ARCHIVES OF ACOUSTICS
31, 4 (Supplement), 287–294 (2006)

REGULARISATION PROBLEMS AT THE DETERMINATION OF THE ACOUSTIC POWER OF SOUND SOURCES

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(*received June 15, 2006; accepted September 30, 2006*)

This paper presents exploratory problems of regularisation of the inverse method for the investigation of the characteristics of acoustic sources at industrial conditions. The solution of problem is to find an effective method for the determination of optimal regularisation parameters in acoustic inverse problems. The sound power of the sound source distribution can be simply deduced from the measured pressure field and the inversion of the corresponding matrix of frequency response functions. The accuracy of reconstruction of the sound power of the source is crucially dependent on the conditioning of the matrix to be inverted. The success of regularisation depends on the appropriate choice of the regularisation parameter.

Key words: inverse method, regularisation, sound source.

1. Introduction

Application of numerical methods for the estimation of the sound level distribution around industrial objects is based on the determination of the acoustic power of each noise source by means of acoustic pressure measurements – according to the relevant procedures – and estimating its emission level in the measuring point outside a factory. Software based on the noise propagation models in an open space is usually applied. Another approach for solving such problems, utilising inversion methods, is also possible. One of the problems formulated in inversion methods is the reconstruction of differential operator of a known structure, in which unknown coefficients are determined on the basis of information on certain functionals estimated within the solution range. By modelling the process of radiation of vibroacoustic energy from the source to the receiver and knowing the acoustic pressure distribution in measuring points as well as the distribution of noise sources in the factory we can reverse the propagation path model and estimate acoustic parameters of the sound source [2]. Thus, if the reverse problem is described by a matrix notation its solution should be looked for by means of inversing the matrix describing the behaviour of the object. Inversion methods can be applied for the identification of vibroacoustic energy sources, for the estimation of sound radiation inversion by vibrating surfaces as well as for the acoustic estimation of machines on the basis of the analysis of the sound field parameters.

2. Inversion methods in vibroacoustics

A sound pressure value in the observation point p_j results from the applied calculation model [1, 2]:

$$p_j^2 = G(i,j) \cdot N_i\,, \tag{1}$$

where N_i – sound power of the source no i; $G(i,j)$ – value of the transfer function between the sound power of the i-th source and the sound pressure value in the j-th point.

Equation (1) will be written in the matrix notation:

$$\widehat{\mathbf{p}} = \mathbf{G} \cdot \mathbf{N}. \tag{2}$$

If we take into consideration, that the emission pressure of the source and the background noise are measured in the measuring point, Eq. (2) becomes:

$$\widehat{\mathbf{p}} = \mathbf{G} \cdot \mathbf{N} + \mathbf{e}, \tag{2$'$}$$

where $\mathbf{e} = \widehat{\mathbf{p}} - \mathbf{p}$ – error vector, difference between the pressure estimated from the noise propagation model and its value measured in the observation point.

There are two sources of the error vector $\mathbf{e}$: the first – all disturbances occurring in the measuring point and influencing the measured value of the sound pressure, the second – errors resulting from noise distribution modelling on the site under testing. The solution is based on the assumption that we know the positions of M noise sources in the factory and that we measure the sound pressure in the finite number of observation points O. In order to limit the error vector we optimise parameters of individual sources in the model. The following methods are applied: the shortest distance, the least square, or the singular value decomposition (SVD). Often applied criterion is the minimisation of the expression:

$$\mathbf{K} = \mathbf{e}^H\mathbf{e}. \tag{3}$$

One of the analytical tools is the matrix distribution versus the singular value decomposition (SVD). Usefulness of such decomposition is due to the fact that that the matrix of transfer function $\mathbf{G}$ can be expressed in the following form:

$$\mathbf{G} = \mathbf{U\Sigma V}^H\,, \tag{4}$$

where $\mathbf{U}$, $\mathbf{V}$ – orthogonal matrices of the following properties: $\mathbf{U}^H\mathbf{U} = \mathbf{U}\mathbf{U}^H = \mathbf{I}$ and $\mathbf{V}^H\mathbf{V} = \mathbf{V}\mathbf{V}^H = \mathbf{I}$, $\mathbf{\Sigma}$ – diagonal matrix $m \times n$, in which successive singular values satisfy the condition $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{\min(o,m)} \geq 0$, superscript H denotes the complex conjugate transposed.

