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Statistical Analysis of the Equivalent Noise Level

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The authors focus their attention on the analysis of the probability density function of the equivalent noise level, in the context of a determination of the uncertainty of the obtained results in regard to the control of environmental acoustic hazards. In so doing, they discuss problems of correctness in the applicability of the classical normal distribution for the estimation of the expected interval value of the equivalent sound level. The authors also provide a set of procedures with respect to its derivation, based upon an assumption of the determined distribution of the measurement results. The obtained results then create the plane for the correct uncertainty calculation of the results of the determined controlled environmental acoustic hazard coefficient.

Keywords: acoustic measurements, statistical analysis of the obtained results, estimation.

1. Introduction

A widely known and applied method of estimating the equivalent noise level on the grounds of control measurements, and hence, deriving the appropriate corresponding acoustic environmental decision, is based on defining the obtained estimation by way of statistical measurement. Such procedure is aimed at minimizing the limitations of subjective and objective errors with which we are dealing within control measurements. One such procedure is also based upon the method of the control decision uncertainty assessment being the determination of the percentage of intervals coinciding with the assessed expected value of possible control results of the investigated collective. These are determined on the basis of the central limiting theorem, and allow researchers to reduce the problem to being a determination of the conditions of the application of the classical normal distribution. With regard to random variables of asymmetric distributions, a situation which is typical for sound level measurements, in consideration of the asymmetry of the expected value, such intervals, when determined in such a way, may not include the asymmetry of the estimated parameter. Thus, it is necessary to carry out a detailed theoretical analyses leading towards an estimation of the probability distribution of the equivalent sound level. The purpose of this work is to bring about a suitable preparation of the algorithm that will allow researchers to determine this distribution, which, in turn, will enable a reduction of the uncertainty of the realized estimations.

2. Procedure for the determination of the probability distribution of the equivalent sound level

Employing Eq. (1):

$$L_{Aeq} = 10 \log \left\{ \frac{1}{T} \sum_{i=1}^{n} t_i 10^{0.1 L_i} \right\} \tag{1}$$

and allowing Eq. (1) to be written in another form, let:

$$p_i = \frac{t_i}{T}, \qquad i = 1, ..., n,$$

$$T > t_i > 0, \qquad i = 1, ..., n,$$

$$\sum_{i=1}^{n} t_i = T,$$

where t_i – time i interval length, L_i can be interpreted e.g. as random variables, where such values are obtained after the integration of a certain, *a priori* determined time interval t_i (e.g. 1 sec, 5 sec, ...), hence:

$$L_i = 10 \log \left\{ \frac{1}{t_i} \int_{t_i} 10^{0.1L(t)} \, \mathrm{d}t \right\} \tag{2}$$

and noting that these could be also sound levels values averaged for the proper time interval, the notion underlying the considerations taken in the paper is the assumption of the independence of the random variables L_i. With respect to measurements connected with acoustical environmental protection in road traffic, random variables L_i are the measurements presented in the form of a time series. For short time intervals such as one 1-second distributions, these random variables might be dependent. However, measurement data obtained from samples generated over a longer period of time tend to be independent. The problem of aggregating the data produced by way of 1-second samples, so as to remove dependencies between successive control results, and which constitutes the random test for statistical analysis, is presented in the paper by (Batko, Knapik, 2013). This results from the authors' research experience, that often the measurement data sampled in 30-second intervals are independent. The likelihood of the assumption adopted in the paper should not generate essential objections, with regard to the creation of a random test based upon the derived values of the day-evening-night noise indicator L_{den}, where links of forces determining the values of equivalent levels L_{eqd}, L_{eqe}, L_{eqn}, are weakly noticeable. With respect to calculating the long-term average noise indicators of variable emission conditions (Batko, Przysucha, 2011), following the European Commission recommendations (Kucharski, 2011), when there is no complete data for their calculations, this process can be simplified to estimating the equivalent sound level of the basic noise sources, where this is taken as the percentage activity of the noise emission within a specified calendar year. In such a situation, we can treat these variables as being independent and random. The determination of the probability distribution of variable L_{Aeq} is given by Eq. (1).

