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Type A Standard Uncertainty of Long-Term Noise Indicators

Wojciech Michał BATKO, Bartłomiej STĘPIEŃ

AGH University of Science and Technology
al. Mickiewicza 30, 30-059 Kraków, Poland; e-mail: {batko, Bartlomiej.Stepien}@agh.edu.pl

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The problem of estimation of the long-term environmental noise hazard indicators and their uncertainty is presented in the present paper. The type A standard uncertainty is defined by the standard deviation of the mean. The rules given in the ISO/IEC Guide 98 are used in the calculations. It is usually determined by means of the classic variance estimators, under the following assumptions: the normality of measurements results, adequate sample size, lack of correlation between elements of the sample and observation equivalence. However, such assumptions in relation to the acoustic measurements are rather questionable. This is the reason why the authors indicated the necessity of implementation of non-classical statistical solutions. An estimation idea of seeking density function of long-term noise indicators distribution by the kernel density estimation, bootstrap method and Bayesian inference have been formulated. These methods do not generate limitations for form and properties of analyzed statistics. The theoretical basis of the proposed methods is presented in this paper as well as an example of calculation process of expected value and variance of long-term noise indicators L_{DEN} and L_N. The illustration of indicated solutions and their usefulness analysis were constant due to monitoring results of traffic noise recorded in Cracow, Poland.

Keywords: long-term noise indicators, uncertainty, non-classical statistics, kernel density estimation, bootstrap, Bayesian inference.

1. Introduction

Directive 2002/49/EC of the European Parliament requires carrying out a long-term policy of the environment protection against noise in the European Union countries. Its realisation is based on the estimation of long-term noise indicators L_{DEN} and L_N in the areas under protection.

The average A-weighted long-term sound levels L_{DEN} and L_N in dB are determined on the basis of noise annoyance indicators $L_{DEN,i}$ for $i = 1, 2, \ldots, M$ (where: $M = 365$ or $M = 366$ for the leap year) of all days of the calendar year in the day-evening-night periods:

$$L_{DEN,i} = 10\log\left[\frac{1}{24}\left(12\times 10^{0.1L_{D,i}} + 4\times 10^{0.1(L_{E,i}+5)} + 8\times 10^{0.1(L_{N,i}+10)}\right)\right], \qquad (1)$$

where $L_{D,i}$ is the A-weighted sound level, determined from the day-time noise exposure, i.e. from 6:00 a.m. to 6:00 p.m., dB, $L_{E,i}$ is the A-weighted sound level, determined from the noise exposures from 6:00 p.m. to 10:00 p.m., dB, $L_{N,i}$ is the A-weighted sound level, determined for the night periods, i.e. from 10:00 p.m. to 6:00 a.m., dB and night periods $L_{N,i}$ for $i = 1, 2, \ldots, M$ determined by relation:

$$L_{N,i} = 10\log\left(\frac{1}{K}\sum_{i=1}^{K} 10^{0.1(L_{Aeq,T})_i}\right), \qquad (2)$$

where K is the sample size, $(L_{Aeq,T})_i$ is the equivalent sound level for the i-th sample, dB.

The estimation of the long-term indicators of the acoustic hazard for the environment L_{DEN} and L_N is the average value calculated from all calendar days:

$$L_{DEN} = 10\log\left(\frac{1}{M}\sum_{i=1}^{M} 10^{0.1L_{DEN,i}}\right), \qquad (3)$$

$$L_N = 10\log\left(\frac{1}{M}\sum_{i=1}^{M} 10^{0.1L_{N,i}}\right), \qquad (4)$$

forming a set of two indicators $L_{Aeq.LT} = \{L_{DEN}, L_N\}$, requires an access to the results of the whole year sound level monitoring.

The necessity of validation of the obtained results, which requires the analysis of uncertainty budget of estimation, is connected with the process of calculating the average long-term noise indicators determined by values L_{DEN} and L_N.

An essential component of such budget is the standard type A uncertainty defined as the standard deviation of the mean from the inspections results. The rules given in the ISO/IEC Guide 98-3:2008 are used in the calculations. They are based on the classic variance estimators on the condition of assigning the normal distribution and lack of correlation between elements of the sample as well as adequate sample size and observation equivalence to random results of the sampling inspections.

Many authors of publications assume that the above assumptions are fulfilled (Romeu *et al.*, 2006; Makarewicz, Żółtowski, 2008; Makarewicz, Gałuszka, 2011) and use ISO/IEC Guide 98 for uncertainty calculation. However, the acoustics measurement results do not meet these assumptions. Referring to Don and Rees's (1985), Tang and Au's (1999), Batko and Stępień's (2007), Gimenez and Gonzalez's (2009), Gałuszka's (2010) publications, the assumption of the normal distributions of measurement results is wrong. Additionally, Wszołek and Kłaczyński (2006) proved that the road traffic noise probability distributions are not related to any statistical distribution known in the literature. However, the probability density function of the average long-term sound levels indicates asymmetry (Batko, Przysucha, 2011). In practice, there is a necessity of estimating the average long-term noise indicators $L_{Aeq.LT}$ on the basis of environment sampling inspections (Schomer, DeVor, 1981; Gaja *et al.*, 2003; Romeu *et al.*, 2006). Moreover, samples from inspections are small and correlated. Extra-statistical information in relation to the occurrence of certain noise expositions in environment, especially in the night hours (more than one maximum), also discredit this assumption.

Therefore, the use of current methods for assessment of standard uncertainties of controlled noise indicators demands special care. Otherwise, one is prone to essential errors in determining the coverage factor for expanded uncertainty.

Therefore, searching for non-standard procedures of estimation of the average long-term noise indicators L_{DEN} and L_N and their variances, seems to be necessary. This problem was already signalled in the previous papers of the authors (Batko, Stępień, 2009; 2010; 2011; Batko, Przysucha, 2010; 2011; Batko, Pawlik, 2012).

The authors proposed three new algorithms for solving these problems. They are based on non-parametric statistical methods allowing to determine the distribution of a random variable without any information of its belonging to the defined class of distributions and a limited sample size. The authors used kernel density estimation, bootstrap method, and Bayesian inference. Discussion of the algorithms, together with the example illustrating their functioning, will be contained in the paper. The reference base constitutes the results of the constant noise monitoring recorded on one of the main arteries of Cracow, Poland.

