ARCHIVES OF ACOUSTICS
31, 4 (Supplement), 379–384 (2006)

APPLICATION OF THE GOLD RING BUNDLES FOR INNOVATIVE NON-REDUNDANT SONAR SYSTEMS

V. RIZNYK

University of Technology and Agriculture in Bydgoszcz
Al. S. Kaliskiego 7, 85-796 Bydgoszcz, Poland

O. BANDYRSKA, D. SKRYBAYLO-LESKIV

Lviv Polytechnic National University
Lviv, Ukraine
e-mail: rvv@polynet.lviv.ua

(*received June 15, 2006; accepted September 30, 2006*)

The paper involves techniques for configure linear, planar or three-dimensional space-tapered arrays of radar or sonar system, using novel designs based on the Perfect Combinatorial Sequencing Theory, namely the concept of Gold Ring Bundles (GRB)s for finding the optimal placement of array antenna elements in the system with respect to minimizing side lobes, while maintaining or improving on resolving ability and the other significant operating characteristics of the system. It is shown that the method provides many opportunities of the concept for configure of non-uniform array with non-redundant aperture of array systems, including acoustics and hydroacoustics.

Key words: non-redundant sonar system, Novel design, perfect combinatorial sequencing theory, Gold Ring Bundle, non-uniform antenna array, minimizing side-lobes.

1. Introduction

Modern combinatorial design techniques and modeling are well using for finding optimal solution of wide classes of technological problems, for example, the problem of structural optimization of sonar systems relates to finding the best placement of structural elements in spatially or temporally distributed systems with respect to improving the quality indices of the system, including active sonar, and it is closely connected with application of fundamental research in an applied finite-field theory [1]. However, the design based on the traditional mathematical apparatus of the theory is not always applicable for solution problems connected with constructing planar or three-dimensional non-uniform antenna arrays for sonar systems. In this connection a new approach to modeling the systems and processes is needed. In general case it was possible to take in consideration a conceptual model of the system as a sequence of numerical ordered-chain of sub-sequences to be of any length as well as number of terms in the sequence can be of any number too. Unfortunately, these numerical models are not very interest because their data redundancy as well as structural complexity. The problem, is known, to be of very important for configure non-redundant planar and three-dimensional antenna arrays. Such antenna arrays would cover nearly all of the required range of spatial frequencies. However, the classical theory of combinatorial configurations based on finite-field theory can hardly be expected effective for solving 2-D and 3-D problems using methods, based on the theory. Hence, both an advanced theory and regular method for finding optimal solution of this problem are needed. Research into underlying mathematical area involves investigation of novel techniques based on combinatorial models, such as multi-dimensional Gold Ring Bundles [2].

2. Gold Ring Bundles

Let us regard the chain sequence $K_n = \{k_1, k_2, ..., k_i, ..., k_n\}$ as being cyclic, so that element k_n is followed by k_1, we call this a ring sequence.

Table of sums of consecutive terms in ordered-ring sequence $K_n = \{k_1, k_2, ..., k_i, ..., k_n\}$ is demonstrated below (Table 1). A sum of consecutive terms in the ring sequence can have any of the n terms as its starting point p_j, and finishing point q_j, and can be of any length (number of terms) from 1 to $n-1$.

Table 1. Sums of consecutive terms in ordered-ring sequence.

p_j	q_j				
	1	2	...	$n-1$	n
1	k_1	$\sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n-1} k_i$	$\sum_{i=1}^{n} k_i$
2	$\sum_{i=1}^{n} k_i$	k_2	...	$\sum_{i=2}^{n-1} k_i$	$\sum_{i=2}^{n} k_i$
...			...		...
$n-1$	$\sum_{i=n-1}^{n} k_i + k_1$	$\sum_{i=n-1}^{n} k_i + \sum_{i=1}^{2} k_i$	...	k_{n-1}	$\sum_{i=n-1}^{n} k_i$
n	$k_n + k_1$	$k_n + \sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n} k_i$	k_n

So, each numerical pair (p_j, q_j) corresponds to sum $S_j = S(p_j, q_j)$, and can be calculated in case, when $p_j \leq q_j$, by equation:

$$S_j = S(p_j, q_j) = \sum_{i=p_j}^{q_j} k_i \, . \tag{1}$$

In case $p_j > q_j$ a ring sum can be calculated by

$$S_j = S(p_j, q_j) = \sum_{i=1}^{q_j} k_i + \sum_{i=p_j}^{n} k_i, \tag{2}$$

Easy to see from the Table 1, that the maximum number of distinct sums S_n of consecutive terms of the ring sequence is

$$S_n = n(n-1) + 1. \tag{3}$$

An n-stage ring sequence $K_n = \{k_1, k_2, ..., k_i, ..., k_n\}$ of natural numbers for which the set of all S_n circular sums consists of the numbers from 1 to $S_n = n(n-1)+1$, that is each number occurring exactly once is called an "Gold Ring Bundles" (GRB) with $R = 1$.