Let L_i, $i = 1, 2, ..., n$ – be random variables of distributions $f_{L_i}(x)$, $i = 1, 2, ..., n$ determined on carriers $x \in (-\infty, \infty)$, and $X_i = p_i 10^{0.1L_i}$, $i = 1, 2, ..., n$, then the distribution function $F_{X_i}(\cdot)$ of the random variable X_i is given by the equation:

$$\begin{aligned} F_{X_i}(x) &= P[X_i < x] = P[p_i 10^{0.1L_i} < x] \\ &= P\left[L_i < 10 \log \left(\frac{x}{p_i}\right)\right]. \end{aligned} \tag{3}$$

The probability distribution $f_{X_i}(\cdot)$ given by Eq. (4), is then obtained by the differentiation of Eq. (3):

$$f_{X_i}(x) = \frac{10}{\ln 10} \frac{1}{x} f_{L_i}\left(10 \log \frac{x}{p_i}\right) \qquad x \in (0, +\infty). \tag{4}$$

Thus, let us mark:

$$\begin{aligned} &Y_1 = X_1, \\ &Y_2 = X_1 + X_2 = Y_1 + X_2, \\ &... \\ &Y_n = X_1 + X_2 + ... + X_n = Y_{n-1} + X_n. \end{aligned} \tag{5}$$

As we can see, the properties of the convolutions of probability distributions indicate that the probability distribution of the sum of independent random variables X, Y is expressed by their convolution (Billingsley, 1979).

$$f_{X+Y}(s) = \int_{-\infty}^{s} f_X(x) f_Y(s-x) \, \mathrm{d}x. \tag{6}$$

Applying equation (6) to the above functions, generates a set of equations for the distribution of the variables:

$$f_{Y_1}(x) = f_{X_1}(x), \quad x \in (0, +\infty), \tag{7}$$

$$\begin{aligned} f_{Y_2}(x_1) &= \int_0^{x_1} f_{X_1}(x) f_{X_2}(x_1 - x) \, \mathrm{d}x \\ &= \int_0^{x_1} \frac{10^2}{\ln^2 10} \frac{1}{x(x_1 - x)} f_{L_1}\left(10 \log \frac{x}{p_1}\right) \\ &\quad \cdot f_{L_2}\left(10 \log \frac{x_1 - x}{p_2}\right) \mathrm{d}x, \\ &\quad x_1 \in (0, +\infty), \end{aligned} \tag{8}$$

$$\begin{aligned} f_{Y_3}(x_2) &= \int_0^{x_2} f_{Y_2}(x_1) f_{X3}(x_2 - x_1) \, \mathrm{d}x_{1_1}, \\ &\quad x_2 \in (0, +\infty), \end{aligned} \tag{9}$$

$$\begin{aligned} f_{Y_n}(x_{n-1}) &= \int_0^{x_{n-1}} f_{Y_{n-1}}(x_{n-2}) \\ &\quad \cdot f_{X_n}(x_{n-1} - x_{n-2}) \, \mathrm{d}x_{n-2}, \\ &\quad x_{n-1} \in (0, +\infty). \end{aligned} \tag{10}$$

Subsequently, the random variable given by Eq. (1) is thus expressed by the following equation:

$$L_{Aeq} = 10 \log Y_n \tag{11}$$

and its probability distribution (12) is equal to:

$$f_{L_{Aeq}}(s) = \frac{\ln 10}{10} 10^{0.1s} f_{Y_n}\left(10^{0.1s}\right), \qquad s \in (-\infty, \infty). \tag{12}$$

The data obtained from the measurement of environmental noise such as that of the random variables of traffic (L_i), are limited to having a positive value, because we do not take into account the generated auditory sensations. The limitation to utilizing only positive values of random tests of sound levels, comes about from the fact that such considerations are directed towards the measure determination for the probabilistic analyses of environmental noises. In such situations, negative sound levels will not occur. During the measurement of environmental noise, researchers must take into consideration the presence of background sound that allows only the capture of positive decibels. For such considerations, the need exists to make appropriate adjustments of the areas in which the L_i variables are defined. This implies a change in the limits of integration in the design of (8)–(10), (12).

