

Distortion in Electrodynamic Loudspeakers Caused by Force Factor Variations

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The non linearities in the motor of an electrodynamic loudspeaker are still a discussed topic. This paper studies the influence of the force factor variation with the coil displacement on the harmonic and inter-modulation distortions. The real variation is described at least by a linear and a quadratic term. The effect of each term is studied separately, as they don't influence the same kind of frequencies, harmonics or inter-modulation. Both terms considered together result in enhanced effects. The dissymmetry of the BL variation with regard to the coil centered position has also peculiar effects. This paper presents the method developed to calculate the power of each harmonic and inter-modulation frequency. This allows to compare the obtained values and thus the induced nonlinearities.

Keywords: electrodynamic loudspeaker, non linearity, force factor.

1. Introduction

When describing the functioning of an electrodynamic loudspeaker, the reference model has been established by THIELE (1978) and SMALL (1972). This model describes the loudspeaker as a linear system. However, various measurements and studies have proven the loudspeaker to be a non linear system (SMALL, 1984; KLIPPEL, 2006; VANDERKOOY, 1989). Indeed, the parameters of the model are in fact time varying variables (RAVAUD *et al.*, 2009). The aim of this paper is to describe the non linear behavior of the electrodynamic loudspeakers. Among

the various methods available to do this, the chosen approach is the two-tone stimulus one (KLIPPEL, 2001; CZERWINSKI *et al.*, 2001). Indeed, this method is fairly representative of the perceived acoustical quality.

Furthermore, one of the effects of a non linear behavior is the generation of new spectral components. They can be harmonic when they are integer multiples of the applied fundamentals. But they can be non harmonic when they result of the inter-modulation of two frequencies. Then, they are the sums and differences of the fundamentals and their harmonics. The most audible defects are related to the inter-modulation distortion (VOISHVILLO *et al.*, 2004).

Moreover, the most important requirement for loudspeakers is a high transduction fidelity, which corresponds to a transduction as linear as possible. Therefore, harmonic and inter-modulation frequencies must be reduced. Consequently, the first step is to analyze carefully the distortion causes (DOBRUCKI, 1994; GANDER, 1986; MERIT *et al.*, 2009; RAVAUD *et al.*, 2010).

2. Modelling and method

2.1. Linear model of the electrodynamic loudspeaker

The electrodynamic loudspeaker can be represented by a lumped parameter model using electromechanical parameters that define the low frequency performances. This means that the model describes the behavior at frequencies below the first deformation mode of the membrane.

According to this model, two coupled differential equations describe the electrodynamic loudspeaker. One of them is the electrical differential equation given by:

$$u(t) = R_e i(t) + L_e \frac{di}{dt} + Bl \frac{dx}{dt}. \quad (1)$$

The other one is the mechanical differential equation given by:

$$Bli(t) = M_{ms} \frac{d^2x}{dt^2} + R_{ms} \frac{dx}{dt} + K_{ms} x(t). \quad (2)$$

The parameters used in Eqs. (1) and (2) are the following:

- $i(t)$ – coil current [A],
- $u(t)$ – input supply voltage [V],
- $x(t)$ – coil position [m],
- Bl – force factor [T·m],
- R_{ms} – mechanical damping parameter and drag force [N·s·m⁻¹],
- K_{ms} – stiffness [N·m⁻¹],
- M_{ms} – coil equivalent mass [kg],
- R_e – coil electrical resistance [Ω],
- L_e – coil inductance [H].

The differential equation of the moving coil displacement is deducted from the previous equations (Eqs. (1) and (2)) and is expressed as follows:

$$u(t) = \frac{M_{ms}L_e}{Bl} \frac{d^3x}{dt^3} + \frac{M_{ms}R_e + R_{ms}}{Bl} \frac{d^2x}{dt^2} + \frac{R_{ms}R_e + K_{ms}L_e + Bl^2}{Bl} \frac{dx}{dt} + \frac{K_{ms}R_e}{Bl} x(t). \quad (3)$$

This equation allows the calculation of the coil displacement and then the acceleration is deducted. Moreover, the acoustic pressure is proportionnal to the acceleration,

$$a = \frac{d^2x}{dt^2}.$$

For the numerical values of the parameters of the considered loudspeaker, the impedance and the acceleration are evaluated versus the frequency (Figs. 1 and 2).

