ARCHIVES OF ACOUSTICS
31, 4 (Supplement), 239–245 (2006)

THE EFFECT OF MODAL LOCALIZATION ON REVERBERANT ENERGY DECAY IN A CASE OF TWO ACOUSTICALLY COUPLED ROOMS

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(*received June 15, 2006; accepted September 30, 2006*)

In the paper a modal analysis was used to describe a reverberation phenomenon in an irregularly shaped room. A theoretical model was limited to low sound frequencies, when eigenmodes are lightly damped, thus they may be approximated by normal acoustic modes of a hard-walled room. A utility of this method was demonstrated in a numerical example where the room in a form of two acoustically coupled rectangular subrooms was considered. A reverberation time was evaluated individually for each subroom from time decay of acoustic pressure amplitude for different distributions of absorbing materials of room walls and various positions of sound source under the condition that a total room absorption remained constant. Calculation results have shown a great influence of modes localization on a reverberant energy decay for a large difference between the absorption coefficients of walls in subrooms, because in this case for frequencies of some localized modes a substantial increase in the reverberation time was observed.

Key words: room acoustics, modal analysis, reverberation time, modes localization.

1. Introduction

The reverberation is a phenomenon which plays a major role in every aspect of room acoustics and yields a main criterion for the assessment of acoustic quality of enclosures. There are various theories for predicting the reverberation time: the geometric theory, the wave theory, the ray-tracing techniques and the statistical or the power flow methods. The geometrical room acoustics at best applies to enclosures with dimensions large compared to the wavelength. In this theory diffraction phenomena are neglected, since a propagation in straight lines is its main postulate. Likewise, interference of sound waves is not considered. A theory more reliable and adequate from the physical point of view but more difficult is that based upon the wave acoustics. The wave theory has a practical application for room dimensions which are comparable with the sound wavelength.

A usage of analytical methods in the wave acoustics is limited to the simplest room shapes such rectangular, triangular and cylindrical ones. In practice it is not uncommon to find that a room actually consists of several partial rooms which are coupled to each other. Examples of coupled rooms are theatres with boxes which communicate with a main room through small apertures only, or churches with several naves and chapels, thus the acoustic properties of coupled rooms have been investigated intensively in recent years [1, 9]. An application of the wave theory to complex room geometries, such as fractal cavities, rooms with irregular shapes or coupled rooms, was possible through numerical methods.

In the present paper, a combination of classical modal analysis with numerical implementation was used to predict a reverberation time in the room consisting of two connected rectangular subrooms (Fig. 1). In a theoretical model it was assumed that a system is lightly damped so coupling terms in a solution of wave equation were neglected and the pressure variable was expanded in "hard box modes". In a numerical example the reverberation time was determined for different distributions of absorbing materials on room walls under condition that a total room absorption remained constant. It was found that a substantial increase in a reverberation time, occurring for a large difference between the absorption coefficient of walls in subrooms, is the result of eigenmodes localization.

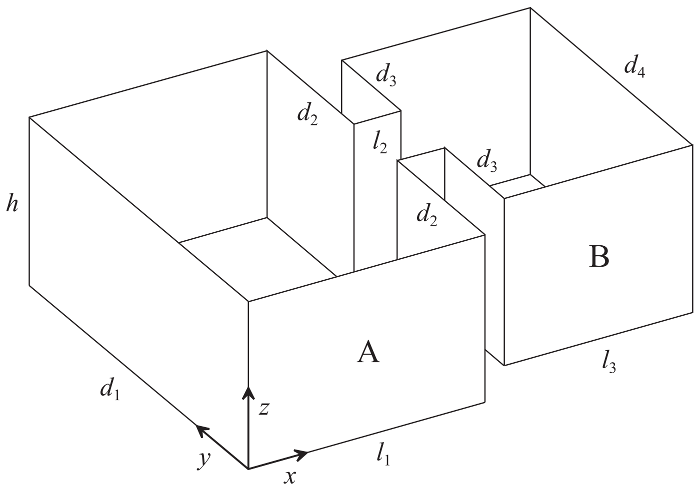

Fig. 1. Analyzed room consisting of two connected rectangular subrooms denoted by A and B.

