

## DIGITAL MONO-PULSE METHOD IN CYLINDRICAL ANTENNA

W. LEŚNIAK, J. MARSZAL, R. SALAMON  
M. RUDNICKI, A. SCHMIDT

Gdańsk University of Technology  
Faculty of Electronics, Telecommunications and Informatics  
Department of Marine Electronics Systems  
Narutowicza 11/12, 80-952 Gdańsk, Poland  
e-mail: marszal@eti.pg.gda.pl

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The article presents a method designed to improve the accuracy of cylindrical antenna multi-beam sonar performance. The method is based on the mono-pulse method used in radars and sonars. It provides good bearing accuracy in a single sounding action. The only change required is the generation of two deflected beams and some simple computations. The new approach to mono-pulse technique, with application of digital signal processing is presented in the paper. The computer simulations proved, that good bearing accuracy can be obtained even for a relatively low signal to noise ratio.

**Key words:** passive sonar, cylindrical array, signal processing, spatial spectrum.

### 1. Introduction

One of the methods for improving sonar-bearing accuracy is the mono-pulse method, originally developed for radars [1, 2]. It is designed to produce two deflected and intersecting receiving beams, which are then observed for echo signal amplitudes. The beams are rotated to keep identical signal amplitude in both beams. The angle determined by the intersecting beams is the probable bearing of the target. Rotating the array or using one of the beamforming methods can rotate the beams.

Mono-pulse bearing is potentially more accurate than using a single beam pattern. In the latter, poor accuracy can be attributed to the small gradient of the beam pattern in the range of near angles of the beam's acoustic axis. Because the gradient is small, relatively big changes in the angle of wave incidence cause small changes in the amplitude of the signals being received. As a result, just watching signal amplitude changes could not possibly assess the wave arrival angle. The method is clearly unreliable, in particular when signal to noise ratio is small.

The mono-pulse method looks at signals coming from the intersecting beam patterns. The gradients of the patterns are high at the intersection and around it and if adequately selected, the intersection ensures maximal gradients. Even small changes of

wave incidence angles cause big changes in echo signal amplitude, ensuring a much better assessment of the bearing.

When beams are rotated mechanically or electronically, the procedure for smoothing amplitudes is time consuming. This is because, each time the beams are deflected, some time has to be allowed until the next echo pulse reaches the receiver. For a wide sonar range, the bearing could take up to fifty seconds or even several minutes and the target may change position. When things go completely wrong, and the amplitudes cannot be equalised, the method is rendered ineffective.

Today's multi-beam sonars can eliminate the defect that comes with the mono-pulse method. Sonar beams of any gradient are generated in a digital beamformer based on a single series of echo signal samples from antenna transducer outputs. As a result, there is no need to deflect beams in several or about a dozen transmissions. The time to establish the bearing is reduced to the necessary minimum. The procedure to imitate antenna deflection really comes down to determining a big number of slightly deflected beams and comparing signal amplitudes with these beams. The objective is to identify the point where these beams intersect with the smallest difference between the amplitudes of the signals.

The algorithm presented further in the article offers significant computational efficiencies and replaces the procedure for determining a high number of beams. The original algorithm was developed for a linear antenna whose beam patterns are described with analytical formulas [3].

## 2. Mono-pulse method in a sonar with a cylindrical array

The number of beams generated in the horizontal plane by beamformer cooperating with cylindrical antennas is the same as the number of columns of ultrasonic transducers forming the antenna. The spacing between column centres is equal to or nearly half the length of the wave in the central frequency of the echo signal spectrum. To generate one beam, signals are used coming from a few or about a dozen columns that form a section of the cylinder. The axis of the beam is perpendicular to the cylinder's chord, and for an odd number of columns it is perpendicular to the central column. As a result, all beams are perpendicular to all columns of the cylindrical antenna, and the number of beams equals the number of columns. All beams have identical width. Adjacent beams usually intersect at  $-3$  dB. The sonar's beamformer works together with the cylindrical antenna by compensating echo signal delays which are produced between the centre of the central column and the perpendicular projection of the centre onto the secant of the cylinder section or the tangent parallel to it.

If we mark the complex echo signal sample from the  $N$  column as  $x_n$ , and phase compensation ratio by  $w_{n,k}$ , the complex signal in  $K$  beam is computed using the following algorithm:

$$y_k = \sum_{n=-N+k}^{N+k} w_{n,k} x_n, \quad k = 0, 1, 2, \dots, \quad n = -N, \dots, +N, \quad (1)$$

where

$$w_{n,k} = e^{j(2\pi f_0 R/c)\{1-\cos[(n-k)\phi]\}} \quad (2)$$

In this formula  $f_0$  means the central frequency of echo signal spectrum,  $c$  – velocity of acoustic wave in water, and  $\phi$  – angular distance between adjacent antenna columns. To generate the beam  $2N+1$  antenna columns are used, and the angular width of the cylinder section generating a single beam is  $2N\phi$ .