2. Classical algorithm

Estimation of long-term noise hazards indicators, described Eqs. (3) and (4), requires an access to the results of the whole year sound level monitoring.

In practice, it is not possible to meet such a requirement. Therefore, estimations of indicators are usually done on the basis of highly limited random samples. They are obtained as results of environmental sampling inspections. Sample size n is very small and the range is from few to several elements (Schomer, DeVor, 1981; Gaja *et al.*, 2003; Romeu *et al.*, 2006).

In the classical approach, assumed are: the normality of measurements results, adequate sample size, lack of correlation between elements of the sample and observation equivalence. In this case, the rules given in the ISO/IEC Guide 98-3:2008 are used to determine the type A standard uncertainty of the long-term noise indicators.

Equations (3) and (4) determined expected values of the long-term noise indicators estimated on the basis of the limited random sample.

$$\overline{L}_{Aeq.LT} = 10\log\left(\frac{1}{n}\sum_{i=1}^{n} 10^{0.1L_{Aeq.LT,i}}\right), \tag{5}$$

where n is the sample size, $L_{Aeq.LT,i}$ is the index level for the i-th sample, dB.

The type A standard uncertainty of the long-term noise hazard indicators were determined by Eq. (6):

$$s\left(\overline{L}_{Aeq.LT}\right) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(L_{Aeq.LT,i} - \overline{L}_{Aeq.LT}\right)^2}. \tag{6}$$

3. Non-classical algorithms

In Sec. 2 the algorithm compatible with ISO/IEC Guide 98 called the classical approach was presented. The non-classical statistical methods were used to estimate the expected value of the environmental noise hazard indicators and their type A standard uncertainty. Section 3 will present detailed algorithms of point estimation of characteristics of the long-term noise indicators.

The estimation of the expected value and type A standard uncertainty of the long-term noise indicators

$L_{Aeq.LT}$ was done on the basis of simple random samples n – elements, which were sampled from the investigated population. In the present paper, the investigated populations are the results of daily noise annoyance indicators during the day-evening-night periods $L_{DEN,i}$ and night periods $L_{N,i}$ which were determined by Eqs. (1) and (2). The values were determined on the basis of the results recorded during the whole calendar year by the constant noise monitoring system. Simple random samples of size $n = 5, 10, 15$ (simulating the number of controlled days on the basis of which the levels: L_{DEN} and L_N will be estimated) were sampled from the above mentioned population.

3.1. Kernel density estimation

The first developed non-classic estimation algorithm of the expected value and type A standard uncertainty was the algorithm which makes use of non-parametric kernel density estimation (Rosenblatt, 1956; Parzen, 1962).

The estimation of probability density function of noise annoyance indicators calculated on the basis of a simple random sample x of size n takes the form of the following equation (Kulczycki, 2005):

$$\widehat{f}(L_{Aeq.LT}) = \frac{1}{n \cdot h} \sum_{i=1}^{n} K\left(\frac{x - L_{Aeq.LT,i}}{h}\right), \tag{7}$$

where K is the normal kernel function, h is the smoothing parameter called the bandwidth, calculated "automatically" on the basis of the algorithm (Bowman, Azzalini, 1997) which was implemented in Matlab package, $L_{Aeq.LT,i}$ are elements of the simple random sample x.

On the basis of the kernel density estimator (7) the expected values of the long-term noise indicators from dependency were determined (Kulczycki, 2005):

$$\overline{L}^{K}_{Aeq.LT} = \int_{-\infty}^{+\infty} x\, \widehat{f}(L_{Aeq.LT})\, \mathrm{d}x, \tag{8}$$

their type A standard uncertainty takes the form (Kulczycki, 2005):

$$s_K\left(\overline{L}^{K}_{Aeq.LT}\right) = \sqrt{\frac{1}{n} \int_{-\infty}^{+\infty} \left(x - \overline{L}^{K}_{Aeq.LT}\right)^2 \widehat{f}(L_{Aeq.LT})\, \mathrm{d}x}, \tag{9}$$

where $\overline{L}^{K}_{Aeq.LT}$ is the estimate of the expected value of the long-term noise indicators, dB, $\widehat{f}(L_{Aeq.LT})$ is the kernel density estimator of noise annoyance indicators (7).

3.2. Bootstrap method

Another non-classical statistical method which was used to estimate the point characteristics of the long-term noise indicators is the bootstrap method.

In the present paper the values of the estimated noise indicators were determined on the basis of $B = 10.000$ bootstrap replications. The greater number of bootstrap replications does not give more accurate results of estimation. The sequence: $L^*_{Aeq.LT,1}, \ldots, L^*_{Aeq.LT,i}, \ldots, L^*_{Aeq.LT,B}$ was obtained as a result of the values: $L_{Aeq.LT,1}, \ldots, L_{Aeq.LT,i}$, $L_{Aeq.LT,n}$ generated B-times, i.e. sampling independently from samples of size n and calculating each time the statistics value $L^*_{Aeq.LT,b}$ from Eq. (5). This sequence was used for determining histograms which illustrate the bootstrap distribution of L_{DEN} and L_N.

The bootstrap estimates of the long-term noise indicators are (Efron, Tibshirani, 1993):

$$\overline{L}^{*}_{Aeq.LT} = \frac{1}{B} \sum_{b=1}^{B} L^*_{Aeq.LT,b}, \tag{10}$$

where $L^*_{Aeq.LT,b}$ is the level of the b-th bootstrap estimate of index $L_{Aeq.LT}$, dB, B is the number of bootstrap replications.

The bootstrap estimate of the type A uncertainty can be determined as follows (Efron, Tibshirani, 1993):

$$s_B\left(\overline{L}^{*}_{Aeq.LT}\right) = \sqrt{\frac{1}{B-1} \sum_{b=1}^{B} \left(L^*_{Aeq.LT,b} - \overline{L}^{*}_{Aeq.LT}\right)^2}. \tag{11}$$

3.3. Bayesian estimation

The third non-classical statistical method which was used to estimate the expected value and type A standard uncertainty of the long-term noise hazard indicators in environment is the Bayesian inference algorithm based on Bayes formula (Osiewalski, 2001; Gamerman, Lopes, 2006; Candy, 2009):

$$p\,(\theta|x) = \frac{f(x)}{p(x)} = \frac{p\,(x|\theta)\, p(\theta)}{\displaystyle\int_{\Omega} p\,(x|\theta) p(\theta)\, \mathrm{d}\theta}, \tag{12}$$

where $p(\theta)$ is called the prior density (before measurement), $p(x|\theta)$ is called the sampling density or likelihood (more likely to be true), $p(x)$ is called the marginal data density or evidence (normalizes the posterior to assure its integral is unity), Ω is called the parameters space.