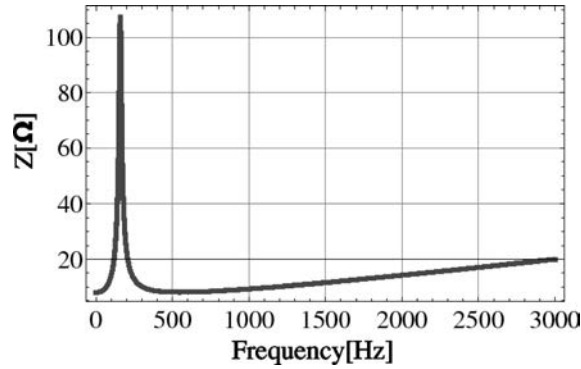


Fig. 1. Impedance of the loudspeaker versus the frequency.

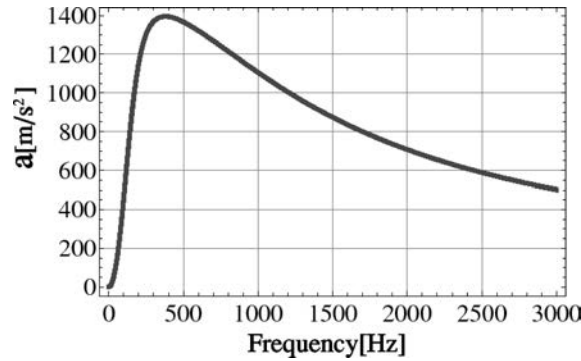


Fig. 2. Acceleration of the loudspeaker moving part versus the frequency for a 10 V Rms input signal.

2.2. Nonlinearities of the electrodynamic loudspeaker

The effects which are generated by a nonlinear system are the following:

- Generation of new spectral components in the output signal which are identified as harmonic or inter-modulation components.
- Nonlinear relationship between the input and output amplitudes of fundamental and distortion components (nonlinear amplitude compression).
- Generation of a non zero mean value of the coil displacement.

The sources of these non linear effects in an electrodynamic loudspeaker are searched for both experimentally and mathematically. Measurement results show that the motor nonlinearities (and among them the force factor variation with the coil position) are responsible for the largest effects. Therefore, the remainder of this paper deals with the study and influence of the force factor.

2.3. Method Description

As said, the force factor, Bl , varies with the coil displacement.

So, the equation of the coil displacement (Eq. (3)) is considered as a nonlinear third order differential equation when the expressions of the force factor variation with the displacement are introduced in the equation. Then, getting analytical solutions for a nonlinear third order differential equation is rather complicated. Therefore, a numerical method is applied to obtain the solution.

A two-tone stimulus,

$$u(t) = V_1\sqrt{2}\sin(2\pi f_1 t) + V_2\sqrt{2}\sin(2\pi f_2 t),$$

excites the nonlinear system: the input signal is constituted of two fundamental frequencies f_1 and f_2 .

Then, the solution of the nonlinear differential equation is evaluated numerically and the result is sampled. Furthermore, a fitting is carried out: the fitting function is written as a sum of harmonics and inter-modulation components of both fundamental frequencies. The amplitude of each frequency component is adjusted by using a mean square method to find the best correspondence between the numerical solution of the differential equation and the evaluation with the fitting function.

When the force factor varies with the coil displacement, harmonic and inter-modulation frequencies are observed in the output. The presented method allows the calculation of the acceleration level of each fundamental and distortion spectral components.

