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THE METHODS OF INVERSION AND THE MAXIMUM LIKELIHOOD ESTIMATION IN ACOUSTIC TESTS OF INDUSTRIAL SOURCES IN THE ENVIRONMENT

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Determining an acoustic model of a factory is a topic connected to the noise sources identification in the factory, their localization and acoustic parameters. In order to determine the source parameters correctly, it is necessary to use the whole available knowledge accessed in acoustic measurements and the knowledge obtained *a priori* (e.g. on the basis of producers' catalogues of equipment). To solve this problem the method of the acoustic inversion was used with the maximum likelihood estimation. The biggest advantage of this method is the possibility to use the information concerning accuracy of the determined *a priori* sound power of noise sources and indicating the importance of the sound pressure level values in measurement areas. The theoretical assumptions of the method, its adjustment to the industrial conditions as well as tosome computer simulations are presented in the paper.

Key words: acoustic inversion, maximum likelihood, sound sources.

1. Introduction

Machines and equipment used in industry are generally characterised by a high vibroactivity level. The decrease of noise levels of industrial sound sources can be achieved by an application of relevant acoustic covers. Since such covers are usually very expensive, it is essential to be able to design the most effective covers in relation to the costs incurred. Thus, the estimation of excessive noise caused by machines and equipment as well as by industrial objects is one of the basic aims of the vibroacoustic research.

Estimation of the sound pressure level around the factory normally consists of determination of sound power of each source by measuring a sound pressure levels around an individual source and then assessing its participation in the total sound level in the measurement point outside the factory.

There are also inversion methods of sound sources power determination based on measuring sound pressure level in several points around a factory [1, 2].

2. Acoustic modelling of a factory

Determining an acoustic model of a factory requires identification of noise sources, their localisation and acoustic parameters. Those sources should be approximated by substitute sound sources, with characteristics corresponding to acoustic field parameters around a factory. In simple cases this can be done by determining the acoustic power of each source separately and locating it in the spatial plan of the factory and then assessing the influence of this source on the sound pressure level in measurements points in the environment outside the factory. However, at high complexity of a noise, e.g. large number and variety of sources such as installations, large equipment operating in technological lines, pipelines, belt conveyor flights, etc., the sound power estimation of individual sources is extremely difficult. Having the access to the software describing the propagation model in open space, knowing values of sound pressure levels in many observation points located in the surrounding environment as well as having information concerning the structure of noise sources inside the factory it is possible to reverse the task and determine acoustic powers of sources located inside the factory.

The dependence between the sound power values of the sound sources system and the sound pressure values in observation points is given by the following formula [2]:

$$\mathbf{p}^2(\mathbf{r}, f) = \mathbf{G}(\mathbf{d}, f) \cdot \mathbf{N}(f), \tag{1}$$

where $\mathbf{p}^2(\mathbf{r}, f)$ – $[p_1^2(\mathbf{r}_1, f), p_2^2(\mathbf{r}_2, f), ..., p_M^2(\mathbf{r}_M, f)]^{\mathrm{T}}$ is a vector determining the function of sound pressure square, p in M observation points $\mathbf{r} = (x_i, y_i, z_i)$, $i = 1, ...M$ for the given frequency f; $\mathbf{N}(f)$ – vector, which determines (the estimated) acoustic power value of individual sound sources within the given frequency; $\mathbf{G}(d, f)$ – determines the value of the transfer function between the acoustic power value of individual source and the sound pressure value in individual observation points, for frequency f.

Standard ISO 9613-2 [4] describes sound propagation in open space. The sound pressure level in the observation point can be determined from the dependency:

$$L_{p_i} = L_{N_i} + D_{\Omega} + D_{\Theta} - A_d - A_E - A_Z - A_{\mathrm{atm}} - A_{\mathrm{gr}}, \tag{2}$$

where:

- L_{p_i} – equivalent sound level, originated from the i-th source, in the observation point,
- L_{N_i} – equivalent sound power level of the i-th source,
- D_{Ω} – correction for the radiation angle,
- D_{Θ} – correction for the directional radiation response,
- A_d – correction for the distance influence,
- A_E – correction for screening,
- A_Z – correction for influence of plantations, industrial installations, etc.,
- A_{atm} – correction for air absorption,
- A_{gr} – correction for ground influence.

After solving logarithm from Eq. (2) we obtain the dependency for the sound pressure square in the observation point:

$$p_{ij}^2 = G(i, j) \cdot N_i\,, \tag{3}$$

where i – number of source, j – number of observation point, N_0, p_0 – reference values, N_i – sound power of the i-th source, $G(i,j) = \dfrac{p_0^2}{N_0} 10^{(-D_\Omega + D_\Theta - A_d - A_E - A_Z - A_{\mathrm{atm}} - A_{\mathrm{gr}})}$ is a transfer function between the i-th source and the j-th observation point.

This dependency – in the matrix record – can be presented as:

$$\mathbf{p}^2 = \mathbf{G} \cdot \mathbf{N} + \mathbf{e}, \tag{4}$$

where **e** – vector of error (which is committed every time when estimating the sound power of sound source, calculating the transfer function, or measuring the sound level in the observation point).