2. Computational analysis

In a low frequency limit an irregularly shaped room may be treated as a resonator with characteristic acoustic modes determined by the eigenfunctions $\Phi_{mn}(\mathbf{r})$, $\mathbf{r} = (x, y, z)$, and the eigenfrequencies ω_{mn}, where $m = 0$, 1, 2... and $n = 0$, 1, 2... The eigenfunctions Φ_{mn} depend on a room shape and are mutually coupled through the impedance condition on absorptive walls, but in the range of low frequencies, where typical materials are characterized by a low absorption: $\Re\mathrm{e}(Z/\rho c) \gg 1$ (Z is a wall impedance, ρ is an air density, c is a sound speed), it is possible to assume that a distribution of modes amplitude is well approximated by the uncoupled eigenfunctions computed for perfectly rigid room walls [2]. In this case a decaying process of sound pressure is described by the equation [5]

$$\begin{aligned} p(\mathbf{r},t) \;&=\; -\frac{Q_{00}e^{-r_{00}t}}{r_{00}^2+\omega^2} \\ &+V^{1/2}\sum_{m=0}^{\infty}\sum_{=0}^{\infty}\frac{\xi_{mn}\Phi_{mn}(\mathbf{r})Q_{mn}e^{-r_{mn}t}\omega_{mn}\cos\left[(\omega_{mn}^2-r_{mn}^2)^{1/2}t-\alpha_{mn}\right]}{\sqrt{(\omega_{mn}^2-r_{mn}^2)\left[(\omega_{mn}^2-\omega^2)^2+4r_{mn}^2\omega^2\right]}}\,, \end{aligned} \tag{1}$$

where $\xi_{00}=0$ and $\xi_{mn}=1$ for the other values of m and n, V is a room volume, ω is a sound frequency, Q_{00} and Q_{mn} are factors determining a sound source intensity

$$Q_{00}=c^2/V\int\limits_V q(\mathbf{r})\,\mathrm{d}v, \qquad Q_{mn}=c^2/V^{1/2}\int\limits_V q(\mathbf{r})\Phi_{mn}(\mathbf{r})\,\mathrm{d}v, \tag{2}$$

r_{00} and r_{mn} are damping coefficients

$$r_{00}=\rho c^2/V\int\limits_S Z^{-1}\,\mathrm{d}s, \qquad r_{mn}=\frac{1}{2}\rho c^2\int\limits_S \Phi_{mn}^2/Z\,\mathrm{d}s, \tag{3}$$

where S is a surface of room walls, and α_{mn} is the phase shift given by

$$\alpha_{mn}=\tan^{-1}\left[\frac{r_{mn}(\omega_{mn}^2+\omega^2)}{(\omega_{mn}^2-\omega^2)(\omega_{mn}^2-r_{mn}^2)^{1/2}}\right]. \tag{4}$$

In a computational analysis the wall impedance is assumed to be a real number, i.e. the mass and stiffness of the absorbing material are neglected.

For a given frequency, a position and a distribution of sound source, Eqs. (1)–(4) make possible to predict the reverberation time in a room from calculated energy decay curves corresponding to a time history of the sound pressure level. In a computational simulation it was assumed that dimensions of a room are the following (in meters): $l_1=5$, $l_2=1$, $l_3=4$, $d_1=8$, $d_2=3.2$, $d_3=2.2$, $d_4=6$, $h=3$ (Fig. 1). The calculations were performed for two positions of a sound source (in meters): $x_0=2$, $y_0=5$, $z_0=1$ (subroom A) and $x_0=8$, $y_0=5$, $z_0=1$ (subroom B). The eigenfunctions Φ_{mn} and the eigenfrequencies ω_{mn} were computed with an application of the forced oscillator method with a finite difference algorithm [4].

In order to examine an influence of walls absorption on the reverberation time, in numerical analysis it was assumed that the absorption coefficient α_1 of material on walls in the subroom A and the absorption coefficient α_2 of material on walls in the subroom B were selected in this way, so that the total room absorption A remained constant, thus

$$\alpha_A=\frac{A}{S}=\frac{\alpha_1S_1+\alpha_2S_2+\alpha_3S_3}{S}=\mathrm{const}, \tag{5}$$

where S_1 is a surface of walls in the subroom A, S_2 is a surface of walls in the subroom B, α_3 and S_3 are an absorption coefficient and a surface of walls in a part of room connecting subrooms A and B, respectively. For an assumed value of the average absorption coefficient α_A, the surface impedances on room walls were found from the well-known relationship between the absorption coefficient α and the impedance ratio ξ [3]

$$\alpha = \frac{8}{\xi}\left[1 + \frac{1}{1+\xi} - \frac{2}{\xi}\ln(1+\xi)\right], \qquad \xi = Z/\rho c. \tag{6}$$

In the numerical example it was assumed that $\alpha_A = \alpha_3 = 0.15$, so the absorption coefficients α_1 and α_2 in subrooms A and B were changing quantities. The reverberation time was computed in a frequency range bounded from above by the "Schroeder frequency" [8]

$$f_s = c\sqrt{6/A}, \tag{7}$$

which denotes approximately the boundary between discrete room modes below f_s and reverberant room behavior above it. In multimode resonance systems the "Schroeder frequency" marks the transition from individual, well-separated resonances to many overlapping modes.