If the loudspeaker is supposed linear, the values of the electromechanical parameters of the actual loudspeaker are the following:

$$\begin{aligned} Bl &= 10 \text{ [T}\cdot\text{m]}, & R_{ms} &= 1 \text{ [N}\cdot\text{s}\cdot\text{m}^{-1}], \\ K_{ms} &= 10000 \text{ [N}\cdot\text{m}^{-1}], & M_{ms} &= 0.01 \text{ [kg]}, \\ R_e &= 8 \text{ [\Omega]}, & L_e &= 0.001 \text{ [H]}. \end{aligned}$$

The input $u(t) = 10\sqrt{2}\sin(2\pi 300t) + 10\sqrt{2}\sin(2\pi 2000t)$ is applied: this corresponds to 10 V Rms at 300 Hz superposed to 10 V Rms at 2 kHz. For the linear functioning, which means that the force factor is a constant, the coil displacement is respectively 0.385 mm Rms at 300 Hz and 0.0043 mm Rms at 2 kHz. Such an input signal produces two pure sound components, $a(300 \text{ Hz}) = 1350 \text{ m/s}^2$ and $a(2 \text{ kHz}) = 700 \text{ m/s}^2$ (Fig. 2), and no harmonic nor inter-modulation component. The spectrum of the corresponding simulation is shown in Fig. 3.

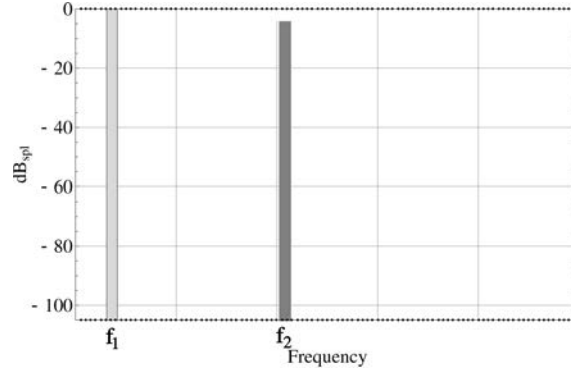


Fig. 3. Output spectrum of the linear loudspeaker excited by a two-tone stimulus $u(t) = 10\sqrt{2}\sin(2\pi 300t) + 10\sqrt{2}\sin(2\pi 2000t)$. The zero dB level corresponds to 1350 m/s^2 . Light gray: First Fundamental, Gray: Second Fundamental.

The characterization of the nonlinear behavior consists in comparing the simulation results of various cases of the force factor variation with the ones obtained in the linear case.

3. The force factor

Indeed, the force factor $Bl(x)$ describes the coupling between the mechanical and electrical sides of the lumped-parameter model of an electrodynamic transducer.

$Bl(x)$ is theoretically a constant value, but in reality it changes with the displacement x . In fact, its variation can be described by a second order equation (Eq. (4)) with the coefficients α for the first order term and β for the second order one.

$$Bl(x) = Bl_0(1 + \alpha x(t) - \beta x^2(t)), \quad (4)$$

where Bl_0 is the force factor value of the ideal linear loudspeaker.

Then, different non linear behaviors are described by giving various values to these coefficients. The influence of each coefficient will be studied separately in the following sections in order to clearly determine their role. But it has to be kept in mind that the real behavior is the combination of both influences. Moreover, it

must be noted here that the force factor itself shouldn't be described as "linear" or "non linear". Indeed, a constant force factor induces a linear functioning of the loudspeaker (leaving other nonlinearity sources aside) whereas a varying force factor induces a nonlinear functioning.

As said previously, the effects of the non linearity are characterized by studying the loudspeaker output when excited by a two-tone signal. Choosing the first frequency $f_1 = 300$ Hz and the second tone $f_2 = 2$ kHz, all of the distortion components are nicely spaced and are easily interpreted.

Indeed, the two fundamental tones are easily identified by their maximal amplitude. The first tone f_1 represents the low frequency producing the substantial voice coil displacement and the second tone f_2 represents a voice or any other musical instrument. However, the difference inter-modulation tones at $f_2 - if_1$ and the summed inter-modulation tones $f_2 + if_1$ with $i = 1, 2, 3, 4, 5$ are found equally spaced around the second tone f_2 . It is interesting to observe the second, third, fourth and fifth order components.

3.1. Effects of the first order coefficient α

Then the force factor is described by:

$$Bl_\alpha(x) = Bl_0(1 + \alpha x(t)). \quad (5)$$

The force factor has a 10% variation with the coil displacement, for a displacement in the interval $[-0.5 \text{ mm}, 0.5 \text{ mm}]$.