This approach requires knowledge of two distributions. The first is the sampling density $p\,(x|\theta)$. It was determined on the basis of a simple random sample x of size n that was selected from the investigated population. The second is the prior density $p(\theta)$, determined on the basis of all available results of constant noise monitoring in the measuring point, excluding the results of the year of the simple random sample x. Both determined prior and sampling density make use of the kernel density estimation presented in Subsec. 3.1.

The Bayesian inference requires the use of numerical methods. In the present paper, the random walk Metropolis-Hastings (random walk M-H) (Metropolis, Ulam, 1949; Metropolis *et al.*, 1953; Hastings, 1970) was used to generate samples x_{BAY} from the posterior distribution. The random sample was obtained as a result of values $\left(L^{\mathrm{BAY}}_{Aeq.LT,1}, \ldots, L^{\mathrm{BAY}}_{Aeq.LT,i}, \ldots, L^{\mathrm{BAY}}_{Aeq.LT,N}\right)$ generated N-times from the posterior density. On the basis of the above mentioned sample the Bayesian estimate of the long-term noise indicators, defined as (Gamerman, Lopes, 2006; Candy, 2009), were determined:

$$\overline{L}^{\mathrm{BAY}}_{Aeq.LT} = \frac{1}{N-S} \sum_{i=S+1}^{N} L^{\mathrm{BAY}}_{Aeq.LT,i}, \tag{13}$$

where $L^{\mathrm{BAY}}_{Aeq.LT,i}$ is the level of the i-th Bayesian estimate of index $L_{Aeq.LT}$ from sample x_{BAY}, dB, N is the total number of samples, and S is the number of samples burned.

Due to this we received a correlated sample x_{BAY} as a result of the sampling. The autocorrelation function had to be designated and the elements that are not correlated had to be selected. We obtained a simple random sample x^*_{BAY} of size k.

The Bayesian estimate type A standard uncertainty was calculated on the basis of the simple random sample x^*_{BAY} from posterior distribution as follows (Gamerman, Lopes, 2006; Candy, 2009):

$$s_{\mathrm{BAY}}\left(\overline{L}^{\mathrm{BAY}}_{Aeq.LT}\right) = \sqrt{\frac{1}{k} \sum_{i=1}^{k} \left(L^{\mathrm{BAY}^*}_{Aeq.LT,i} - \overline{L}^{\mathrm{BAY}^*}_{Aeq.LT}\right)^2}, \tag{14}$$

where $L^{\mathrm{BAY}^*}_{Aeq.LT,i}$ is the level of the i-th Bayesian estimate of index $L_{Aeq.LT}$ from sample x^*_{BAY}, dB, $\overline{L}^{\mathrm{BAY}^*}_{Aeq.LT}$ is the mean of simple random sample x^*_{BAY}, dB, k is the sample size of x^*_{BAY}.

The Markov chain length and number of cycles burned were established separately for each indicator depending on the algorithm convergence to the posterior density. The number of cycles burned ranged from 10.000 to 40.000. The acceptance probability of random walk M-H algorithm ranged from 39% to 48%, which provided a fast convergence of algorithm MCMC to posterior distribution.

4. Numerical experiments

For the scientific research an application in the Matlab software package was developed. In this application, the classical algorithm approach as well as algorithms with non-classical statistical methods: kernel estimation of probability density function, bootstrap resampling method, and Bayesian inference were implemented.

The study of the usefulness and effectiveness of the presented algorithms in practical situations was carried out on the basis of estimation of the expected value and type A standard uncertainty of long-term environmental noise hazard indicators. For this purpose constant monitoring results of traffic noise recorded on one of the main arteries of Cracow, Poland were used.

4.1. Research material

The study of the usefulness and effectiveness of the presented algorithms were carried out on the real measurement data. They were recorded in 2004, 2005, 2008, and 2009 by a constant noise monitoring system. To estimate the expected value and type A standard uncertainty only those days in the year were used in which 24-hour period of A-weighted sound level was recorded. The number of days differed each year of the study, i.e. 331, 317, 314, 334 respectively. In other days there is a break in the recorded data, or their total absence.

Based on the data obtained from Voivodship Inspectorate for Environmental Protection in Cracow noise annoyance indicators during the day-evening-night periods and night periods, presented in Fig. 1, were determined.

4.2. Results of the experiments

The results of estimation of expected values of long-term environmental noise hazard indicators $L_{Aeq.LT} = \{L_{DEN}, L_N\}$ together with designated absolute errors (in parentheses) are presented in Table 1. The type A standard uncertainty obtained using the methods discussed above are presented in Table 2. The obtained results are also presented graphically in Figs. 4–7 marking the estimate of the expected value (marker) and interval $L_{Aeq.LT} \pm s(L_{Aeq.LT})$ (thin vertical line) for all of the presented methods. These graphs provide a comparative visual analysis of the obtained estimates.

The measured value was determined basing on the results of constant noise monitoring (Fig. 1) for each year. Due to a large number of data, only a few examples of distributions are presented in this paper (Figs. 2 and 3).

In Table 1 the estimation results of the expected value of the long-term environmental noise hazard indicators with a greater absolute error than the error of a classical estimate are marked in bold, while in Table 2 the estimation results of the type A standard uncertainty greater than the classical estimate are marked in bold.

The analysis of estimation results presented in the graphical and tabular way shows that there are some discrepancies between the measured values and the values estimated based on the presented methods ranging from 0.0 dB to 1.3 dB. The kernel and bootstrap estimators of the expected value are characterized by more accurate results than the classical estimator.

