PAN
ARCHIVES OF ACOUSTICS
Vol. **39**, No. 2, pp. 199–202 (2014)

DOI: 10.2478/aoa-2014-0024

Verification of the Calculation Assumptions Applied to Solutions of the Acoustic Measurements Uncertainty

Wojciech BATKO[(1)], Renata BAL[(2)]

[(1)] *AGH University of Science and Technology*
al. A. Mickiewicza 30, 30-059 Kraków, Poland; e-mail: batko@agh.edu.pl

[(2)] *Politechnical Institute, State Higher Vocational School in Krosno*
Rynek 1, 38-400 Krosno, Poland; e-mail: renbal@pwsz.krosno.pl

(*received March 28, 2013; accepted April 24, 2014*)

The assessment of the uncertainty of measurement results, an essential problem in environmental acoustic investigations, is undertaken in the paper. An attention is drawn to the – usually omitted – problem of the verification of assumptions related to using the classic methods of the confidence intervals estimation, for the controlled measuring quantity.

Especially the paper directs attention to the need of the verification of the assumption of the normal distribution of the measuring quantity set, being the base for the existing and binding procedures of the acoustic measurements assessment uncertainty. The essence of the undertaken problem concerns the binding legal and standard acts related to acoustic measurements and recommended in: 'Guide to the expression of uncertainty in measurement' (GUM) (OIML 1993), developed under the aegis of the International Bureau of Measures (BIPM). The model legitimacy of the hypothesis of the normal distribution of the measuring quantity set in acoustic measurements is discussed and supplemented by testing its likelihood on the environment acoustic results.

The Jarque-Bery test based on skewness and flattening (curtosis) distribution measures was used for the analysis of results verifying the assumption. This test allows for the simultaneous analysis of the deviation from the normal distribution caused both by its skewness and flattening. The performed experiments concerned analyses of the distribution of sound levels: L_D, L_E, L_N, L_{DWN}, being the basic noise indicators in assessments of the environment acoustic hazards.

Keywords: environmental noise control, statistical analysis, acoustic measurement, uncertainty.

1. Introduction

An estimation of uncertainty of measurements is the required part of every investigation procedure. It comprises validations of measurement procedures, analyses of sources of possible random errors occurring in a measuring process as well as the selection of rules of their processing in dependence of their probability distribution. In uncertainty assessment analyses the attention is, first of all, directed towards: a determination of the standard uncertainty at direct and indirect measurements, analysis of the uncertainty budget and problems of the expansion factor k selection, determining the expanded uncertainty assessment.

A correct application of the uncertainty assessment procedures recommended in the (Guide to the Expression of Uncertainty Measurements 1995) is related to the determined assumptions class, the acceptability of which should be thoroughly analysed. Unfortunately this is often a marginalised operation. We are generally dealing with such situation in uncertainty assessments of environment acoustic hazards control, represented by the results of the noise level measurements.

Using the assumption of the normal distribution as the representative of the mathematical model of the sound level measurement results L_{Ai}; $i = 1, 2, ..., n$ is a rule in investigations, respected in the accredited laboratories and other units realising the basic and technical tests from the field of acoustics. It is obvious that in case of inadequacy of such assumption, the sound level average value or another noise indicator from the test and their standard deviations – representing the control assessment – can not be the best estimation of the measurement and its standard type A uncertainty (Batko, Bal-Pyrcz, 2006).

The majority of researchers, intuitively receives the assumption of the normal distribution of the sound measurement results of the investigated population, (from which the random test for the estimation of the controlled noise indicators is taken). They are connecting these tests with the results of the central limiting Lindeberg-Levy theorem, determining the convergent form of random events distribution, not analysing a mechanism of the sound level measurement result generation. As it is shown in papers (Batko, Przysucha, 2010; 2011; 2012), taking into account this mechanism leads to distributions considerably differing from the normal distribution.