For $\alpha = 200$, the force factor variation is shown in Fig. 4. In a classical electrodynamic loudspeaker the force factor increases when the coil moves towards the motor inside ($x > 0$).

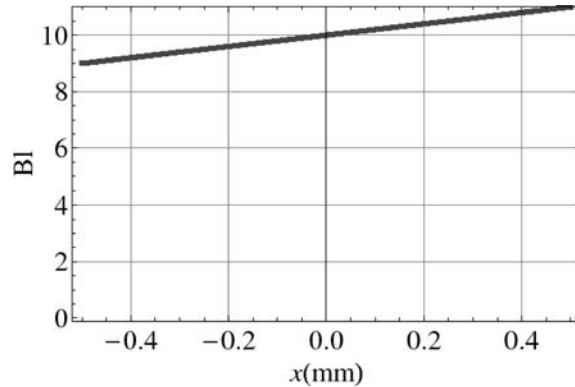


Fig. 4. Variation of the force factor Bl versus the coil displacement x , described by the first order coefficient in $Bl_\alpha(x) = Bl_0(1 + \alpha x(t)) = 10(1 + 200x)$.

The calculation of the acceleration, $a(t)$, contains three terms: the first term is the acceleration at the fundamental frequencies, $a_{\text{fond}}(t)$, the second is the accel-

eration at the harmonic frequencies, $a_{\text{harm}}(t)$, and the last one is the acceleration at the inter-modulation frequencies, $a_{\text{int}}(t)$.

For $\alpha = 200$, $f_1 = 300$ Hz, $f_2 = 2$ kHz and $V_1 = V_2 = 10$ V Rms, the acceleration frequency spectrum is shown in Fig. 5.

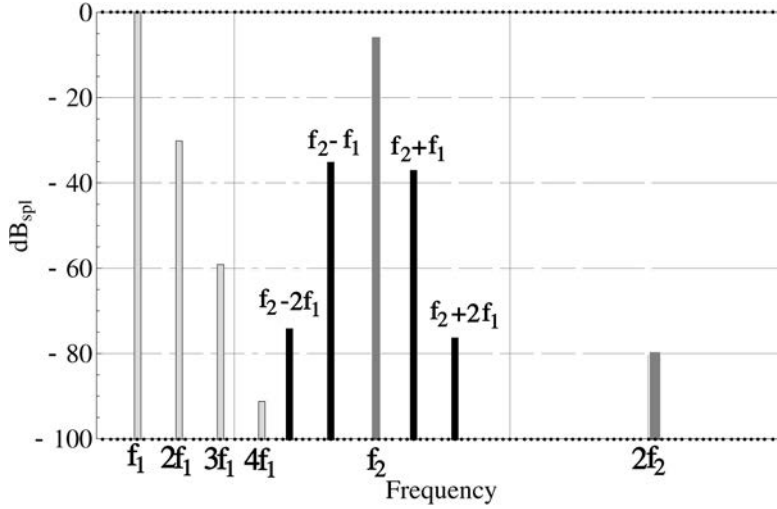


Fig. 5. Spectrum of the acceleration resulting from a two-tone stimulus, with $Bl_\alpha(x) = Bl_0(1 + \alpha x(t)) = 10(1 + 200x)$. Light gray: First fundamental and its harmonics. Gray: Second fundamental and its harmonics. Black: Intermodulation components.

3.1.1. Mean value variation

If the force factor variation is asymmetrical around the rest position then a position offset appears, X_{dc} . As the force factor is higher when the moving part goes into the motor ($x > 0$), the offset position corresponds to the moving part being pushed out of the motor (Fig. 6) (DOBRUCKI, 1988).