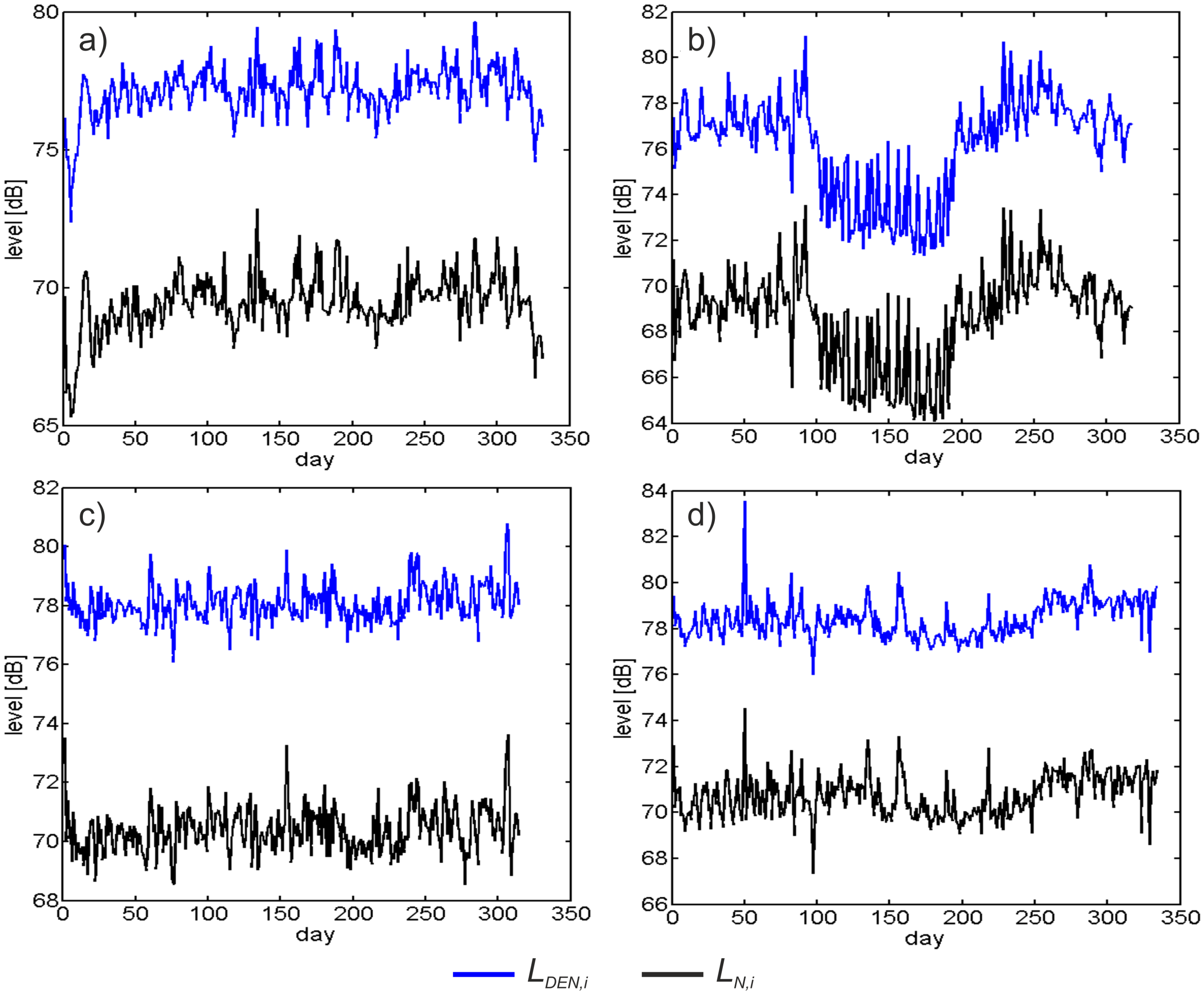

Fig. 1. Time history of noise annoyance indicators during the (a) 2004, (b) 2005, (c) 2008 and (d) 2009.

Only 16.7% (4 values) of the kernel estimates were more distant from the measured value than the classical estimate. One may also note that the distribution of the values burdened with a greater absolute error is random and does not depend on the size of the analyzed sample under which they were calculated. The first three estimators: classical, kernel, and bootstrap are dependent on the representativeness of the sample. Such a conclusion can be drawn from the estimates of the expected value of long-term noise indicators for 2005.

The measured value in 2005 is underestimated due to a decrease registered for noise annoyance indicators (Fig. 1b). In this case the Bayesian estimates are more accurate because the prior distribution $p(\theta)$ was taken into account. The influence of this distribution is visible in all Bayesian estimates of the expected values, therefore, 87.5% of the estimation values are burdened with an absolute error greater than the classic estimator.

Out of the four presented methods for estimating the expected value of long-term noise indicators estimates obtained using the kernel estimator were closest to the measured values (11 estimates). At the opposite pole there is the Bayesian estimator which is characterized by the largest number (21 values) of estimates furthest from the measured values.

While analyzing the table with the results of estimation type A standard uncertainty of long-term environmental noise hazard indicators, it can be seen that the Bayesian estimates have the greatest values. This is because of the fact that inference based on Bayes' theorem (12) introduces additional information about the parameter possessed before observing the data in the form of the prior density. The smallest value of the uncertainty is characterized by a bootstrap estimator. This fact proves that the method is the most accurate for determining the values of long-term noise indicators. The estimates determined by the kernel estimator are characterized by larger values than calculated in the classical way on the basis of the analyzed sample. The estimate values of type A standard uncertainty range from 0.2 dB to 1.0 dB.

Table 1. Estimates and absolute errors of the expected values of long-term noise indicators.