The vector of estimated complex parameters value of the model source can be estimated from the dependency:

$$\mathbf{N} = \mathbf{G}^+\mathbf{p}, \tag{5}$$

where matrix $\mathbf{G}^+ = [\mathbf{G}^H\mathbf{G}]^{-1}\mathbf{G}^H$ is "pseudo-inversion" of matrix $\mathbf{G}$.

We can estimate matrix $\mathbf{G}^+$ using the SVD distribution:

$$\mathbf{G}^+ = \mathbf{V}\boldsymbol{\Sigma}^+\mathbf{U}^H\,, \tag{6}$$

where $\boldsymbol{\Sigma}^+$ – matrix of pseudo-inversion of matrix $\boldsymbol{\Sigma}$, diagonal matrix with elements $(1/\sigma_1,\ 1/\sigma_2,\ \ldots 1/\sigma_M)$.

Thus, the noise source power N:

$$N = \mathbf{V}\,\boldsymbol{\Sigma}^+\mathbf{U}^H\hat{p}. \tag{7}$$

The value of the expression for the square matrix is the accuracy measure of the performed simulations:

$$\kappa(\mathbf{G}) = \|\mathbf{G}\|\|\mathbf{G}^{-\mathbf{1}}\| \tag{8}$$

since $\|\mathbf{G}\| = \sigma_{\max}$ and $\|\mathbf{G}^{-1}\| = 1/\sigma_{\min}$ then

$$\kappa(\mathbf{G}) = \frac{\sigma_{\max}}{\sigma_{\min}}\,. \tag{8$'$}$$

By means of this value we can estimate an error δn committed at the determination of the model parameters:

$$\left\|\frac{\delta n}{n}\right\| \leq \kappa(\mathbf{G})\left\|\frac{\delta p}{p}\right\|. \tag{9}$$

For the matrix, which is not a square one, the condition number $\kappa(\mathbf{G})$ is expressed by:

$$\kappa(\mathbf{G}) = \|\mathbf{G}\|\|\mathbf{G}^+\|, \tag{10}$$

where $\|\mathbf{G}^+\| = 1/\sigma_m$, σ_m – the smallest non-zero **G** value, thus:

$$\kappa(\mathbf{G}) = \frac{\sigma_{\max}}{\sigma_m}\,. \tag{11}$$

Such inversion task is not correctly formulated – in a classical sense – since small changes of investigated functionals can correspond to large changes of solutions. If we superimpose additional restrictions on the allowed set of solutions we can obtain solutions stable in respect of data changes, it means tasks conditionally-correct. Thus, applying various regularisation methods we can consider the approximate solutions based on the approximate data.

3. Selected regularisation methods

Let us consider the possibility of application of the selected regularisation methods for limiting the error vector **e**. Ill-conditioned equation sets, which require regularisation before their solution, occur often during the numerical calculations. Regularisation must eliminate unreliable solutions dominated by noises and errors during the measurements of sound field parameters. The method of the optimal value selection of the regularisation parameter, allowing to obtain maximum information from available data, is one of the most useful in such cases.

We may look for the minimum value of the expression:

$$J = \mathbf{e}^H\mathbf{e} + \beta\mathbf{N}^H\mathbf{N}, \tag{12}$$

where β – regularisation parameter.