Here is an example of a numerical ring sequence with $n = 4$ and $S_n = n(n-1)+1 = 13$, namely $\{1,\ 3,\ 2,\ 7\}$, where $k_1 = 1$, $k_2 = 3$, $k_3 = 2$, $k_4 = 7$.

Table of circular sums for the sequence is given below (Table 2).

Table 2. Table of circular sums for numerical ring sequence {1, 3, 2, 7}.

p_j	q_j			
	1	2	3	4
1	1	4	6	13
2	13	3	5	12
3	10	13	2	9
4	8	11	13	7

Table 2 is calculated in similar way than above, using Eqs. (1) and (2). To see this Table 2 contains the set of all $S_n = n(n-1)+1 = 13$ sums to be consecutive elements of the 4-stage ring sequence {1, 3, 2, 7}, and each sum from 1 to $n-1$ occurs exactly once. So, the ring sequence is an one-dimensional Gold Ring Bundle (1D-GRB) with $n = 4$.

Here is a graphical representation of one-dimensional Gold Ring Bundle (1-D GRB) containing four $(n = 4)$ elements {1, 3, 2, 7}.

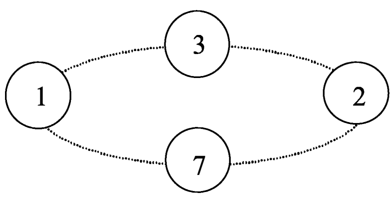

Fig. 1. A graph of one-dimensional Gold Ring Bundle (1D-GRB) containing four $(n = 4)$ elements {1, 3, 2, 7}.

It is known, a number of consecutive elements in a GRB can be of a considerable length and the more length the more number of GRB [2].

3. Two-dimensional Gold Ring Bundles

Let us regard the n-stage ring sequence $K_{2D} = \{(k_{11}, k_{12}), (k_{21}, k_{22}), ..., (k_{i1}, k_{i2}), ..., (k_{n1}, k_{n2})\}$, where we require all terms in each circular vector-sum to be consecutive 2-stage sequences as elements of the sequence. A circular vector-sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length from 1 to $n-1$. An n-stage ring sequence $K_{2D} = \{(k_{11}, k_{12}), (k_{21}, k_{22}), ..., (k_{i1}, k_{i2}), ..., (k_{n1}, k_{n2})\}$, for which the set of all

$$S_2D = n(n-1), \tag{4}$$

circular vector-sum forms two-dimensional grid, where each node of the grid occurs exactly R-times, is named a two-dimensional Gold Ring Bundle (2D-GRB).

Next, we consider two-dimensional n-stage ring sequence GRB with four $(n = 4)$ terms in the ring topology, where $k_1 = (0,2)$, $k_2 = (1,0)$, $k_3 = (1,1)$, $k_4 = (2,2)$ which graph is depicted below (Fig. 2).

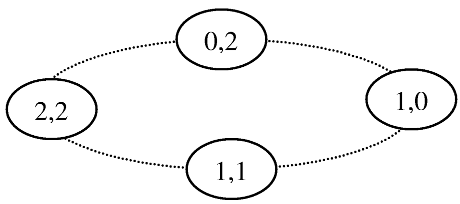

Fig. 2. A graph of two-dimensional Gold Ring Bundle (2D-GRB) containing four $(n = 4)$ elements $\{(0,2), (1,0), (1,1), (2,2)\}$.

We can calculate easy the all circular two-dimensional vector-sums, taking modulo $m_1 = 3$ for the first component of vector-sum and modulo $m_2 = 4$ for the second its component:

$(0,0) \equiv (2,2) + (0,2) + (1,0)$,
$(0,1) \equiv (1,1) + (2,2) + (0,2)$,
$(0,2) = (0,2)$,
$(0,3) \equiv (1,1) + (2,2)$,

$(1,0) = (1,0)$,
$(1,1) = (1,1)$,
$(1,2) \equiv (0,2) + (1,0)$,
$(1,3) \equiv (1,0) + (1,1) + (2,2)$,

$(2,0) \equiv (2,2) + (0,2)$,
$(2,1) \equiv (1,0) + (1,1)$,
$(2,2) = (2,2)$,
$(2,3) \equiv (0,2) + (1,0) + (1,1)$.

So long as the elements $(0,2)$, $(1,0)$, $(1,1)$, $(2,2)$ of the ring sequence themselves are circular vector-sums too, the circular vector-sums set configure the 3×4 matrix as follows:

$$\begin{matrix} (0,0) & (0,1) & (0,2) & (0,3) \\ (1,0) & (1,1) & (1,2) & (1,3) \\ (2,0) & (2,1) & (2,2) & (2,3) \end{matrix}$$

The result of the calculation forms the 3×4 grid which exhausts the circular 2D vector-sums and each of its meets exactly once $(R = 1)$. So, the ring sequence of the 2D vectors $\{(0,2), (1,0), (1,1), (2,2)\}$ is two-dimensional Gold Ring Bundle (2D-GRB) with $n = 4$, $R = 1$, and $m_1 = 3$, $m_2 = 4$.