Endeavours of a wider verification of the acceptability of the assumption of the normal distribution of the noise level measurement results – with which we are dealing in controlling the environment acoustic hazards – were undertaken in the paper. The analytical bases constituted the results of the continuous, of many years, monitoring of the environment acoustic hazards in the city of Krakow. They were used for the verification of a wider acceptability of the hypothesis of the measured values normal distribution in controlling the environment acoustic hazard control. Their discussion constitutes the contents of the hereby paper.

2. Data base of test results

The bases of the verification of the correctness of the assumption of the control results normal distribution constituted the year-long, of many years, recordings of sound level L_{Ai}; $i = 1, 2, ..., n$ carried out every 1-second, in one of the stations of the continuous noise monitoring system. They were used for the verification of the hypothesis of the normal distribution of sound level results during: day time L_{Di} (6:00–18:00), evening L_{Ei} (18:00–22:00) and night L_{Ni} (22:00–6:00), in successive days of the calendar year $i = 1, 2, ..., 365$ and also for the estimation of the day-evening-night level:

$$L_{DEN\,i} = 10 \log\left[\frac{1}{24}\left(12 \cdot 10^{0.1\,L_{Di}} + 4 \cdot 10^{\cdot 1(L_{Ei}+5)} + 8 \cdot 10^{0.1(L_{Ni}+10)}\right)\right]. \tag{1}$$

They also constituted the bases for the estimation of their long-term average noise levels $L_{LT}^{(j)}$:

$$L_{LT}^{(j)} = 10 \log\left[\frac{1}{365}\sum_{k=1}^{365} L_{Aeq\,LT\,k}^{(j)}\right] \tag{2}$$

for day time – $j = 1$, evening – $j = 2$ and night – $j = 3$, during the whole calendar year, which are necessary in the decision taking process related to programs of the environment acoustic protection.

From these values the general populations of the results of the analysed noise indicators [L_D, L_E, L_N, L_{DEN}]; $i = 1, 2, ..., 365$, were formed and the correctness of assumption of the possibility of approximation of their occurrence probability by the normal distribution was analysed for them.

The example of the variability waveform of the sound level $L_{DEN\,i}$; $i = 1, 2, ..., 365$ for one of the analysed years; (determining the general population *for the results of the random control test)*; being the subject of the verification of the hypothesis of the normal distribution of the results is presented in Fig. 1, while the corresponding probability distribution in figure.

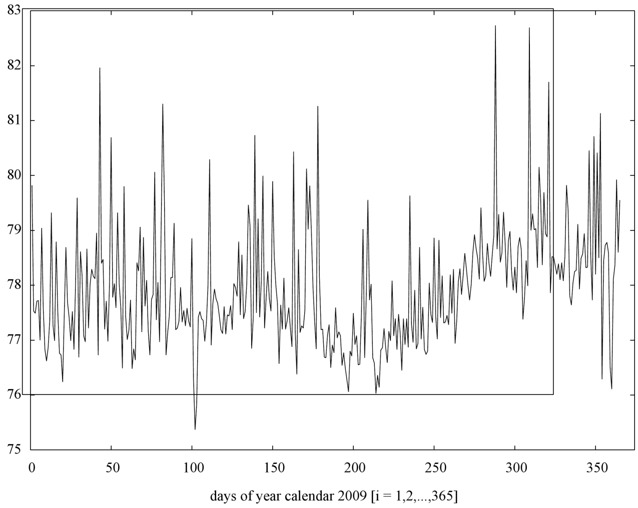

Fig. 1. Variability of L_{DEN} in the year 2009.