Indicator	Measured value [dB]	Sample size n	Classical estimate (error) [dB]	Kernel estimate (error) [dB]	Bootstrap estimate (error) [dB]	Bayesian estimate (error) [dB]
year 2004						
L_{DEN}	77.2	5	77.4 (0.2)	77.4 (0.2)	77.4 (0.2)	**77.6 (0.4)**
		10	77.3 (0.1)	77.2 (0.0)	77.3 (0.1)	**77.7 (0.5)**
		15	77.3 (0.1)	77.3 (0.1)	77.3 (0.1)	**77.6 (0.4)**
L_N	69.5	5	69.7 (0.2)	69.6 (0.1)	69.7 (0.2)	**69.9 (0.4)**
		10	69.3 (−0.2)	69.3 (−0.2)	69.3 (−0.2)	69.6 (0.1)
		15	69.6 (0.1)	69.4 (−0.1)	69.5 (0.0)	**69.8 (0.3)**
year 2005						
L_{DEN}	76.5	5	76.9 (0.4)	76.5 (0.0)	76.8 (0.3)	**77.7 (1.2)**
		10	76.7 (0.2)	76.7 (0.2)	76.7 (0.2)	**77.3 (0.8)**
		15	77.0 (0.5)	76.6 (0.1)	77.0 (0.5)	**77.8 (1.3)**
L_N	68.9	5	68.7 (−0.2)	**68.4 (−0.5)**	68.7 (−0.2)	**69.8 (0.9)**
		10	69.2 (0.3)	69.1 (0.2)	69.2 (0.3)	**69.7 (0.8)**
		15	69.4 (0.5)	68.8 (−0.1)	69.3 (0.4)	**69.9 (1.0)**
year 2008						
L_{DEN}	78.1	5	77.8 (−0.3)	77.8 (−0.3)	77.8 (−0.3)	**77.6 (−0.5)**
		10	78.3 (0.2)	78.2 (0.1)	78.3 (0.2)	77.9 (−0.2)
		15	78.2 (0.1)	78.1 (0.0)	78.2 (0.1)	**77.9 (−0.2)**
L_N	70.4	5	70.2 (−0.2)	70.2 (−0.2)	70.2 (−0.2)	**70.0 (−0.4)**
		10	70.6 (0.2)	70.5 (0.1)	70.6 (0.2)	70.2 (−0.2)
		15	70.6 (0.2)	70.4 (−0.0)	70.6 (0.2)	**70.0 (−0.4)**
year 2009						
L_{DEN}	78.5	5	78.6 (0.1)	78.5 (0.0)	78.6 (0.1)	**77.9 (−0.6)**
		10	78.5 (0.0)	**78.4 (−0.1)**	78.5 (0.0)	**77.9 (−0.6)**
		15	78.4 (−0.1)	**78.3 (−0.2)**	78.4 (−0.1)	**78.0 (−0.5)**
L_N	70.8	5	71.1 (0.3)	71.0 (0.2)	71.0 (0.2)	**70.1 (−0.7)**
		10	71.0 (0.2)	71.0 (0.2)	71.0 (0.2)	**70.4 (−0.4)**
		15	70.7 (−0.1)	**70.6 (−0.2)**	70.7 (−0.1)	**70.1 (−0.7)**

Table 2. Estimates of the type A standard uncertainty of the long-term noise indicators.

Indicator	Sample size n	Classical estimate [dB]	Kernel estimate [dB]	Bootstrap estimate [dB]	Bayesian estimate [dB]
year 2004					
L_{DEN}	5	0.4	0.4	0.4	**0.6**
	10	0.3	**0.4**	0.3	**0.7**
	15	0.2	0.2	0.2	**0.6**
L_N	5	0.5	0.5	0.5	**0.8**
	10	0.2	0.2	0.2	**0.6**
	15	0.3	0.3	0.3	**0.7**
year 2005					
L_{DEN}	5	1.0	1.0	0.7	0.7
	10	0.3	0.3	0.3	**0.7**
	15	0.6	0.6	0.5	**0.8**
L_N	5	1.0	1.0	0.7	0.8
	10	0.4	0.4	0.4	**1.0**
	15	0.6	0.6	0.6	**0.8**
year 2008					
L_{DEN}	5	0.4	0.4	0.3	**0.7**
	10	0.3	0.3	0.3	**0.7**
	15	0.2	0.2	0.2	**0.6**
L_N	5	0.3	0.3	0.3	**0.6**
	10	0.3	0.3	0.2	**0.8**
	15	0.3	**0.4**	**0.4**	**0.9**
year 2009					
L_{DEN}	5	0.4	**0.5**	0.4	**0.7**
	10	0.3	0.3	0.3	**0.6**
	15	0.2	0.2	0.2	**0.6**
L_N	5	0.5	**0.6**	0.4	**0.9**
	10	0.3	0.3	0.3	**0.6**
	15	0.2	**0.3**	0.2	**0.8**

Table 3. Range of expected values and the type A standard uncertainty.

Indicator	Range of classical estimate [dB]		Range of kernel estimate [dB]		Range of bootstrap estimate [dB]		Range of Bayesian estimate [dB]	
	expected values	the type A standard uncertainty	expected values	the type A standard uncertainty	expected values	the type A standard uncertainty	expected values	the type A standard uncertainty
year 2004								
L_{DEN}	0.1	0.2	0.2	0.2	0.1	0.2	0.1	0.1
L_N	0.4	0.3	0.3	0.3	0.4	0.3	0.3	0.2
year 2005								
L_{DEN}	0.3	0.7	0.2	0.7	0.3	0.4	0.5	0.1
L_N	0.7	0.6	0.7	0.6	0.6	0.3	0.2	0.2
year 2008								
L_{DEN}	0.5	0.2	0.4	0.2	0.5	0.1	0.3	0.1
L_N	0.4	0.0	0.3	0.1	0.4	0.2	0.2	0.3
year 2009								
L_{DEN}	0.2	0.2	0.2	0.3	0.2	0.2	0.1	0.1
L_N	0.4	0.3	0.4	0.3	0.3	0.2	0.3	0.3