4. Non-redundant 2-D antenna arrays

The present method relates to constructing thinned planar phased antenna array configurations, which have the antenna or sensor elements positioned in a manner as prescribed by the GRB, using appropriate variant of two-dimensional Gold Ring Bundle for the constructing. The method involves technique for minimizing sizes of antenna arrays prescribed by parameter S of appropriate GRB. The search algorithm allows finding optimal solution in the simplest way based on regarding selected matrix of circular two-dimensional vector-sums on the GRB as well as crossing out operations. These procedures make it possible to configure mask system with the smallest possible number of grids.

Here is example of constructing the planar antenna array configuration based on the two-dimensional Gold Ring Bundle with parameters $n = 4$, $R = 1$, and $m_1 = 3$, $m_2 = 4$ (Fig. 3).

We search needed solution after construction of 2-D matrix of all circular two-dimensional vector-sums on the GRB and regarding each of its with respect to search minimum of the sum using crossing out. The method described is illustrated in Fig. 3.

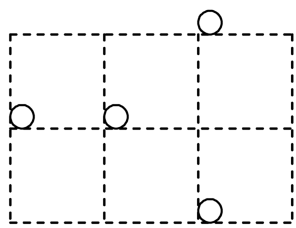

Fig. 3. An antenna array over 3×3 grids reconstructed from the array over 3×4 grids based on the 2-D GRB $\{(0,2), (1,0), (1,1), (2,2)\}$.

The example shows that the grid order based on the GRB can be reduced further without loss of the possibility to construct an antenna array.

5. Non-redundant 3-D aperture constructions

The three-dimensional $(t = 3)$ GRB of order n can be represented as n-stage ring-like sequence $\{(k_{11}, k_{21}, k_{31}), (k_{12}, k_{22}, k_{32}), \ldots, (k_{1n}, k_{2n}, k_{3n})\}$ which give us a set of circular 3-vector-sums on the sequence as $M_1 \times M_2 \times M_3$-matrix exactly R times.

Let the first of six $(n = 6)$ mask elements is the $(0, 0, 0)$ cell of $2 \times 3 \times 5$-matrix cycling. Now, we can obtain coordinates of the remaining five elements accordingly the underlying 3-D perfect distribution cycling modulus $m_1 = 2$, $m_2 = 3$, $m_3 = 5$: $(1, 1, 1)$, $(0, 2, 3)$, $(1, 2, 1)$, $(1, 1, 3)$, $(1, 2, 2)$.

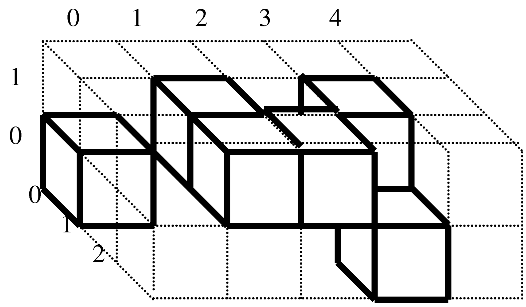

Fig. 4. A non-redundant $2 \times 3 \times 5$-matrix, based on the 3-D Gold Ring Bundle with parameters $n = 6$, $R = 1$, and $m_1 = 2$, $m_2 = 3$, $m_3 = 5$.

Now, to obtain configuration with smaller grids, we can exclude all right-hand columns (Fig. 4), and one can be reconstructed on smaller matrix $2 \times 3 \times 4$.

It exists *a priori* an infinite set of GRBs, and its parameters can be of any large number. Underlying technique can be used both for configure sonar systems with high quality indices due to all spacing vectors between their elements are different in order to avoid of interference of components of the same spatial frequency, and for development methods of non-redundant 3-D mask construction.

6. Conclusions

Two- and three-dimensional Gold Ring Bundles (GRB)s are perfect combinatorial models of non-redundant planar or 3-D space-tapered arrays of sonar. These models provide the optimal its structure from the point of the convenience to reproduce the maximum number of combinatorial varieties in the system with the limited number of elements. Method allows finding optimal solution in the simplest way using selected matrix of circular two- or three-dimensional vector-sums on the GRB as well as crossing out operations. These procedures make it possible to configure high performance arrays of sonar systems, including synthesis of wide-aperture equipped with non-redundant sets of opening for space and underwater acoustics.

References

[1] HALL M. Jr., *Combinatorial theory*, Waltham, Mass., Blaisdel Publishing Company, 1967.

[2] RIZNYK V. V., *Multi-dimensional systems based on perfect combinatorial models*, IEE, Multidimensional Systems: Problems and Solutions, pp. 5/1–5/4, 1998.