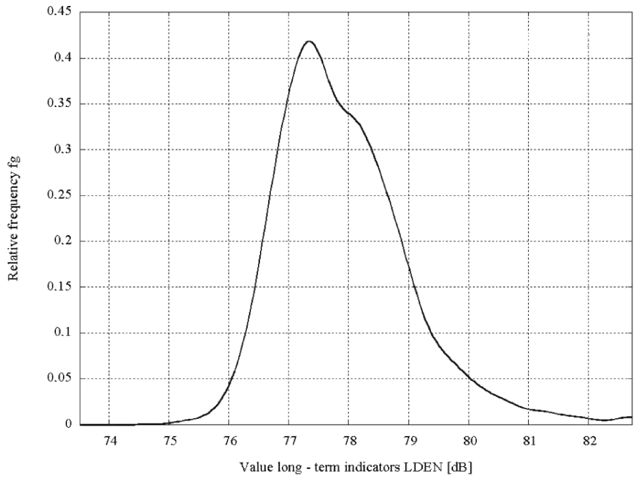

Fig. 2. Density probability distribution function of sound levels L_{DEN}.

3. Verification of the hypothesis of the normal distribution of the controlled noise indicators

Several statistic tests for the verification of the hypothesis H_0 of the results normal distribution are known. Due to the, often observed in the acoustic measurements practice, asymmetry of the probability distribution of the acoustic investigations results, thick tailings occurrence, the Authors directed attention to the Jarque-Bery test (Jarque, Bera, 1987).

This test is based on measures of: skewness (asymmetry coefficient) and curtosis (flattening coefficient). Simultaneously this test takes into account deviations from normality caused by: skewness $\sqrt{b_1} = m_3/\sqrt{m_2^3}$ and flattening (curtosis) $b_2 = m_4/m_2^2$ of the distribution, which are determined by moments: $m_k = \frac{1}{n}\sum_{i=1}^{n}(x_i - x)^k$, $k = 2, 3, \ldots$.

The Jarque-Bery statistics is presented by the equation:

$$JB = n\left[\frac{\sqrt{b_1}}{6} + \frac{(b_2-3)^2}{24}\right]. \tag{3}$$

At the assumption of the rightness of the zero hypothesis H_o , stating that the tested distribution is the normal one, the JB statistics expressed by Eq. (3), has an asymptotic distribution χ^2 of two degrees of freedom.

Its first component (4):

$$JB_1 = n\frac{(\sqrt{b_1})^2}{6} \tag{4}$$

takes into account the distribution skewness, while the second one (5) takes into account the flattening.

$$JB_2 = n\frac{(b_2-3)^2}{24}. \tag{5}$$

When the rightness of the zero hypothesis H_o is assumed the skewness coefficient equals zero, the flattening coefficient equals 3, and the related to them statistics (4) and (5) have the asymptotic distribution χ^2 of one degree of freedom.

Examples of the calculation results related to using the Jarque-Bery test, for the verification of the acceptability of the hypothesis of the normal distribution of the acoustic measurements results, illustrated in Fig. 1, are given in Table 1. These data are necessary in the verification process of the hypothesis of the possibility of attributing the normal distribution form to the measurements results of noise levels: L_D, L_E, L_N, L_{DEN}.

Table 1. Calculation results related to using the Jarque-Bery test.

Year 2009	L_D	L_E	L_N	L_{DEN}
Expected value	74.5	74.3	69.8	77.9
Standard deviation	0.92	1.91	1.30	1.10
Curtosis	0.65	3.22	5.9	2.18
Skewness	0.45	1.61	1.4	1.13
JB test	18.95	316.52	663.29	150.2

Assuming the rightness of the hypothesis H_o, that the tested distributions are normal ones, the JB statistics is compared with the critical value for the χ^2 distribution of two degrees of freedom, at the assumed significance level of error making: $\alpha = 0.05$. For these parameters the critical value of the Jarque-Bery statistics equals: $\chi^2 = 5.991$.

In the case when:

a) $JB \leq \chi^2 = 5.991$ – there is no reason to reject the zero hypothesis H_o assuming that the tested distribution is the normal distribution;

b) $JB > \chi^2 = 5.991$ – the zero hypothesis should be rejected.