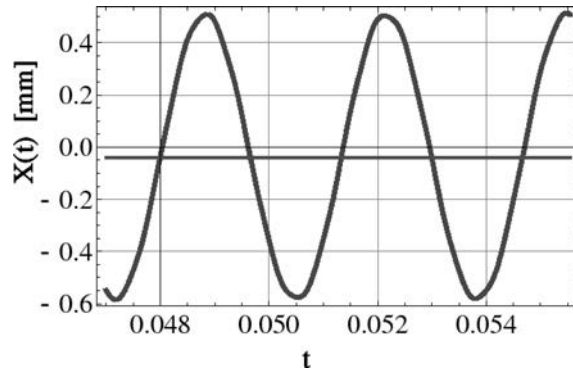


Fig. 6. Displacement for a two-tone stimulus of 10 V Rms each. The offset of the voice-coil displacement is highlighted by the line at $X_{dc} = -0.04$ mm.

This offset doesn't correspond to an acoustical emission but it generates a pre-constraint on the suspension which won't work in a symmetrical way. As a consequence, this asymmetrical functioning will create distortions.

The value of X_{dc} for a nonlinearity due to $\alpha = 200$ is around 0.04 mm. In this case, the force factor, Bl , varies from 9 to 11 for a displacement, x , from -0.5 mm to 0.5 mm (Fig. 4).

3.1.2. Amplitude of the fundamentals

When the multi-tone input $u(t) = V\sqrt{2}\sin(2\pi f_1 t) + V\sqrt{2}\sin(2\pi f_2 t)$ is applied to the system, with both inputs of same level 10 V Rms then the output fundamental amplitudes are the same as in the linear case.

3.1.3. Harmonics

As shown in Fig. 5, the harmonic levels of first and second fundamentals decrease with the increasing harmonic rank. However, the amplitudes of the harmonics of the first fundamental are greater than these of the second one.

3.1.4. Inter-modulation

Figure 5 shows that the inter-modulation components are quite significant. Indeed, they have the same level as the harmonics of the lower fundamental frequency, which is only 30 dB below the fundamental frequency level. Moreover, they contribute dramatically to the sound quality deterioration.

The two highest inter-modulation components are

$$a_{\text{int}}(f_2 - f_1) \quad \text{and} \quad a_{\text{int}}(f_2 + f_1).$$

3.2. Variation due to second order coefficient β

The force factor is modelled by $Bl_\beta(x) = Bl_0(1 - \beta x^2(t))$ and is shown in Fig. 7 for $\beta = 4 \cdot 10^5$.

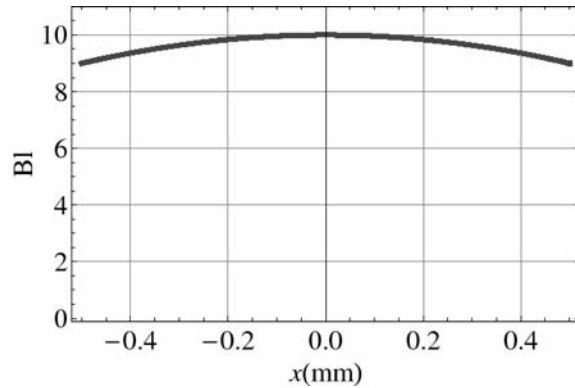


Fig. 7. Variation of the force factor $Bl_\beta(x) = Bl_0(1 - \beta x^2) = 10(1 - 4 \cdot 10^5 x^2)$.

The force factor variation is symmetrical about the rest position.

The force factor has a 10% variation with the coil displacement, for a displacement between -0.5 mm and 0.5 mm.

For $f_1 = 300$ Hz, $f_2 = 2$ kHz and $V_1 = V_2 = 10$ V Rms, the frequency spectrum is shown in Fig. 8.