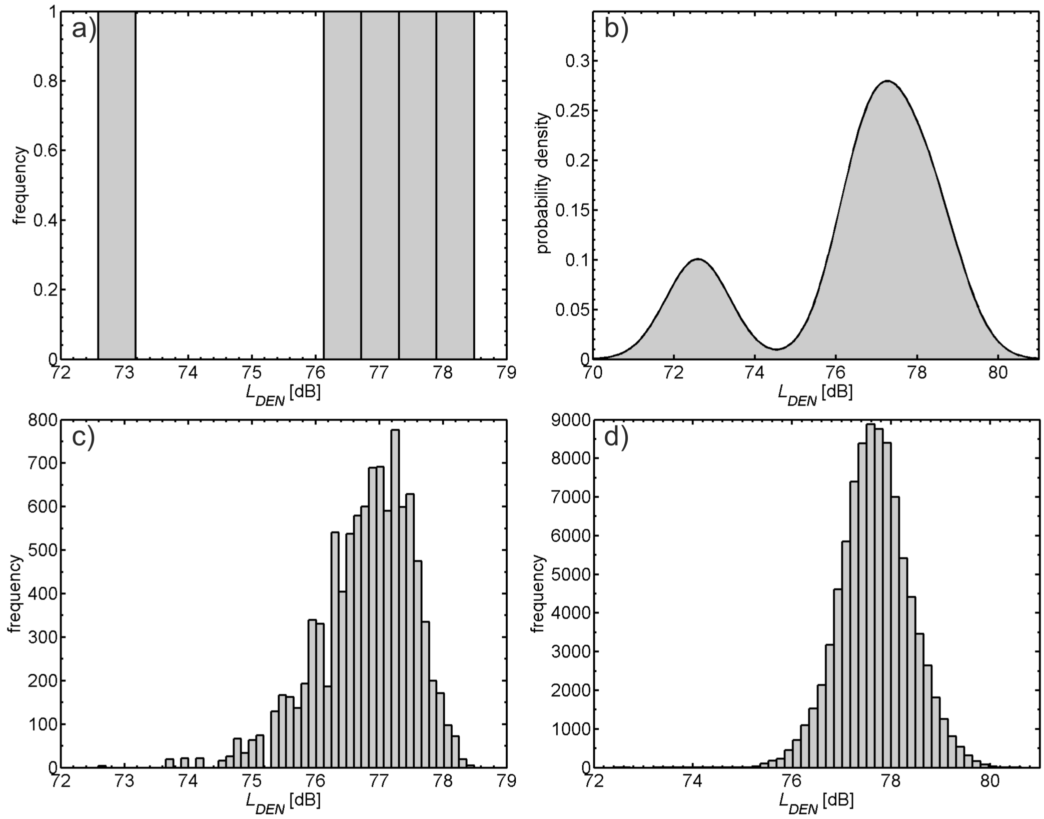

Fig. 2. Probability distributions of L_{DEN} in 2005 obtained from the analyzed sample of size $n = 5$: a) analyzed sample, b) kernel density estimator, c) bootstrap distribution, d) *a posteriori* distribution.

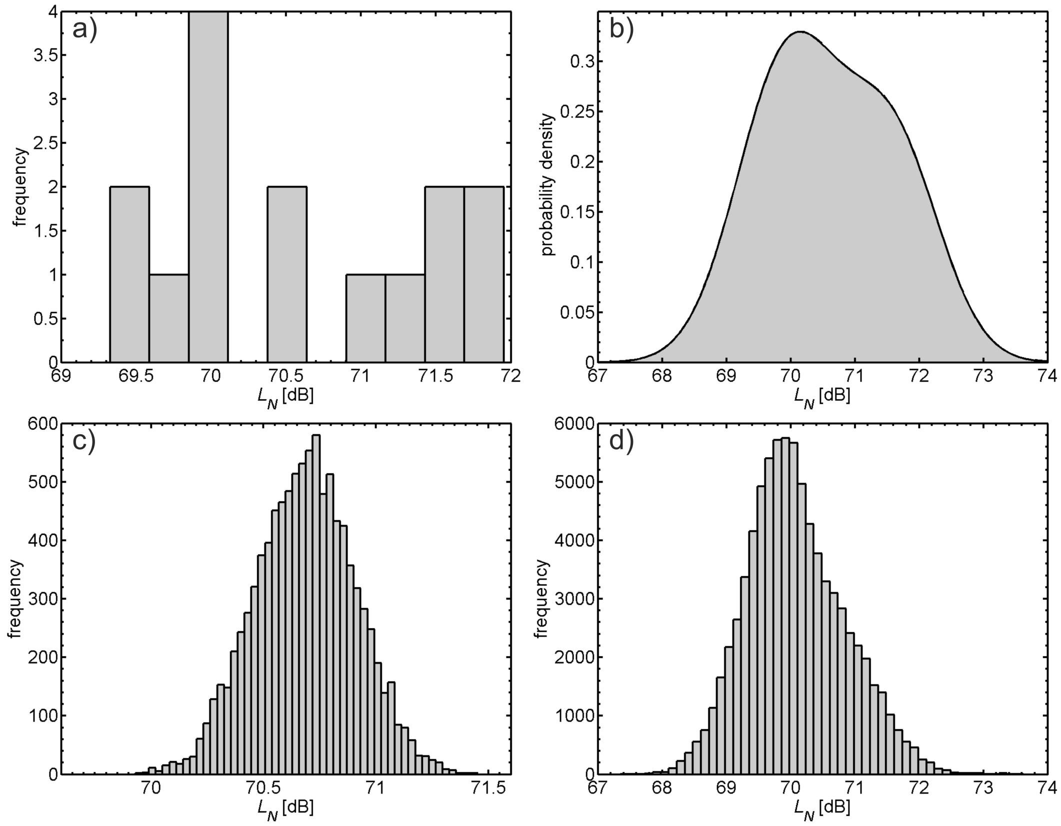

Fig. 3. Probability distributions of L_N in 2009 obtained from the analyzed sample of size $n = 15$: a) analyzed sample, b) kernel density estimator, c) bootstrap distribution, d) *a posteriori* distribution.

