the plane D. In order to calibrate the whole system and determine the acoustic pressure, before the experiment we measured the light intensity at the on-axis point of the far field versus the voltage applied to the transducer. It was realised with the aid of a photomultiplier PMT (H5783P; Hama-

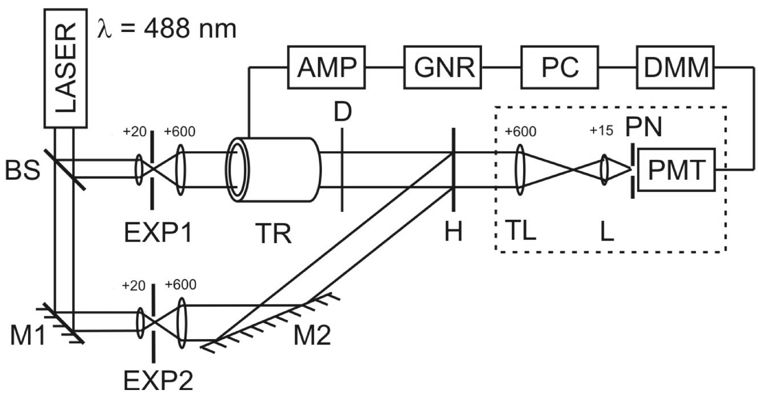

Fig. 1. The experimental set-up. BS – beamsplitter; EXP1, EXP2 – expanders in the object and reference beams, respectively; M1, M2 – mirrors; TR – cylindrical transducer; D – diffuser; H – holographic plate; TL – transforming lens; L – lens; PN – pinhole; PMT – photomultiplier; DMM – digital multimeter; PC – computer; GNR – synthesizer/function generator; AMP – power amplifier.

matsu) and a digital multimeter DMM (PM 2534; Philips). The detailed description of the subsystem used for this purpose is given elsewhere (e.g. [7]).

4.2. Results and discussion

We have applied the technique of holographic interferometry (the method of holographic addition [2, 11]). During the first exposure the ultrasound was switched off, while during the second exposure the transducer was generating the standing cylindrical ultrasonic wave, thus modulating the refractive index in the object beam. It should be noted that the total exposure time was estimated to cover the linear recording range of the holographic plate. During the reconstruction process, the CCD camera (ST-6V OPTO-HEAD; SBIG) and the objective were used. The transducer operated at its fundamental resonance frequency ($F = 482.75$ kHz; $v_{\max} = 9.3$). The picture of reconstructed hologram (virtual image) is presented in Fig. 2a.

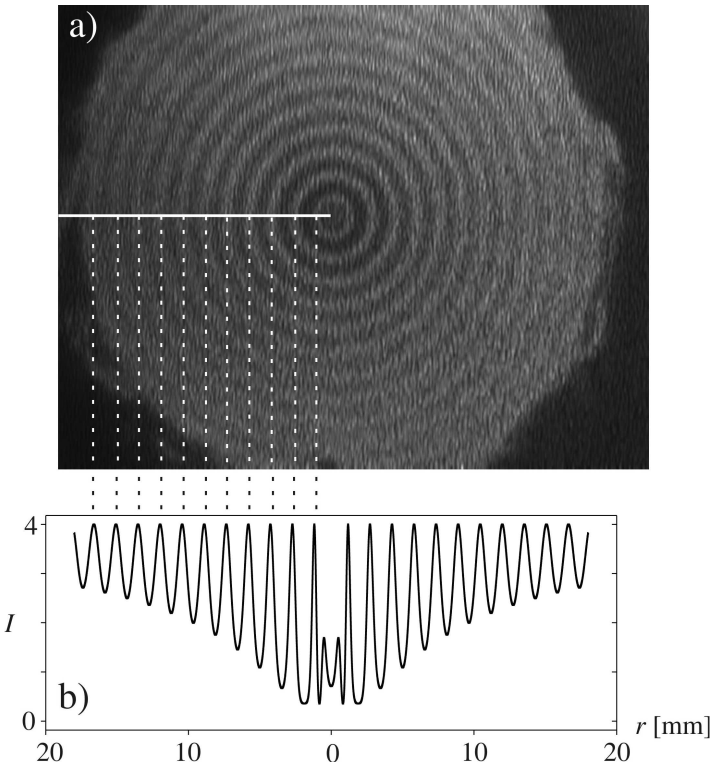

Fig. 2. Virtual image of the reconstructed hologram (a). Theoretical cross-section of the light intensity of the reconstructed hologram (b).

It is easily to recognise the axial symmetry of ultrasonic field, and the interferometric fringes appear as concentric rings. The interference pattern of the reconstructed hologram has been numerically simulated using Eq. (3), in which the experimental conditions were applied ($F = 482.75$ kHz; $v_{\max} = 9.3$). Figure 2b reveals the light intensity distribution versus radial position as a cross-section of the simulated virtual image based on Eq. (3). What is more, the brightest regions represent the nodal points of the ultrasonic field's spatial distribution ($J_0(Kr) = 0$ in Eq. (2) and $J_0(0) = 1$). We indicated the corresponding maxima of light intensity by dashed lines in Fig. 2.

Table 1. Radial positions of ultrasonic nodal points.

r_{th} [mm]	1.19	2.72	4.28	5.81	7.36	8.91	10.45	12.00	13.56	15.10	16.67
r_{exp} [mm]	1.12	2.64	4.28	5.84	7.44	8.91	10.42	11.98	13.58	15.10	16.78

In Table 1 we compared also the radial positions of nodal points calculated according to Eq. (3) for experimental conditions with the results obtained from experimental studies. We can observe a good agreement of experimental data with theoretical predictions. It must be also noted that there is no one particular nodal distance for cylindrical ultrasonic wave as the Bessel function of the first kind of zero order can be approximated by a periodic function only for large values of the argument.

5. Conclusions

It has been shown that time-average holographic interferometry permits visualisation of acoustic field distribution inside vibrating piezoelectric shell. One can imagine that considered technique might be successfully used for the identification of particular mode. We showed qualitative agreement of assumptions made in previous reports [6–9]. Moreover, when the shell is diffusely illuminated (the diffuser is placed in front of the shell), it is possible to observe three-dimensional structure of ultrasonic field inside cylindrical transducer.

Acknowledgments

This work was supported by the University of Gdańsk (grant # BW/5200-5-0212-6) as well as by the European Social Fund and Polish Government (project # Z/2.22/II/2.6/002/05 "Scholarship programme for Ph. D. students of the University of Gdańsk engaged in the research on the development of Pomerania's innovative economy").

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