Sound power values of individual sound sources (vector **N**) are the most difficult to determine. In the case of small dimension sound sources located far from each other it is possible to use the existing standards. An error in determining the sound power of machines or devices – at the requirements met by those standards – is usually quite small. Unfortunately, in the case of a high concentration of large dimension sound sources, significantly differing in the sound power and mutually penetrating sources the error in determination their sound power can exceed 10–20 dB.

The sound pressure level, emitted by individual sources is estimated in such a way as to obtain the calculated sound pressure level – from all sound sources in a certain control point – equals the measured value (calibration and verification of a model). This requirement, in the majority of cases (when observation points are more numerous than measuring points), is not satisfied. In the remaining cases this requirement can be satisfied, however, the calculation result is burdened with a considerable error (since surely certain errors will be committed at estimating the sound pressure level in observation points or at estimating the transfer function – even the best standard is not able to describe adequately the reality).

Inversive determination of the sound power levels of individual machines is very sensitive to disturbances in the observation points.

When there are more results of the sound pressure level measurements in the observation points than sound sources, it is possible to try to make the model calibration on the basis of the Least Squares Method (LSM). Sometimes this method leads to the determination of the negative values of the sound power. In such cases the Non-Negative Least Square (NNLS) should be applied.

Apart from the error minimisation the method is not useful for practical applications in the calibration of the majority of acoustic models. Measurements performed close to the sound source (where the sound pressure levels are the highest and in consequence differences between the measured and calculated values are the biggest) are the most important in this method. On the contrary, for the acoustic model calibration the most essential are measurements performed at the far side (not in the vicinity of the factory but in the vicinity of the nearest housing estate). Acoustic measurements are in these points burdened usually with the biggest error due to the most significant influence of an background noise.

As an example we would like to use the assessment of the influence exerted on the environment by the electric power station of 400 kV. The electric power station usually consists of several high power transformers (usually 360 MVA). Those sources are of the dimensions approximately $3 \times 4 \times 3$ m. Determining their sound power levels is usually performed without any trouble since their operation is uniform and dimensions not too large. The accuracy of estimating their sound power level is ± 3 dB. In addition there are usually also air compressors of small dimensions. They operate periodically (several minutes in 24 hours). The error at estimating their acoustic power level does not exceed usually 1.5–2 dB. The third and the most significant sound source is the corona effect originated from electrical installations. The origin of this effect is the large number of cables and electrical line installations with current of 400 kV voltage. Measuring sound levels in the vicinity of cables is not possible due to the danger of break down and the high intensity of electric field (10 kV/m), which can be the reason of erroneous readouts of measuring instruments. An error at estimating the sound power levels of the corona effect can exceed 10 dB.

3. Maximum likelihood method

The solution of this problem is the application of the maximum likelihood method for the calibration of such acoustic model. The biggest advantage of the method is the possibility to use information concerning the accuracy of the estimated acoustic powers of sound sources as well as indicating the significance of sound pressure level values in the measuring points.

Density of probability of the estimated sound power value can be calculated from the following formula [3]:

$$\sigma(\mathbf{N}) = \eta(\mathbf{N}) \int \rho(\mathbf{p}^2_{\text{true}}) \cdot \theta(\mathbf{p}^2_{\text{true}}|\mathbf{N})\, \mathrm{d}\mathbf{p}^2_{\text{true}}, \tag{5}$$

where $\rho(\mathbf{p}^2_{\text{true}})$ – density of probability of the correct value determination of the sound pressure square; $\eta(\mathbf{N})$ – density of probability of the *a priori* determined source sound power; $\theta(\mathbf{p}^2_{\text{true}}|\mathbf{N})$ – density of probability of the conditional determination of the correct value of the sound pressure square, when knowing the correct value of the source sound power.

Assuming, that all distributions of the density of probability are Gaussian distributions (this assumption – in the case of the density of probability of determining the sound pressure square – being far from the true one, simplifies the calculations significantly) the density of probability of the correctly determined sound power can be presented in the form [3]:

$$\sigma(\mathbf{N}) = D\, e^{-S(\mathbf{N})}, \tag{6}$$

where D – constant,

$$S(\mathbf{N}) = \frac{1}{2}\Big([\mathbf{GN} - \mathbf{p}^2]^{\mathrm{T}}\mathbf{C}^{-1}_{\rho\theta}[\mathbf{GN} - \mathbf{p}^2] + [\mathbf{N} - \mathbf{N}_0]^{\mathrm{T}}\mathbf{C}^{-1}_{\eta}[\mathbf{N} - \mathbf{N}_0]\Big)\,; \tag{7}$$

$\mathbf{C}_{\rho\theta}$ – covariance matrix of measurement and model errors (at the assumption of independence of the measurement results, which sometimes can be difficult to achieve) is equal:

$$\mathbf{C}_{\rho\theta} = \begin{bmatrix} \sigma_{\rho_1}^2 + \sigma_{\theta_1}^2 & 0 & ... & 0 \\ 0 & \sigma_{\rho_2}^2 + \sigma_{\theta_2}^2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \sigma_{\rho_n}^2 + \sigma_{\theta_n}^2 \end{bmatrix}, \tag{8}$$