The results presented in Table 1 indicate that the hypothesis of the normal distribution of the controlled noise indicators should be rejected. Similar results, with respect to the applied test verifying the hypothesis of the possibility of attributing the normal distribution form to the control data of levels: L_D, L_E, L_N, L_{DEN} in the years 2000–2012 – were obtained.

Investigations concerning the question whether the variability of equivalent sound levels from 5-minute samples, taken in 24-hour periods, are subjected to the normal distribution were also performed applying the Jarque-Bery test. Quite often, in the controlled investigations of the environment acoustic hazards, the measured values determined in such way constitute the random test used in the estimation of the controlled noise indicators and in their uncertainty assessments. In relation to such determined value the assumption is taken, that they originate from the population of results of the normal distribution. Thus, the acceptability verification of this assumption was necessary.

The calculated values of the JB statistics, together with its components, are given in Table 2.

Table 2. Calculated values of the JB statistics.

24 hours, every 5 minutes	L_{eqD}	L_{eqE}	L_{eqN}
Expected value	74.7	74.1	68.9
Standard deviation	1.72	1.47	3.32
Curtosis	12.05	24.43	0.35
Skewness	2.79	4.54	0.06
JB test	667.1	1082.0	47.4

These results indicate that the hypothesis of the normal distribution of the results of the general test of controlled assessments – at the significance level of error making $\alpha = 0.05$ – should be rejected. Similar results were obtained in analogous verification calculations on 3-minute samples.

All investigations carried out by the Authors explicitly indicated, that the currently assumed uncertainty assessments of the results of controlling environment acoustic hazards could be burdened with the accusation of their improper application, due to the impossibility of accepting the realization assumptions.

4. Conclutions

The results obtained by the Authors, can be a strong motivation for the necessity of using – in the acoustic hazards control – the indicated tool for the verification of the hypothesis of the normal distribution of the acoustic measurements results. Negative indications of the applied test, in relation to the wide research material from the environment acoustic monitoring, can become the justified motivation of the verification need of the current solutions of the uncertainty assessments of the performed estimations. They document deficiencies of the current solutions applied in acoustic investigations based on recommendations contained in the Guide to the Expression of Uncertainty Measurement (Guide to the Expression of Uncertainty Measurement 1995).

On the grounds of the performed investigations it can be stated, that there is the observable gap between the assumptions acceptability and formal constrains supplied by the to date research experiments from the environment acoustic monitoring.

Unacceptability of the assumption of the possibility of approximating the acoustic measurements results by the normal distribution, can be also a reason for wider searching for the proper analytical form of the density probability distribution function for the controlled noise indicators mentioned in papers (Batko, Przysucha, 2010; 2011; 2012). This form is related to the propagation law of the density distribution function of the control process input variables and to the distribution of the output value of the estimated indicator in the noise indicator control process.

References

1. Batko W., Bal-Pyrcz R. (2006), *Uncertainty analysis in the assessment of long – term noise indicators*, Archives of Acoustics, **31**, 4, 253–260.
2. Batko W., Przysucha B. (2010), *Determination of the Probability Distribution of the Mean Sound Level*, Archives of Acoustics, **35**, 4, 543–550.
3. Batko W., Przysucha B. (2011), *Random distribution of long-term indicators of variable emission conditions*, Acta Physica Polonica A, **119**, 6-A: Acoustic and biomedical engineering, 1086–1090.
4. Batko W., Przysucha B. (2012), *Uncertainty assessment of index M*, Acta Physica Polonica A, **121**, 1-A: Acoustic and biomedical engineering, 156–159.
5. Guide to the Expression of Uncertainty Measurement. International Organization for Standardization, ISBN 92-67-10188-9, (1995), Polish Edition: *Wyrażanie niepewności pomiaru – Przewodnik*.
6. Jarque C.M., Bera A.K. (1987), *Tests of Observations and Regression Regression Residualed*, International Statistical Review, **55**, 163–172.