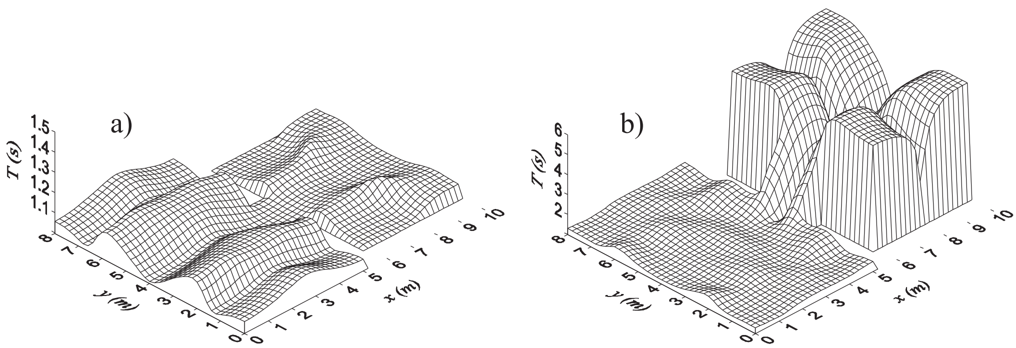

Fig. 2. Reverberation time at distance $z = 1.8$ m from room floor for two distributions of absorption material on room walls: a) $\alpha_1 = \alpha_2 = 0.15$, b) $\alpha_1 = 0.24$, $\alpha_2 = 0.016$. Point harmonic sound source of frequency 52 Hz located in subroom A.

The plots in Fig. 2 depict calculation results obtained for the frequency $f = 52$ Hz for two different distributions of absorption material and a sound source located in the subroom A. From these data it results, that for $\alpha_1 = \alpha_2$ (the uniform distribution of absorption material on room walls) the reverberation time T varies very slightly in an observation plane (Fig. 2a). However, when the absorption coefficient α_2 is much smaller than α_1 one observe surprisingly large values of T in the subroom B (Fig. 2b). In order to investigate this effect in more detail, for a given sound frequency f from the distribution of T in an observation plane the average values T_A of reverberation time in subrooms A and B were computed for assumed localizations of the sound source. Calculation results obtained in this way are shown in Fig. 3. The presented graphs illustrate the effect of modal localization on the reverberant energy decay because, as was shown by a detailed analysis of calculation data, peaks of the time T_A occur for frequencies of eigenmodes which are localized in the subroom B. The modal localization is the direct result of an irregular geometry of lateral walls in the analyzed system of two coupled subrooms, since in a rectangular room all eigenmodes are delocalized [6].

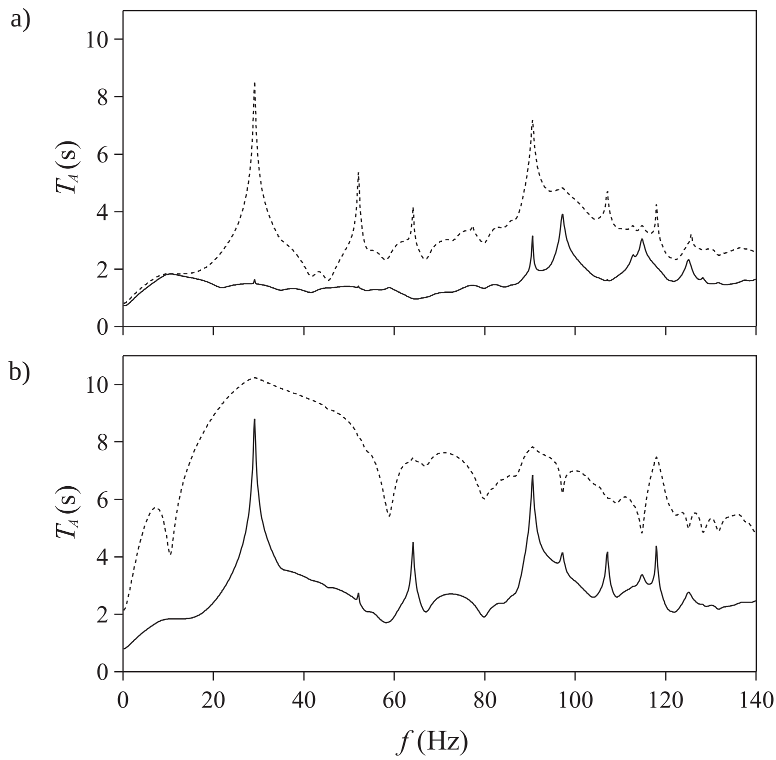

Fig. 3. Frequency dependence of average values of reverberation time in subroom A (solid lines) and subroom B (dashed lines) for absorption coefficients: $\alpha_1 = 0.24$, $\alpha_2 = 0.016$, and sound source located in: a) subroom A, b) subroom B. Calculation results obtained at distance $z = 1.8$ m from room floor.