$\sigma_{\rho_i}^2$ – variance of the errors distribution of noise measurements in point i, $\sigma_{\theta_i}^2$ – variance of the model error in i-measurement point, $\mathbf{C}_{\eta}$ – covariance matrix of errors of *a priori* estimation of the sound power (assumptions of independence are here also difficult to achieve) equals:

$$\mathbf{C}_{\eta} = \begin{bmatrix} \sigma_{N_1}^2 & 0 & ... & 0 \\ 0 & \sigma_{N_2}^2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \sigma_{N_m}^2 \end{bmatrix}, \tag{9}$$

$\sigma_{N_i}^2$ – variance of distribution of *a priori* determined sound powers.

Searching for the likelihood solution it is tried to find the maximum of the density of probability function $\sigma(\mathbf{N})$ (6), which is equivalent to looking for the minimum of the following function: $S(\mathbf{N})$ (7).

$$\frac{\partial S(\mathbf{N})}{N_i} = \frac{1}{\sigma_{N_i}}(N_i - N_{0i}) + \sum_{k=0}^{n} \frac{\left(\tilde{p}_k - \sum\limits_{j=1}^{m} G_{jk} N_j\right) G_{ik}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} = 0. \tag{10}$$

As the result we obtain a set of linear equations:

$$\mathbf{A}\,\mathbf{N} = \mathbf{B}, \tag{11}$$

where:

$\mathbf{N} = [N_1,\ N_2,\ ...,\ N_m]^{\mathrm{T}}$ is the vector of the sound powers being looked for,

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sigma_{N_1}} + \sum\limits_{k=0}^{n} \frac{G_{1k}G_{1k}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} & \sum\limits_{k=0}^{n} \frac{G_{2k}G_{1k}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} & ... & \sum\limits_{k=0}^{n} \frac{G_{mk}G_{1k}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} \\ \sum\limits_{k=0}^{n} \frac{G_{1k}G_{2k}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} & \frac{1}{\sigma_{N_2}} + \sum\limits_{k=0}^{n} \frac{G_{2k}G_{2k}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} & ... & \sum\limits_{k=0}^{n} \frac{G_{mk}G_{2k}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} \\ ... & ... & ... & ... \\ \sum\limits_{k=0}^{n} \frac{G_{1k}G_{mk}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} & \sum\limits_{k=0}^{n} \frac{G_{2k}G_{mk}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} & ... & \frac{1}{\sigma_{N_m}} + \sum\limits_{k=0}^{n} \frac{G_{mk}G_{mk}}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{1}{\sigma_{N_1}^2} N_{01} + \displaystyle\sum_{k=0}^{n} \dfrac{G_{1k} p_k^2}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} \\ \dfrac{1}{\sigma_{N_2}^2} N_{02} + \displaystyle\sum_{k=0}^{n} \dfrac{G_{2k} p_k^2}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} \\ \cdots \\ \dfrac{1}{\sigma_{N_m}^2} N_{0m} + \displaystyle\sum_{k=0}^{n} \dfrac{G_{mk} p_k^2}{\sigma_{\rho_k}^2 + \sigma_{\theta_k}^2} \end{bmatrix}.$$

It should be emphasised that the given above set of equations is a universal one. There are no requirements concerning the number of the measurement points. Even without performing any measurements the set of equations will have the solution (in such case a trivial solution is obtained and the knowledge will be the same as it was before performing calculations). With the lack of *a priori* knowledge (when the number of measurements cannot be smaller than the number of the estimated sound powers) of the classic Maximum Likelihood Estimation (MLE) equations will be obtained. Assuming that all measurements are burdened with errors of the same density of probability and we have none *a priori* knowledge we will obtain a typical set of equations of the LSM method.

Table 1. Results of sound power calculations.

Source	Sound power level [dB]	Accuracy [dB]		Sound power level after correction [dB]	Difference [dB]
ATR1	98.3	– 3.0	+ 1.8	97.1	−1.2
ATR2	98.3	– 3.0	+ 1.8	96.2	−2.1
S220	87.1	– 1.0	+ 0.8	88.0	0.9
S400	87.1	– 1.0	+ 0.8	85.4	−1.7
U220	91.0	– 10.0	+ 2.8	86.3	−4.7
U400	95.7	– 10.0	+ 2.8	90.6	−5.1

4. Conclusions

Problem of acoustic modelling of large factories is usually very complex. There are algorithms of operations (and computer software) allowing to obtain such models, however, they meet the quality requirements for relatively simple cases only. In more complex cases, when there is a large number of various and mutually penetrating sources, complicated site configuration, many buildings, the acoustic measurements results are burdened with large measuring errors and the obtained models are not precise. We can improve the accuracy of models by performing their additional calibration on the basis of the inversion method. Assuming additional algorithms is usually connected with providing additional information, which should be efficiently utilised by the calibration procedures.

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