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UNCERTAINTY ANALYSIS IN THE ASSESSMENT OF LONG-TERM NOISE INDICATORS

W. BATKO

AGH University of Science and Technology
Faculty of Mechanical Engineering and Robotics
Department of Mechanics and Vibroacoustics
Al. Mickiewicza 30, 30-059 Kraków, Poland
e-mail: batko@uci.agh.edu.pl

R. BAL–PYRCZ

State Higher Vocational School in Krosno
Institute of Technology
Wyspiańskiego 20, 38-400 Krosno, Poland

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This study addresses the assessment conditions of long-term noise indicators based on irregular noise monitoring data. Variations of L_{DEN} estimates (day – evening – night noise indicator) on different days of the week were examined. Two hypotheses are verified: that mean estimates shall be identical on all days of the week and that variances of thus obtained estimates are homogeneous. The data for statistical analyses were noise levels recorded in one year by an on-line noise monitoring system in Kraków.

Key words: environmental noise, acoustic monitoring, statistical analysis.

1. Introduction

In the context of the approximation of the Polish legal system to the EU legislation having relevance to assessment and monitoring of environmental noise it is required that current assessment and uncertainty analysis methods be modified accordingly. New standards are imposed by the Directive 2002/49/WE of the European Parliament [3] relating to the assessment and management of environmental noise, which provides the procedure for deriving the L_{DEN} and L_{night} (the noise indicator for sleep disturbance).

Monitoring of noise indicator variations requires shall be supported by probabilistic analyses, as the random factor is of major importance. Application of such tools as hypothesis testing affords us the means to evaluate the reliability of outcomes derived from the available database of measurement data.

This study investigates whether the distributions of the random vector L_{DEN} are identical on particular days of the week when measurements are taken. It is required to check the statistical significance of variations of their expected values and variances. Statistical treatment is given to a database of noise level observations registered during one year by an online road traffic noise monitoring system installed in Kraków, in Krasińskiego Avenue [1], as a part of a program of road traffic noise hazard assessment.

2. The mathematical formalization of the task

In order to select the scenario of noise indicator measurements $L_{\mathrm{DEN}}\ i = 1, 2, 3, ...$, for the purpose of long-term noise indicator assessment L_{DEN} [2], it has to be determined whether the set of L_{DEN} observations on particular days and in various times of the year make up a set of homogeneous elements, which would imply that differences between them are purely random. Our task, therefore, is to find out if specified sets (samples) of L_{DEN} levels collected on particular days of the week and in specified times of the calendar year should be homogeneous and identical to the set of measurements data collected throughout the whole year.

The analysis of the problem is supported by a model where the outcomes are generated in accordance with the formula:

$$L_{\mathrm{DWN}ij} = \mu + a_i + \varepsilon_{ij}, \tag{1}$$

where μ – common value for all sets, equal to their mean value, a_i – impact factor present only on the i-th day of the week, ε_{ij} – random disturbances assumed to be independent: $\mathrm{cov}(\varepsilon i,\ \varepsilon j,\ i) = 0$ for each pair k, and $k \neq k'$.

These sets of outcomes associated with the accepted classification to account for hypothetical disturbances of noise level variations will generate random samples of the size: $n_1, n_2, \ldots, n_r$ with $\sum_{i=1}^{r} n_i = n$.

For thus formulated model, the expected value of the variable $L_{\mathrm{DEN}i}$ in the i-th group is equal to:

$$E(L_{\mathrm{DWN}\,i}) = \mu_i = \mu + a_i, \qquad i = 1, \ldots,\ r, \tag{2}$$

where μ is the mean value of the given measurement data set.

To state that these assessments are equivalent it is required that a null hypothesis be verified that mean L_{DEN} levels on particular days should be equal:

$$\mathrm{H}_0 :\ \mu_{\mathrm{Mo}} = \mu_{\mathrm{Tu}} = \mu_{\mathrm{We}} = \mu_{\mathrm{Th}} = \mu_{\mathrm{Fr}} = \mu_{\mathrm{Sa}} = \mu_{\mathrm{Su}}$$

with relation to the alternative that at least two averages differ from one another.

An alternative hypothesis has it that at least two mean values should be different.

The procedure allowing us to find out if the null hypothesis is true in the light of experimental data involves the decomposition of general variance of the sample into two components which measure the variance between samples and within the sample.