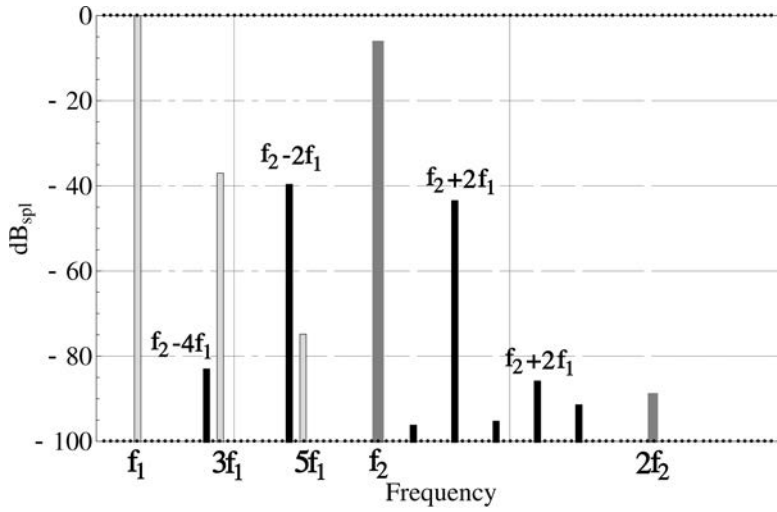


Fig. 8. Spectrum of the acceleration resulting from a two-tone stimulus, with $Bl_\beta(x) = Bl_0(1 - \beta x^2(t)) = 10(1 - 4 \cdot 10^5 x^2)$. Light gray: First fundamental and its harmonics. Gray: Second fundamental and its harmonics. Black: Intermodulation components.

3.2.1. No mean value

The force factor variation due to β has a symmetrical shape. Therefore, the movement remains centred about the rest position.

3.2.2. Amplitude of the fundamentals

In this case, the output fundamental amplitudes decrease slightly (1%).

3.2.3. Harmonics

The harmonic components are the odd multiples of the lower fundamental: $3f_1$, $5f_1$ (Fig. 8).

3.2.4. Inter-modulation

The components $a_{\text{int}}(f_2 - 2f_1)$ and $a_{\text{int}}(f_2 + 2f_1)$ are the most significant and have the same level as the third harmonic of the lower frequency, which is 40 dB below the fundamental level.

3.3. Variation due to first and second order coefficients

The actual force factor is modelled by $Bl_{\alpha,\beta}(x) = Bl_0(1 + \alpha x(t) - \beta x^2(t))$ with $\alpha = 200$ and $\beta = 4 \cdot 10^5$.

The force factor has a 20% variation with the coil displacement for a displacement between -0.5 mm and 0.5 mm (Fig. 9).

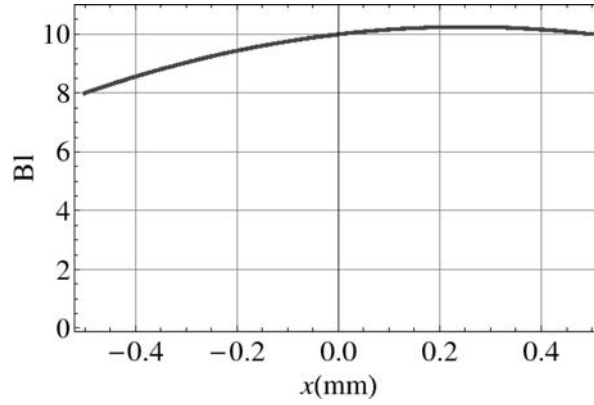


Fig. 9. Variation of the force factor $Bl_{\alpha,\beta}(x) = Bl_0(1 + \alpha x(t) - \beta x^2(t)) = 10(1 + 200x - 4 \cdot 10^5 x^2)$.

For $f_1 = 300$ Hz, $f_2 = 2$ kHz and $V_1 = V_2 = 10$ V Rms, the spectrum is shown in Fig. 10.

This variation range of Bl is characteristic of a loudspeaker of average quality.