The axis of the beam determined as above is perpendicular to column number  $k$ . The mono-pulse method requires that two beams are determined which are deflected from the beam with the echo signal (central beam). The simplest solution is to use two natural beams adjacent to the central beam. The signals in these beams are described with the relations (1) and (2). But this is not a good solution, because the point where the beams intersect is too low. Because the usable signals are small so is the signal to noise ratio. For a small signal to noise ratio the likelihood of error increases. This is why the optimal solution is to generate two deflected beams, but the angle the central beam and the axis should be more or less equal to half the width of beam  $\alpha$ . The point of beam intersection is then high enough, and the deflection of beam slopes is adjacent to the intersection point close to the maximum. As a result, bearing error is small even for a relatively small signal to noise ratio.

Figure 1 shows modules of patterns of the central beam and two deflected beams. The angle of beam deflection is equal to  $\alpha/2$ , where  $\alpha$  is a 3-decibel beam width. The design of the antenna makes sure that beam width  $\alpha$  is equal to the angular distance between column centres  $\phi$  (the linear distance between the columns is equal to half the wave length).

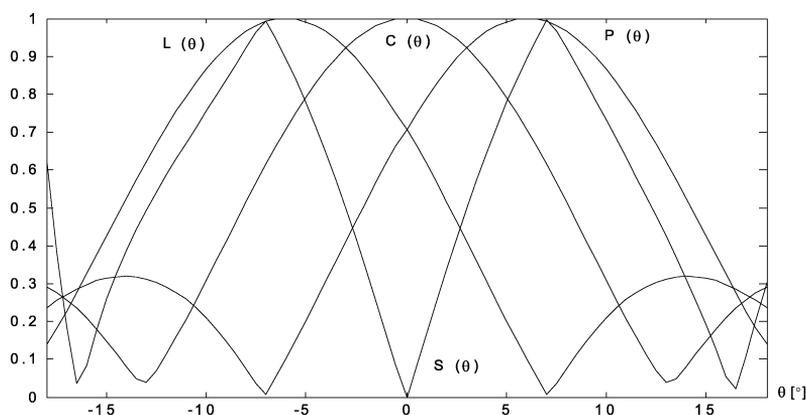


Fig. 1. Modules of beam patterns of the central beam  $C(\theta)$ , left beam  $L(\theta)$ , right beam  $P(\theta)$  and function  $S(\theta)$ .

The signals in deflected beams are determined from these formulas:

$$y_k^L = \sum_{n=-N+k}^{N+k} w_{n,k}^L x_n \quad - \text{left beam}, \quad (3)$$

$$y_k^P = \sum_{n=-N+k}^{N+k} w_{n,k}^P x_n \quad \text{– right beam,} \tag{4}$$

where

$$w_{n,k} = e^{j(2\pi f_0 R/c)\{1-\cos[(n-k)\phi+\alpha/2]\}} \quad \text{– left beam,} \tag{5}$$

$$w_{n,k} = e^{j(2\pi f_0 R/c)\{1-\cos[(n-k)\phi-\alpha/2]\}} \quad \text{– right beam.} \tag{6}$$

Signals  $y_k^L$  and  $y_k^P$  are functions of the angle of incidence  $\theta$ , because the values depend on the beam patterns as shown in Fig. 1.

To determine the angle of incidence of wave  $\theta$  the following function is convenient [3]:

$$S(\theta) = \frac{|y_k^P| - |y_k^L|}{|y_k^P| + |y_k^L|}. \tag{7}$$

Figure 1 shows the module of above function. As you can see, it is almost linear for angles  $\theta$ , which fit within the 3-decibel beam width. With the known value of function  $S(\theta)$  we can determine the wave incidence angle. It was demonstrated [3] that for a linear antenna the mean value of the wave incidence angle  $\bar{\theta}$  is approximately equal to:

$$\bar{\theta} \cong \arcsin [S(\theta) \sin(\alpha/2)]. \tag{8}$$

Figure 2 shows the relation between the mean angle  $\bar{\theta}$  computed from formula (8) and the wave incidence angle  $\theta$  for different signal to noise ratios at the input.