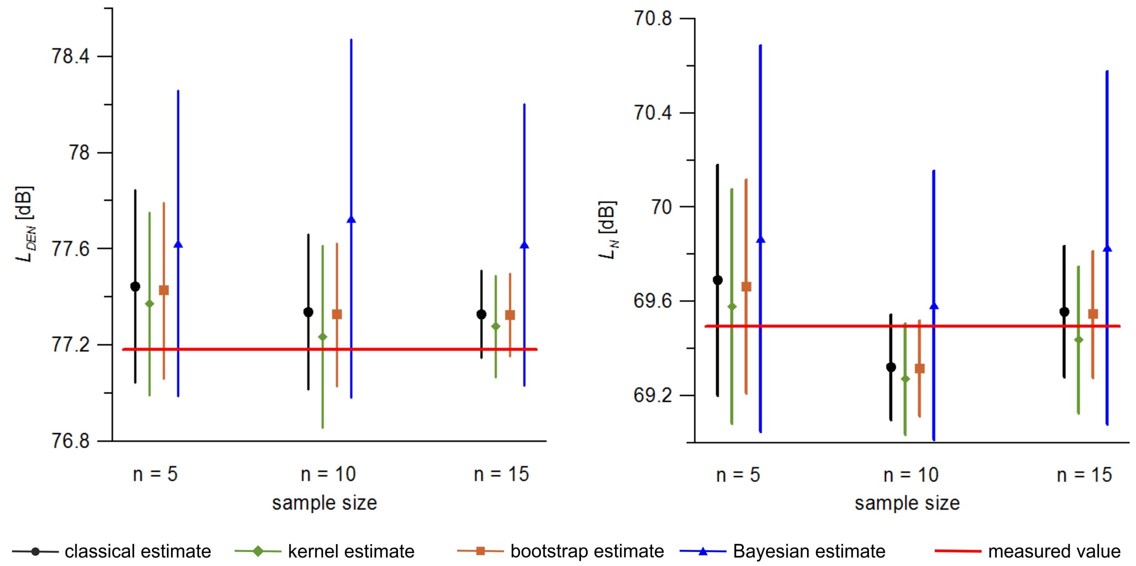

Fig. 4. Estimates of expected values and type A standard uncertainty of long-term noise indicators in 2004.

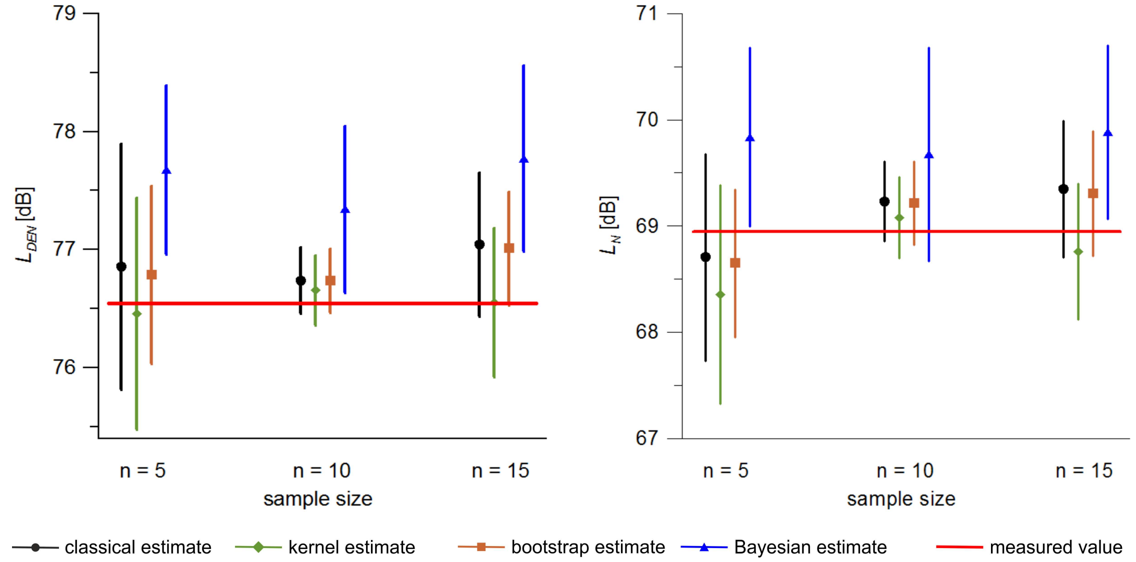

Fig. 5. Estimates of the expected value and type A standard uncertainty of long-term noise indicators in 2005.

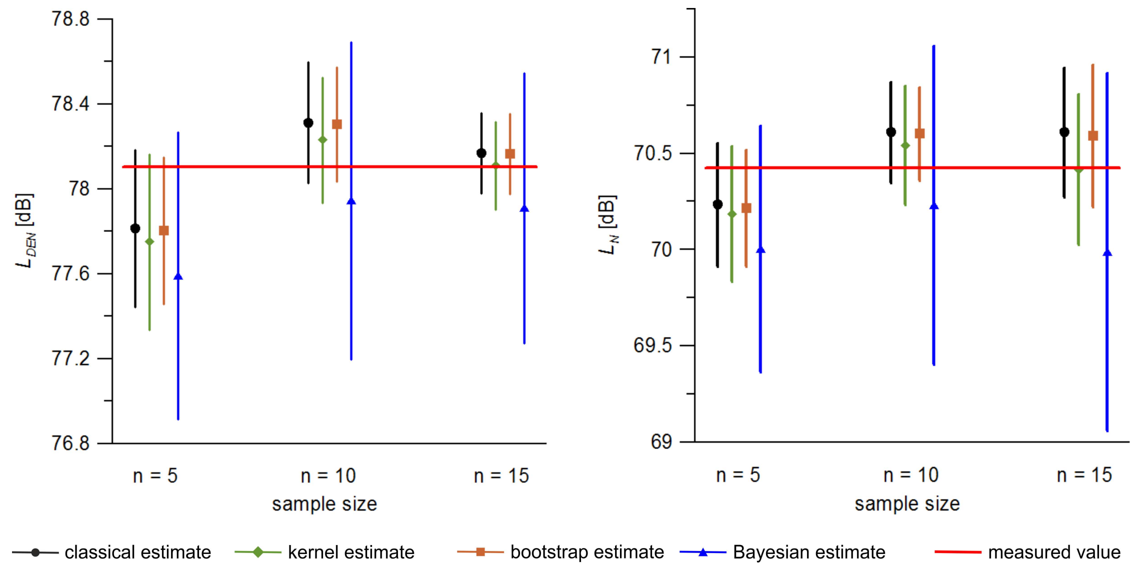

Fig. 6. Estimates of expected value and type A standard uncertainty of long-term noise indicators in 2008.