Accordingly, an arithmetic mean of all outcomes is computed:

$$\overline{L}_{\mathrm{DWN}} = \frac{1}{n} \sum_{i=1}^{7} \sum_{k=1}^{n_i} L_{\mathrm{DWN}\,k\,i} \tag{3}$$

and the mean value for the i-th group:

$$\overline{L}_{\mathrm{DWN\,i}} = \frac{1}{n_i} \sum_{k=1}^{n_i} L_{\mathrm{DWN}\,k\,i}\,. \tag{4}$$

Total sum of deviations contains two terms:

$$\mathrm{SSE} = \sum_{i=1}^{r} \sum_{k=1}^{n_i} \left(L_{\mathrm{DWN}\,k\,i} - \overline{L}_{\mathrm{DWN}\,i}\right)^2, \tag{5}$$

SSE – associated with variation within a group

$$\mathrm{SSB} = \sum_{i=1}^{r} n_i \left(\overline{L}_{\mathrm{DWN}\,i} - \overline{L}_{\mathrm{DWN}}\right)^2, \tag{6}$$

SSB – associated with variation between groups.

If a null hypothesis is rejected, there are no grounds to assume the sample selection procedure (choosing the specific days of the week or periods of time at which L_{DEN} measurements are taken) to be insignificant for long-term noise indicator L_{DEN} assessment. This happens when variation between groups is sufficiently large in relation to inter-group variation. The assessment uses the statistics:

$$FS = \frac{\mathrm{MSB}}{\mathrm{MSE}}, \tag{7}$$

where MSB – Mean Square Between, MSE – Mean Square Error (within)

$$\mathrm{MZM} = \frac{\mathrm{ZM}}{r-1}; \qquad \mathrm{MZW} = \frac{\mathrm{ZW}}{n-r}. \tag{8}$$

If H_0 were true, the statistics would have the F distribution with $v1 = (r-1)$ and $v2 = (n-r)$ degrees of freedom. As MZM and MZW are unbiased estimators of variance in population, the statistics should assume small values of unity. The critical region is given by the equation: $P(F \geq F_{a,v1,v2}) = \alpha$.

While testing the homogeneity of measurement data sets two hypotheses have to be verified: that their expected values are equal and their variances are homogeneous (providing the level of scattering around the mean value).

This task can be brought down to the verification of the null hypothesis:

$$\mathrm{H}_0:\ s_1^2 = s_2^2 = s_3^2 = s_4^2 = s_5^2 = s_6^2 = s_7^2 \tag{9}$$

against an alternative hypothesis:

$$\mathrm{H}_1:\ s_i^2 \neq s_j^2 \ \text{ for at least one pair of indices } i,\ j. \tag{10}$$

Accordingly, the Barlett's test is performed with the test statistics:

$$\lambda = \frac{M \cdot \ln 10}{1 + \dfrac{1}{3(r-1)}\left[\displaystyle\sum_{i=1}^{r} \frac{1}{(n_i - 1)} - \frac{1}{n-r}\right]}, \tag{11}$$

where $M = (n-r) \cdot \log MZW - \sum\limits_{i=1}^{r} (n_i - 1) \log s_i^2$, s_i^2 – variance of L_{DEN} variable for observations in one of the 7 groups.

If the hypothesis H_0 were true, the statistics λ would have the χ^2 distribution with $v = r - 1$ degrees of freedom. The critical region is given by the formula: $P(\lambda \geq \chi^2 a, y) = \alpha$.

When group sizes n_i are equal or similar, the hypothesis might be verified using the Cochran test, based on the test statistics:

$$F = \frac{(r-1)C}{1-C} \quad \text{where} \quad C = \frac{\max\limits_{i} s_i^2}{\sum\limits_{i=1}^{r} s_i^2}. \tag{12}$$

This statistics has the F distribution with the number of degrees of freedom of the numerator $\nu_1 = \dfrac{n}{r} - 1$ and denominator $\nu_2 = \left(\dfrac{n}{r} - 1\right)(r-1)$.

The critical region is determined by the formula:

$$P(\lambda \geq \chi^2 a, y) = \alpha.$$

3. Results of testing

Assessment data of long-term noise indicator L_{DEN} supported by noise level measurements on various days of the week were compared in accordance with the outlined methodology basing on variance analysis [4]. Characteristics of the data sets summarised in the form of tables and the outcomes – relevant parameters are shown in Fig. 1.

Verification procedures were applied to evaluate the significance of long-term noise indicator estimates L_{DEN} in various options. The FS statistics was used, given by formula (7).

It appears that the statistics (7) assumes the value $FS = 1.32$. The tables of statistics distribution show that for the given significance level $\alpha = 0.05$ the value of $FS(6;\,249) = 2.14$. The rejection range $\mathrm{H}_0 \in (2.14, \infty)$.