If we consider the identical probability distributions of sound levels, we can see that L_i, as well as $p_i = 1/n$ were assumed in Eq. (1), and we gain the logarithmic mean of sound levels – for which, apart from the probability distribution, the useful recurrent equations allowing for its estimation were determined in the paper by (Batko, Przysucha, 2010).

However, if we limit ourselves to two sources and to random variables of cut distributions, we will be dealing with the case described in paper by (Batko, Przysucha, 2011).

3. Conclusions

The statistical methodology widely applied in investigations of environmental acoustic control and its related decision-taking with respect to the degree of the acoustic hazard present within controlled areas, requires the search for new mathematic formalisms in order to cope with the uncertainty assessment of the performed recognition. By way of properly identifying the form of the density function of the equivalent sound level probability distribution, it will be possible for researchers to optimise the control process and to reduce the risk of erroneous environmental decisions. The hereby paper is focused on the statistic analysis of the equivalent sound level when dependent upon the characteristics of the input probability distributions of sound levels. In so doing, uncertainty aspects related to external factors such as e.g. meteorological conditions, or the type of areas where measurements are made, or the uncertainty of measurements – are omitted. These uncertainty aspects can be found in the papers by (Makarewicz, 2011) and (Makarewicz, Gołębiewski, 2006).

The mathematical formalism proposed in the paper is a good representative of the class of the possible detailed solutions. Similar considerations concerning uncertainty at the equivalent noise level determination are contained in the paper by (Heiss, 2001). The author presents in this, the determination method of the equivalent sound level variance, as well as its uncertainty assessment, while omitting the analytical estimation of the probability density function of random variable L_{eq}. However, in this work, the proposed solution of the probability density function estimation, together with its numerical implementation, has the advantage of having a higher universality. This allows researchers to determine not only the expected value of the investigated noise indicator and its uncertainty (determined by variance), but also its other probabilistic characteristics (e.g. the third and the fourth moment) that are important in assessing the asymmetrical confidence intervals for the analysed variable. This solution also creates conditions for a better analysis and description of the investigated variable L_{eq}. This, we feel, is important in designing relevant numerical experiments dedicated to, for example, the efficiency analysis of various distribution characteristics for the description of the probabilistic variability of the analyzed variable L_{eq}. The presented solution, based on the distribution propagation method, corresponds with instructions contained in the new document: JCGM 101:2008 (BIPM, 2008) of the Guide of Uncertainty in Measurement, GUM (issued by the International Standards Organisation, ISO). It is also in agreement with the given there-in suggestions, concerning calculations of the control result uncertainty by way of the identification of the probability density distribution function describing the analysed value. In contrast to the contained proposed estimation of its distribution by means of the Monte Carlo simulation from the mathematical model of input values, the solution proposed in our paper, is directed towards analytical research. This approach is more difficult to realize than the one that is shown within the paper by BIPM (2008), but is much more universal.

What is more, its numerical implementation will allow researchers to calculate the uncertainty of the estimated noise indicator. Such work is useful for the analysis of the error that comes about in processing the acoustic measurement results, as well as for programming the conditions that enable a control of the environment acoustic hazard – including the number of measurements that ensure the required accuracy of control assessments (for example, in the problem described in the paper (Gómez Eskobar *et al.*, 2012), regarding the process of generating controlling acoustical maps, and in determining the uncertainty of the random control measurements for small samples).

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