The Tichonov's method is one of the most often used regularisation technique. The solution is of an adequately residual norm and after meeting additional restrictions it will be not further than the expected unknown solution. The combination of a residual norm square and the additional restriction defined as the weighted value can be a measure of difference between the measured data and identified values.

The following expression is the error criterion [3]:

$$J = \|\mathbf{GN} - \widehat{\mathbf{p}}\|^2 + \beta\|\mathbf{LN}\|^2, \tag{13}$$

where β – regularisation parameter, $\mathbf{L}$ – identity matrix.

Tichonov's regularisation method does not provide an accurate solution. Regularisation parameter β is the only introduced value at the defining the regularisation matrix $\mathbf{L}$. In this case matrix $\mathbf{L}$ reflects a range of weights of unknown boundary values. When $\beta = 0$ we will obtain the solution consistent with the least square method but unstable when without the regularisation. On the other hand the large value of β favours solutions of small dimensions. Thus, parameter β controls the degree at which the regularised solution will be more fitted to the obtained results or to the solution range.

When we select the continuous parameter β at the Tichonov's regularisation, this regularisation becomes not objective. The proper selection of the regularisation parameter allows obtaining the highest possible accuracy of the solution, which however, is unknown to us. Parameter determination methods should not require too much information on error distribution at the sound pressure measurements. Effectiveness of the Tichonov's regularisation method depends on the proper selection of parameter β, which causes deviation between disturbance errors and the regularisation ones. Obviously, we are dealing here with two contradicting requirements. Large value of the regulation parameter β is preferred in the case of a numerical problem, however, for an increased estimation accuracy the applied parameter β should be as small as possible.

Another method of the regularisation parameter determination is the graphical method, called Curve L method. It is based on plotting – in a logarithmic scale – the regularised parameter values versus the minimum error. Norm $\|\mathbf{LN}\|$ of the regularised solution is plotted versus the residual norm $\|\mathbf{Gp} - \beta\|$ for all possible regularisation parameters. The curve – of the shape similar to the letter L (for logarithmic plots) – is plotted for the regularised parameter β range (Fig. 1).

Thus, the optimal regularisation parameter value marked β_{LCV} corresponds to coordinates of the curve L corner. The horizontal part of the curve characterises too smooth solutions (over-regularised), while the perpendicular part shows solutions dominated by errors (under-regularised). The solutions found in-between these extremes represent the required compromise. Optimal value of the regularising parameter occurs in the corner of L curve. By selecting the proper β value we can control the filtration degree of the

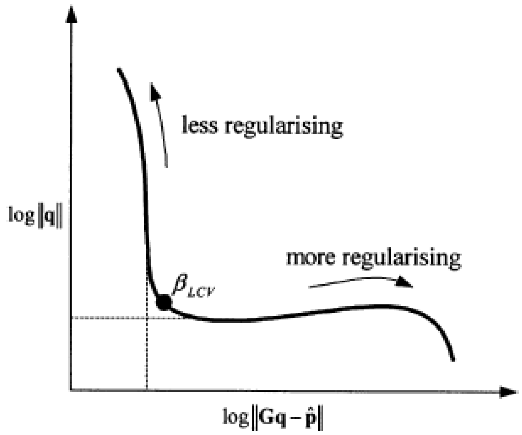

Fig. 1. Curve L [3].

solution. Searching for the optimal deviation between the disturbances error and regularisation error is the base of the Generalised Cross Validation Method – (GCV). The value of β_{GCV} is needed here for a minimisation of the generalised cross validation function determined as [3]:

$$\mathrm{GCV}(\beta) = \frac{\left(\dfrac{1}{m}\right) \|\mathbf{I} - \mathbf{B}(\beta)\widehat{\mathbf{p}}\|^2}{\left[\left(\dfrac{1}{m}\right) \mathbf{T}_r\{-\mathbf{B}(\beta)\}\right]^2}, \tag{14}$$

where m – number of measuring points, $\mathbf{T}_r$ – matrix trace (sum of orthogonal elements); $B(\beta)$ – influence of the matrix defined by:

$$\mathbf{B}(\beta) = \mathbf{G}\left(\mathbf{G}^H\mathbf{G} + \beta\mathbf{I}\right)^{-1}\mathbf{G}^H. \tag{15}$$