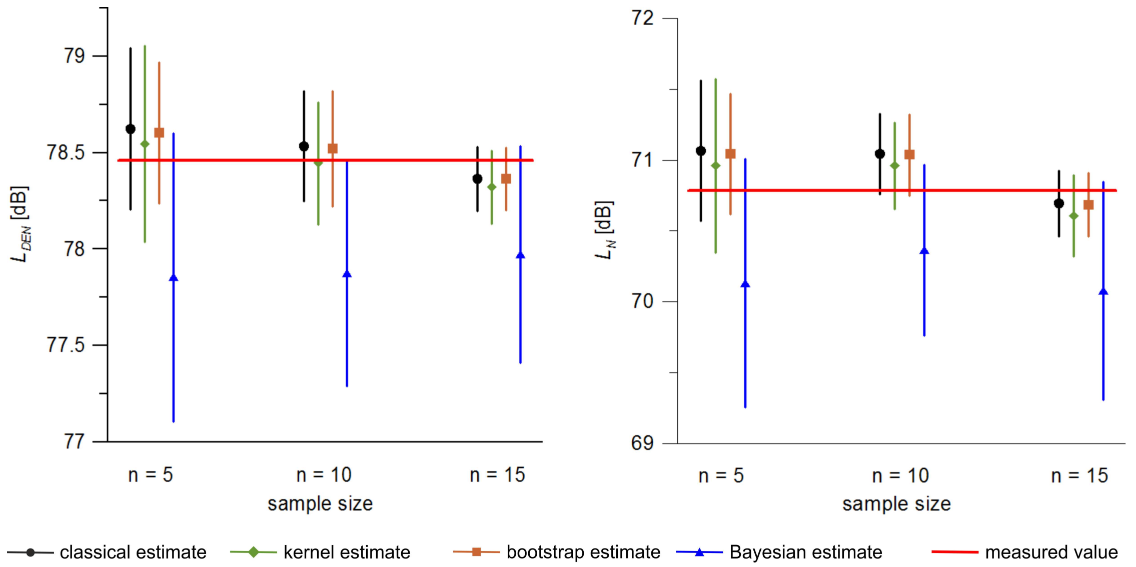

Fig. 7. Estimates of expected values and type A standard uncertainty of long-term noise indicators in 2009.

Moreover, a range of expected values and standard uncertainty for each methods (Table 3) was analyzed. Expected values are in the range of 0.1 dB to 0.7 dB. The smallest values of the range are characterized by the Bayesian inference method, while the largest are characterized by the classical estimator.

The values of the index of dispersion (range) of type A standard uncertainty range from 0.0 dB to 0.7 dB. The most stable results of the estimations are characterized by the Bayesian inference as well as the expected value. The largest values of the range of uncertainty of measurement results are determined for the kernel estimation.

5. Conclusions

This paper is dedicated to the problem of estimation of the environmental noise hazard indicators and their uncertainty. The rules given in the ISO/IEC Guide 98 are currently used. These rules are based on the classic variance estimators, under the following assumptions: the normality of measurements results, adequate sample size, lack of correlation between elements of the sample and observation equivalence. However, such assumptions in relation to the acoustic measurements are rather questionable. That is the reason why the authors indicated the necessity of implementation of non-classic statistic solutions. The estimation idea of seeking density function of long-term noise indicators distribution by the kernel density estimation, bootstrap method, and Bayesian inference was formulated. These methods do not generate limitations for form and properties of the analyzed statistics. This paper presents three non-classical algorithms of estimation of expected values and variance of long-term noise indicators L_{DEN} and L_N. An example of the calculation process which makes it possible to determine estimators was presented.

The results of the numerical experiments presented in Subsec. 4.2 allow to formulate the following conclusions:

- The Bayesian estimates of expected values are more reliable in the case of lack of sample representativeness from which inference is made.
- The most stable results of estimation of the expected value of long-term indicators, i.e. the lowest range value, are characterized by the Bayesian inference method as opposed to the kernel estimation method.
- The long-term environmental noise hazard indicators and acoustic measurements results do not come from a normal distribution.
- In the case type A standard uncertainty the lowest range values are characterized by the Bayesian inference method as opposed to the kernel estimation method.
- The classical and kernel methods are strongly dependent on the structure of the analyzed sample.
- The Bayesian inference is the most resistant to the lack of sample representativeness through insertion of the prior distribution $p(\theta)$ to inference.
- The non-classical approach, mainly the Bayesian and bootstrap inference can be regarded as convenient and effective inference tools for investigated time series with unknown parameters.
- Application of non-classical statistical methods can bring many important methodological and empirical insights to the probabilistic analysis of environmental noise.

- The interpretation assumptions accompanying Bayesian and bootstrap methods are more efficient than the kernel method and classical estimation analysis applied up to the present.
- The presented methods can be used not only to determine characteristics of long-term noise indicators but also all the parameters associated with acoustic measurements.

Making non-classical inference (bootstrap and Bayesian) we do not follow the classical rules of inference. We must remember that the parameter is a random variable and all the information we have is distribution. However, the mean and standard deviation of the mean are only some characteristics of the distribution.

In Table 2 classical and kernel estimates are standard deviation of the mean, on the other hand Bayesian and bootstrap estimates are standard deviation of distributions. If the classical and kernel estimates are multiplied by $\sqrt{5}$, $\sqrt{10}$, $\sqrt{15}$ it turns out the Bayesian and bootstrap estimates show the lowest values of type A standard uncertainty. Using the non-classical statistical methods we obtain a reduce of the standard deviation estimator in relation to the classical estimator:

- for the bootstrap method from 0.5 to 1.8 dB for L_{DEN} and from 0.4 to 1.7 dB for L_N,
- for the Bayesian inference from 0.2 to 1.5 dB for L_{DEN} and from 0.0 to 1.5 dB for L_N.

For the kernel estimator we obtain an increase of the standard deviation estimator in relation to the classical estimator from 0.0 to 0.3 dB for L_{DEN} and from 0.0 to 0.4 dB for L_N. It is the least efficient non-classical estimator presented in the present paper.

Taking into account the above statement the authors propose to consider the standard deviation of bootstrap or posterior distribution as the type A standard uncertainty.

The present paper does not cover all aspects related to the widely discussed topics. It presented an alternative to the current methods of determining the expected value and type A standard uncertainty of long-term noise indicators. The numerical experiment results presented in this paper refer only to one measuring cross-section located in a dense urban area. Further research should focus on testing the algorithms in other measurement conditions (other road profiles and building types). The presented algorithms should be tested also on other sources of noise.

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