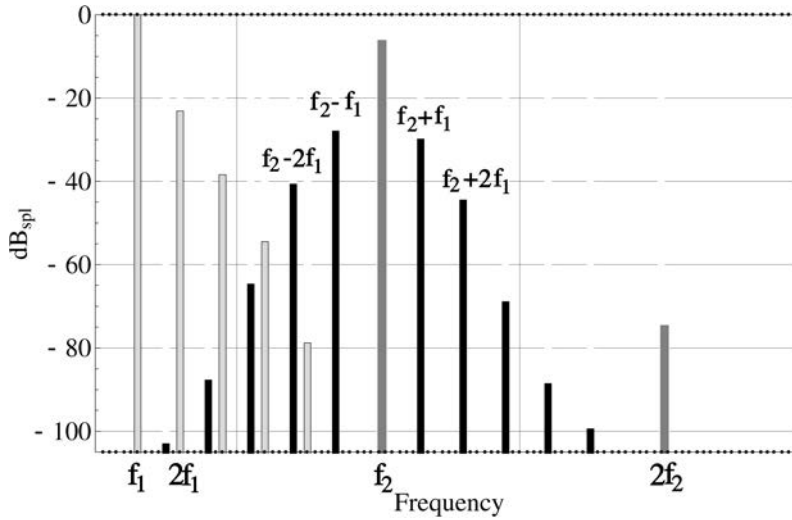


Fig. 10. Spectrum of the acceleration resulting from a two-tone stimulus, with $Bl_{\alpha,\beta}(x) = Bl_0(1 + \alpha x(t) - \beta x^2(t)) = 10(1 + 200x - 4 \cdot 10^5 x^2)$. Light gray: First fundamental and its harmonics. Gray: Second fundamental and its harmonics. Black: Intermodulation components.

Figure 10 shows the spectrum of the output signal: both effects previously described are enhanced. An offset is observed for the moving part position: its value is the same as the one observed in the case with α alone.

4. Distorsion analysis

This section presents the values of the total harmonic distortion coefficient, THD (Eq. (6)), as it is usually done to characterize the non linearity effects. The values are calculated for the three cases of non linearities of the force factor studied in this paper.

$$\text{THD} = \frac{\sqrt{\sum_{i=2}^5 a_{\text{harm}}^2(i f_1)}}{a_{\text{fond}}(f_1)}, \quad (6)$$

$$\text{IMD} = \frac{\sqrt{\sum_{i=1}^5 (a_{\text{int}}^2(f_2 - i f_1) + a_{\text{int}}^2(f_2 + i f_1))}}{a_{\text{fond}}(f_1)}. \quad (7)$$

As a result, the distortion caused by the linear variation, α , of the force factor is twice as high as the distortion caused by the quadratic variation, β . Moreover, when both variations are simultaneously taken into account, the resulting distortion is far higher than the sum of each distortion considered separately (Table 1). This statement is valid when the harmonic distortion alone is taken into account (THD) but also when the inter-modulation distortion is taken into account (IMD).

Table 1. Most significant spectral components and distortion coefficients for different non linearity models of the force factor.

Model of nonlinearity	$a(f_2 - f_1)$ [dB]	$a(f_2 - 2f_1)$ [dB]	THD [%]	IMD [%]
$Bl_\beta(x)$	negligible	-40	1.4	1.2
$Bl_\alpha(x)$	-35	-74	3.1	2.2
$Bl_{\alpha,\beta}(x)$	-27	-40	7.1	5.3

It is emphasized that from the acoustic quality point of view, the most important effect is the appearing of inter-modulation components in the frequency spectrum. Indeed, when the linear variation, α , is considered, the most significant component corresponds to the difference of the fundamental frequencies, $f_2 - f_1$. For a 10% variation of the force factor the corresponding acceleration, and thus the acoustical level, is only 35 dB lower than the level of the fundamental frequency. Moreover, when the force factor varies quadratically, β , the most significant component corresponds to the frequency $f_2 - 2f_1$ and is 40 dB lower than the fundamental frequency level.

Furthermore, when both variations are considered, the higher acoustical level corresponds to the frequency $f_2 - f_1$ and is 27 dB lower than the fundamental frequency level.

5. Conclusion

This paper studies the influence of the force factor variations with the coil displacement. These variations are modelled by a second order polynomial. A method is presented to evaluate the frequency spectrum of the output signal. The study is carried out for each variation separately, linear and quadratic, and for both of them simultaneously. As a result, a position offset of the moving part position is pointed out in the case of a first order variation of the force factor. Moreover, harmonic frequencies of all orders appear as well as sum and difference inter-modulation frequencies of all orders. The second order variation alone produces only odd-order harmonic frequencies and sum and difference inter-modulation frequencies of f_2 and even multiples of f_1 . Both types of variation simultaneously, which corresponds to the real case, enhance the described effects.

Eventually, the most damaging effect on the acoustical quality is caused by the first order variation, far above the quadratic variation.

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