Thus calculated value of the statistics is beyond that range, hence there are no reasons to reject the accepted hypothesis H_0 that the outcomes of long-term noise indicator L_{DEN} assessment are independent of the selection of a control sample, meaning that measurements are taken at specified times of the week. That implies that times the measurements are taken and associated random disturbances will not affect the variation of the L_{DEN} indicator assessments.

Table 1. Size and variance of control data sets.

I	n_i	S_i^2
1 (Monday)	35	1.40
2 (Tuesday)	38	0.55
3 (Wednesday)	37	0.59
4 (Thursday)	34	0.55
5 (Friday)	34	0.32
6 (Saturday)	34	0.64
7 (Sunday)	37	0.98
Total	249	0.73

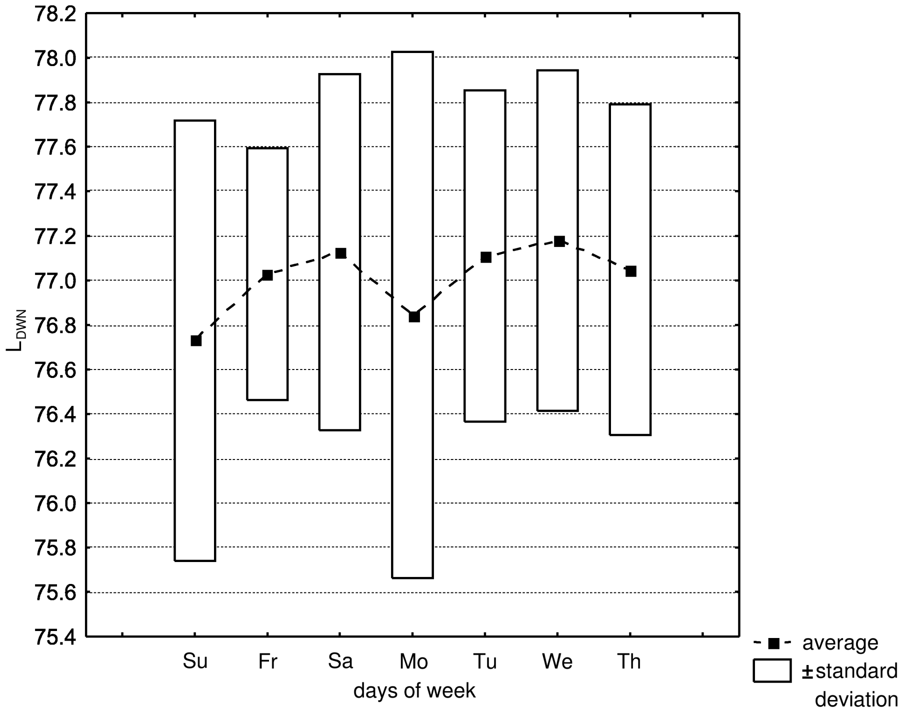

Fig. 1. Estimated value of the long-term noise indicator L_{DEN}.

Homogeneity of variance of L_{DEN} assessments on particular days of the week was analysed using the Barlett's test (11). The critical value of the chi-square tests corresponding to the relevant measurement condition would approach $\chi^2_{0.05.6} = 1.635$ for the significance level $\alpha = 0.05$ and 6 degrees of freedom. Hence the critical region becomes the interval $(1.635, \infty)$. The computed value of the test statistics (11) falls into the critical region, which implies there are no grounds to accept the null hypothesis of homogeneous variances of sets of L_{DEN} assessments derived from the measurements data collected on various days of the week.

This result is borne out by the Cochran test, too. The research data would yield the following numerical values: $\mathrm{Max}\, s_i^2 = 1.40$, $\Sigma = 5.0267$. The result of the Cochran test is $C = 0.27797$. Transforming C onto F: $F = 2.309899$. For the degrees of freedom $v1 = 34.6$ and $v2 = 207.4$ and the significance level $\alpha = 0.05$, the critical level of the significance test would approach $F_{0.05\ 34.6.207} = 1.39$, equivalent to the probability $P(F \geq 1.39) = 0.05$. Thus computed value falls in the range $(1.39, \infty)$ and hence the null hypothesis of homogeneity of variance shall be rejected.