Error of disturbances caused by the regularisation of the error criterion J with addition of β is being estimated by the expression occurring in a denominator (14). Square sum of the residue of the regularised solution is presented in a numerator. Since a numerator is less than unity it increases the $\mathrm{GCV}(\beta)$ value, if parameter β is increased. Thus, the GCV function estimates both errors in solution and the inaccuracy expressed by matrix $\mathbf{G}$ will be inverted by being included into the selected regularisation parameter. Figure 2 presents an example of the GCV function and the regularisation parameter β_{GCV} leading to GCV. The method provides right results when disturbances are similar to a white noise, it means when they are independent and of an identical variance. The GCV method allows determination of optimal estimates on the basis of measurement results only, however it requires huge number of calculations, which makes it quite impractical.

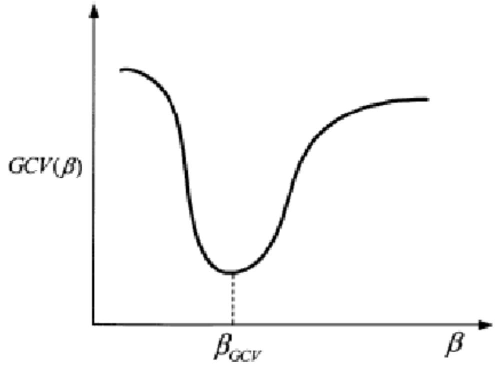

Fig. 2. Function GCV course [3].

4. Results of an experiment

59 measurements of the sound pressure on the hemisphere surface, which was surrounding the machine placed on a sound reflecting surface – were performed in the experiment. The actual machine was modelled by 4 monopole noise sources arranged in space. A sound power of the machine was determined with taking into consideration mutual configuration of substitute sources and observation points as well as the Green's function for substitute sources. Elements of matrix **G** for omni-directional sources are given by the following dependency [2]:

$$G_{mo} = \frac{\mathrm{Exp}(-ikr_{mo})}{r_{mo}}, \tag{16}$$

where r_{mo} – distance between the m-th source and the o-th observation point; k – wave number.

The plot showing the accuracy of determining the parameters of the model (using formula (8′)) as well as the characteristics of machine radiation is shown in Fig. 3.

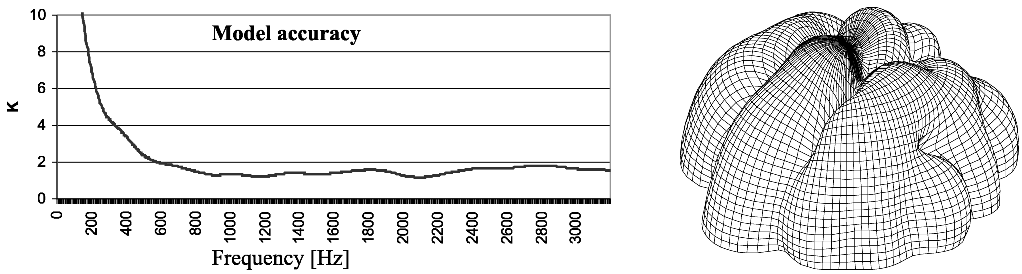

Fig. 3. Accuracy coefficient of a sound power determination and the radiation characteristics [2].

Figure 4 presents the results of fitting sound power of 4 noise sources for the regularisation parameter β while using criterion (13) for the selected frequency for two different machines. The results are confirmed our previous studies [2] the accuracy of estimation the parameters of substitute sources.