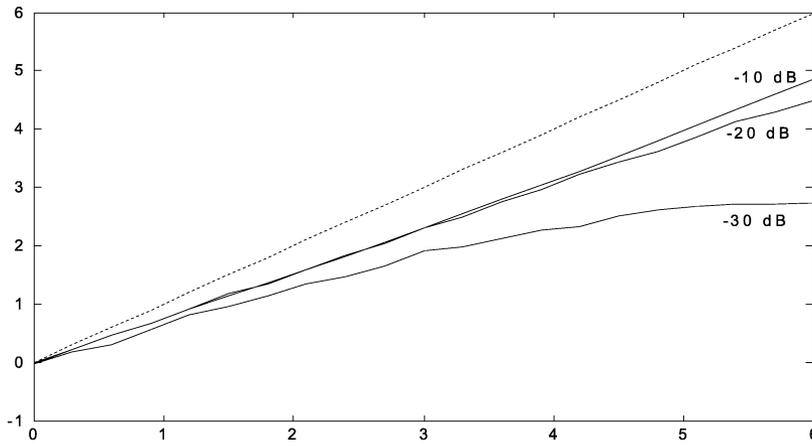


Fig. 2. Relation between mean bearing and the wave incidence angle for different SNR.

As the signal to noise ratio increases the relation becomes linear in high angle range. But the gradient of the function is not right, because it should be the same as that of the straight line marked with the dotted line. This is why an adjustment ratio must be introduced to change the gradient. When this is done, formula (8) takes this form:

$$\bar{\theta} \cong a \cdot \arcsin [S(\theta) \sin(\alpha/2)]. \tag{9}$$

If we keep all data from Fig. 2, and insert  $a = 1.25$  we obtain the relations shown in Fig. 3.

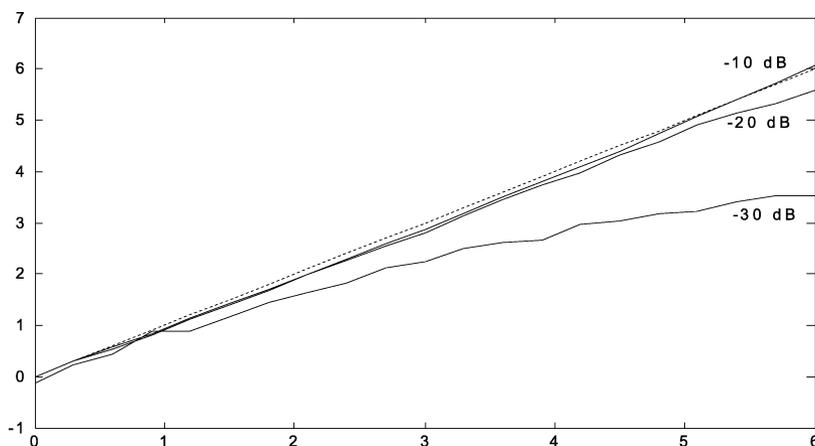


Fig. 3. Relations between mean bearing and wave incidence angle for different SNR, after adjustment of the curves.

### 3. Effects of noise on bearing error

Echo signals in all sonars are the sum of the usable signal and noise. Noise at the output of antenna transducers is the sum of acoustic noise and electric noise generated by the receiver. In both cases we can assume that noise in the channels of antenna columns is not correlated [3, 4]. Under this assumption, computer simulations were conducted with the objective to determine the mean value and dispersion of the bearing in the function of the signal to noise ratio at the input and the wave incidence angle. The computations were made for 1000 random samples each time. The usable signal was a sinusoidal signal at frequency  $f_0$ , and the noise was Gaussian noise in the band  $B = 4f_0$ . These conditions are for the passive sonar; active sonars can have much bigger signal to noise ratios. The input signals  $y_k^L$  and  $y_k^P$  for the purpose of the simulation were the height of the signal spectrum line with noise at frequency  $f_0$ .

Figures 2 and 3 show that the mean bearing error increases as the wave incidence angle increases and the signal to noise ratio drops. The smallest error is for angle  $\theta = 0^\circ$ , which is an angle at which the beam patterns intersect. It is also the direction of the acoustic axis of the central beam. As a reminder, for these angle the gradient of the central beam is the smallest, and the bearing error is the biggest. The signal to noise ratio has practically no effect on this positive result (within reason). The details are presented in Fig. 4.

The situation in Fig. 4 is for a maximal wave incidence angle contained in the 3-decibel central beam width. If exceeded, the target is detected in the adjacent beam. For this beam two deflected beams are generated and the incidence angle moves to the new left beam. In this beam it is to fit within the range  $(-6^\circ, +6^\circ)$ .