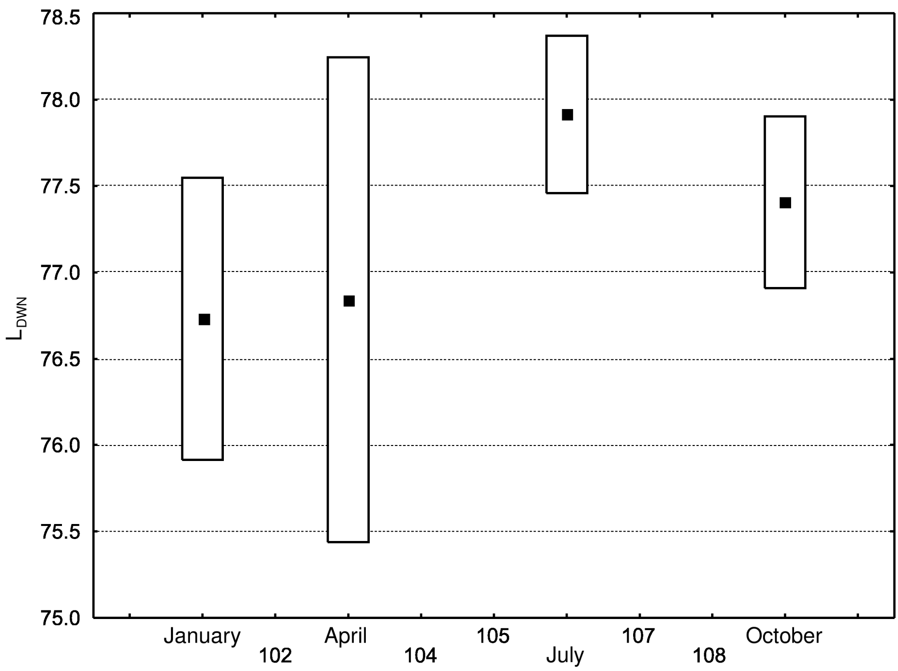

Fig. 2. Assessments of long-term noise indicator L_{DEN} on the basis of measurement data in various months of the year.

Similar analyses were performed to find out how the selection of month in which measurements were taken should affect the variation of long-term noise indicator L_{DEN} assessments. Four months were considered in this analysis, most characteristic of the given season, when the atmospheric conditions are most diverse: January, April, July, October. The null hypothesis to be verified has it that the mean L_{DEN} levels in particular months should be equal: H_0; $\mu_{\mathrm{Jan}} = \mu_{\mathrm{April}} = \mu_{\mathrm{July}} = \mu_{\mathrm{Oct}}$. The alternative hypothesis is that at least two mean values should be different. The computed F statistics $F = 12.647$. Tabulated distribution $FS(3;\, 109) = 2.687$ for the significance level $\alpha = 0.05$. The rejection range $\mathrm{H}_0 \in \langle 2.687, \infty)$. The calculated value of the statistics falls in the critical range, hence there are grounds to reject the assumption that the classification procedure (i.e. selection of a month) should not affect the variations of L_{DEN}. It is reasonable to conclude, therefore, that the final outcome of L_{DEN} measurements shall depend on the season.

The hypothesis that variance is homogenous in particular months is verified using the Barlett's and Cochran tests with the confidence level $\alpha = 0.05$. In each case the null hypothesis of homogenous variance would be rejected.

4. Final comments

There are scant reports on actual requirements for long-term noise indicator L_{DEN} estimation. The analyses outlined in this study might be a starting point for further research, covering particular sources of noise and types of areas to be protected. The methodology of verification of statistical hypotheses provides us solutions for designing the scenarios of control tests and for selection of times when measurements ought to be taken.

Statistic analyses relating to the significance of variations of long-term noise indicator L_{DEN} assessments reveal that the fact that measurements are taken on a specified day of the week does not have a major bearing on the expected value of L_{DEN}.

One cannot claim the insignificance in estimation of standard deviation of L_{DEN}, associated with the uncertainty of L_{DEN} assessments. The Barlett's and Cochran test data reveal that the hypothesis of no significance of L_{DEN} variance fluctuations on particular days of the week (from Monday to Friday) shall be rejected. Hence the statistic accuracy of L_{DEN} assessments of on different days of the week will be different.

When a particular month is to be selected for noise measurements, it appears that differences between expected values in selected months are statistically significant and have a bearing on L_{DEN} levels computed accordingly.

The issue of key importance is how to control the estimation procedure so as to achieve the required accuracy levels since this accuracy is associated with the standards deviation. In other words, how to derive standard deviations, what shall be the correct method of computing the estimator and what should be the sample size to ensure correct estimations of L_{DEN} noise indicator?

Results of pilot studies outlined in this paper reveal that the expected value of the long-term noise indicator L_{DEN} shall not vary for a sample 249 days and for randomly chosen 31 days. It has to be emphasised, however, that randomly chosen 31 days are taken from the sample of 249 days in a year. No differences are reported between all days (working days and holidays), working days only and holidays only, hence it is reasonable to suppose that noise generation measured by a long-term noise indicator L_{DEN} will be the same, both on working days and holidays.

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