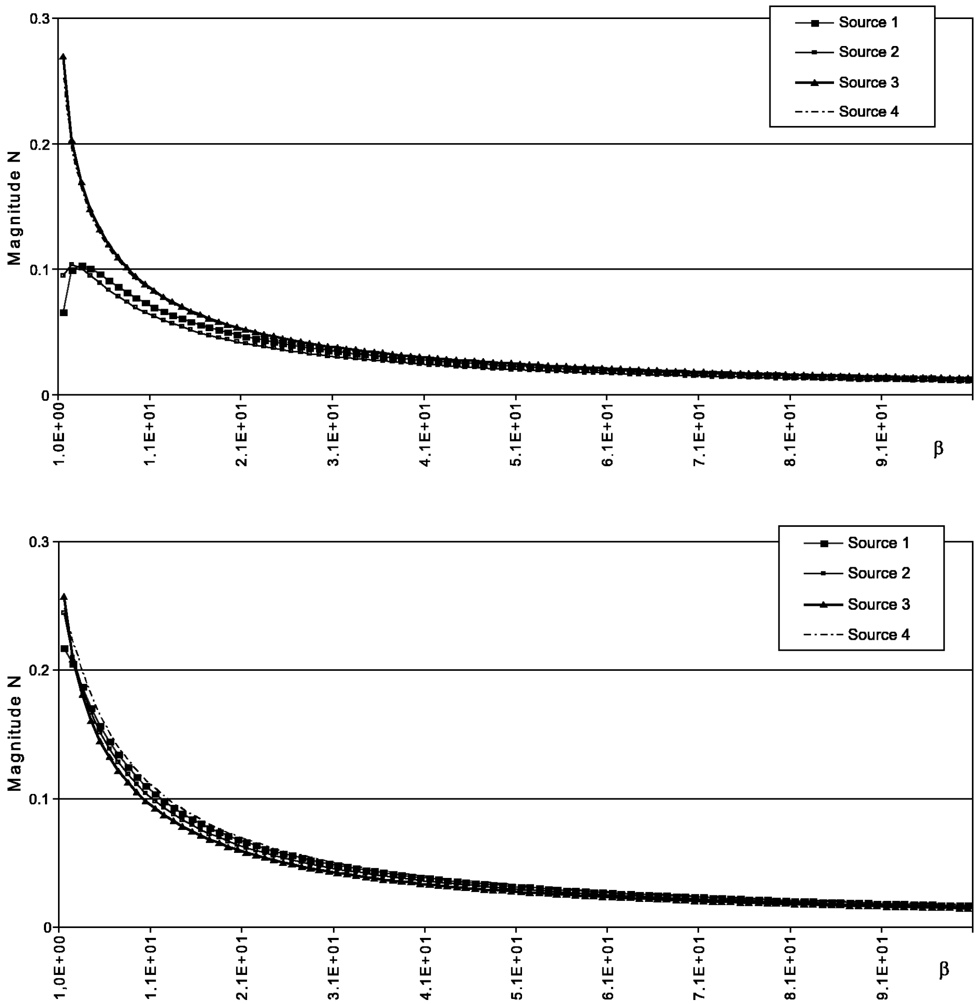

Fig. 4. Example of the sound power estimation while using regularisation formula (13) for two different machines.

5. Conclusions

Inversion methods can be applied in investigations of vibroacoustic processes due to the advancements in the calculation possibilities and process modelling as well as due to the development of effective methods of obtaining and processing large amounts of data. Inversion tasks are usually not correctly formulated – in a classical sense. As a consequence small changes in investigated functionals can correspond to large changes in solutions. In cases of an ill-conditioned problem the regularisation methods are applied for its solution. Inverse determination of sound power levels of individual machines – on the basis of sound pressure measurements in the observation points – is very sensitive to disturbances in receiving points.

Acknowledgments

The presented hereby work was conducted within the research programs 4T07B00329 and 4T07C00830 of the Ministry of Science and Higher Education realised in the Department of Mechanics and Vibroacoustics UST.

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