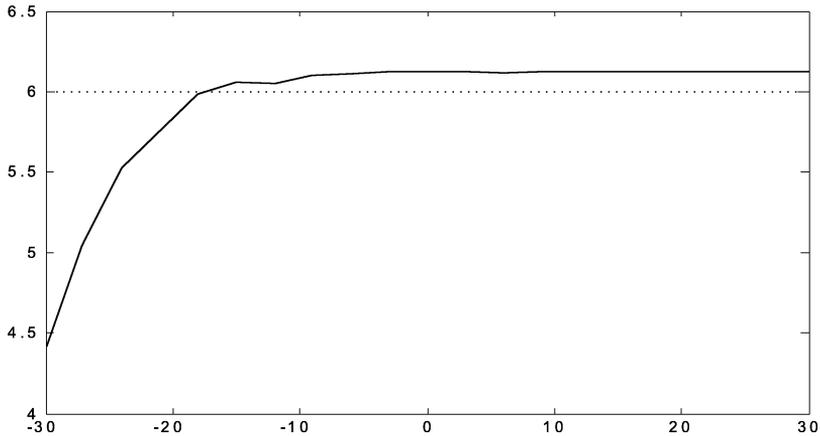


Fig. 4. Mean bearing in the function of signal to noise ratio for wave incidence angle  $\theta = 6^\circ$ .

For a big wave incidence angle and a high signal to noise ratio the mean bearing error becomes almost constant. This is because a relatively simple algorithm was used; it is based on simplifications, which cannot be fully justified for a cylindrical antenna and the wave deflection angle. However, the error in this example is about  $0.2^\circ$ , which is 60 times less than the conventional bearing error for a single beam, which is equal to a 3-decibel beam width (about  $12^\circ$ ).

The second way to measure the error is bearing standard deviation. The results of bearing standard deviation computations are given in Fig. 5. As expected, standard deviation drops when signal to noise ratio increases. For small signal to noise values errors in assessing the bearing are comparable with beam width, which renders the method completely ineffective. The good thing, however, is that the wave incidence angle has little effect on bearing standard deviation.

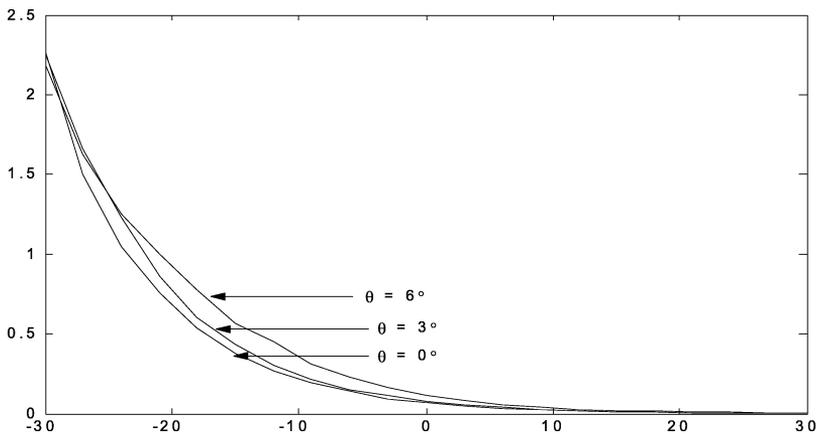


Fig. 5. Standard deviation of bearing in the signal to noise ratio function.

#### 4. Conclusion

The results of the simulation demonstrate that the proposed method for improving bearing accuracy is effective for wide wave incidence angles. The bearing error is small, when the signal to noise ratio is big enough. Compared with the classical mono-pulse method, which uses the wave incidence angle to determine the point of intersecting beam patterns (here  $\theta = 0^\circ$ ), the mean bearing error is relatively small for wider angles. Bearing standard deviation has little to do with the wave incidence angle, which does not speak to its disadvantage compared with the classical mono-pulse method. The main advantage of the method is the computational efficiency compared with the method, which generates a big number of beams with small angular deflection. When compared with the method using antenna rotation, the advantage of this method is that the time for determining bearing is much shorter.

The results suggest that the work should be continued to improve the properties of the method. The first step will be to reduce the error of mean bearing by modifying the algorithm (9). Next, the effects of changing the algorithm will be studied  $S(\theta)$ . Early results show that by replacing the sum of modules in formula (7) with the module of the sum smaller bearing errors are obtained.

#### References

- [1] SKOLNIK M. I., *Radar handbook*, Second Edition, Mc Graw Hill, 1990.
- [2] SHERMAN S. M., *Monopulse principles and techniques*, Dedham MA, Artech House, 1984.
- [3] SALAMON R., *Systemy hydroakustyczne*, Wydawnictwo Gdańskiego Towarzystwa Naukowego, Gdańsk 2006.
- [4] FANGHUI L., LIXIANG R., TENG L., LIP T., *High-speed data acquisition and digital signal processing of mono pulse Doppler radar system*, Proceedings of ICSP2000, pp. 1500–1506 (2000).