As may be seen in Fig. 3, a frequency dependence of the reverberation time is strongly influenced by a source position. When it is located in the subroom A, that walls are covered by a material with a high absorption, sharp peaks of the reverberation time occur in the subroom B (Fig. 3a), where an energy of localized modes is concentrated, and the sound damping is much smaller than in the subroom A. If a sound source is in the subroom B, strong peaks of T_A are observed in the subroom A since the energy of modes localized in the subroom B is weakly damped in the subroom A. In this case, in a frequency dependence of T_A in the subroom B there are not sharp peaks because a mode localized in this subroom has a high amplitude in a steady-state [7], thus it dominates the reverberant energy decay in a wide frequency range (Fig. 3b). The same behavior of a frequency dependence of the reverberation time can be noted when the absorption coefficient α_1 is much smaller than α_2 (Fig. 4). However, in this case an amount of sharp peaks of T_A appears to be somewhat larger than before showing that in the subroom A there is a stronger localization of modes.

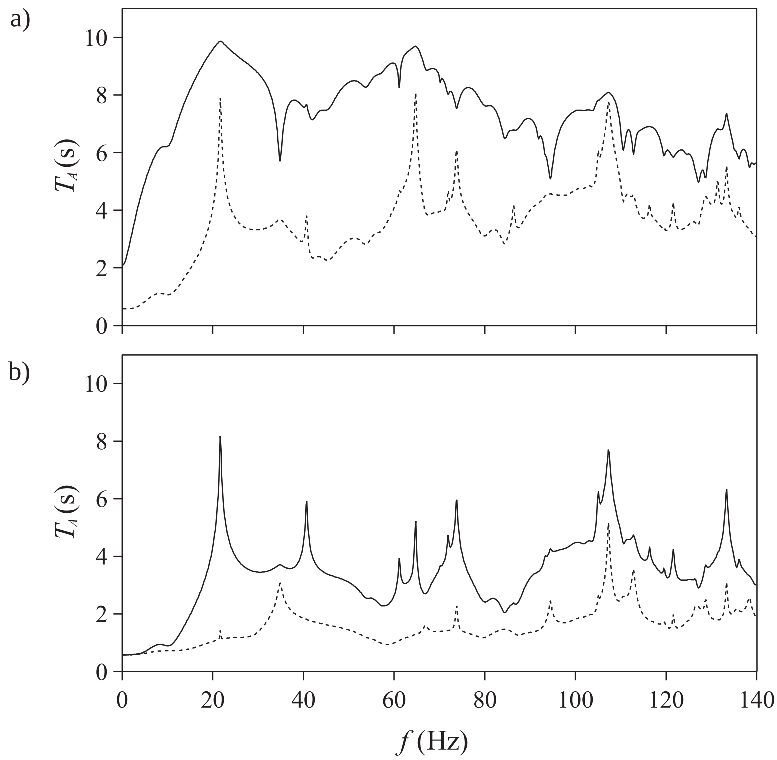

Fig. 4. Frequency dependence of average values of reverberation time in subroom A (solid lines) and subroom B (dashed lines) for absorption coefficients: $\alpha_1 = 0.02$, $\alpha_2 = 0.34$, and sound source located in: a) subroom A, b) subroom B. Calculation results obtained at distance $z = 1.8$ m from room floor.

3. Conclusions

In a low frequency limit a process of reverberant energy decay in a room consisting of two acoustically coupled subrooms was investigated theoretically with the aid of modal analysis. In a numerical example the reverberation time was predicted individually for each subroom under the condition that a total room absorption remained constant. Results of numerical simulation have shown that a location of absorption material on room walls and a position of sound source have a great influence on the distribution and the average value of reverberation time in the subrooms. As was found it is the result of the modal localization which appears in enclosures of irregular geometry such as the analyzed system of coupled subrooms. This effect entails an unwanted, substantial increase in the reverberation time in the case of a large difference between absorption coefficients of walls in the subrooms.

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