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Decomposition of Acoustic and Entropy Modes in a Non-Isothermal Gas Affected by a Mass Force

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Diagnostics and decomposition of atmospheric disturbances in a planar flow are considered in this work. The study examines a situation in which the stationary equilibrium temperature of a gas may depend on the vertical coordinate due to external forces. The relations connecting perturbations are analytically established. These perturbations specify acoustic and entropy modes in an arbitrary stratified gas affected by a constant mass force. The diagnostic relations link acoustic and entropy modes, and they are independent on time. Hence, they provide an ability to decompose the total vector of perturbations into acoustic and non-acoustic (entropy) parts, and to establish the distribution of energy between the sound and entropy modes, uniquely at any instant. The total energy of a flow is hence determined in its parts which are connected with acoustic and entropy modes. The examples presented in this work consider the equilibrium temperature of a gas, which linearly depends on the vertical coordinate. Individual profiles of acoustic and entropy parts for some impulses are illustrated with plots.

Keywords: acoustics of non-uniform media; initialization of hydrodynamic field.
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1. Introduction

Theoretical and numerical models that describe dynamics of gases and liquids affected by external forces, are of great interest in geophysics, meteorology, and wave theory (Brekhovskikh, Godin, 1990; Pedloski, 1987; Gordin, 1987; Borovikov, Kelbert, 1985; Leble, 1990). The main aim of this study is the diagnostics and decomposition of the wave and non-wave modes. This is helpful in interpretation of experimental data, and may be useful in validation of numerical models (Leble, Vereschagina, 2016). Especially, it is important in establishing location of the sources of a wave, and modelling of the atmosphere's warming (Karpov *et al.*, 2016). It is authors' belief that the analytical models are more desirable than numerical methods, which are usually time-consuming, require high-performance computer, and special attention to underlying algorithms, their convergence, and stability. On the other hand, reasonably simple analytical models, also when complimented by a numerical approach, are much more efficient. Theory should base on the conservation equations and rely upon physically justified boundary conditions and simplifications. Some problems of practical usage are considered in this study. Among them, it is to distinguish the types of fluid motion from the total perturbations and associated with them energies.

The external forces and sources of energy and momentum make the background of a fluid non-uniform. Hence, equilibrium thermodynamic parameters depend on spatial coordinates, which drastically complicates the definition of linear modes (motions of infinitely small magnitude) taking place in such non-uniform media. The number of roots of dispersion equation, if it is possible to determine them, agrees with the number of types of motion, and equals to the number of scalar conservation equations (Pedloski, 1987). Each of conservation equations represents a PDE which contains the first-order derivative with respect to time. In the case of isothermal gas in equilibrium with pressure and density depending exponentially on the coordinate, and in the simplest case of a planar flow, the dispersion relations may be introduced over the total wave-length range. The non-exponential case needs either consideration of the atmosphere as a layered medium or, for the short waves, making use of the Wentzel, Kramers, and Brillouin (WKB) method (Brekhovskikh, Godin, 1990). Either way, there are three types of motion in one dimension: two acoustic modes of different direction of propagation, and the entropy mode. The entropy mode is not stationary in a fluid that conducts heat, but also in multidimensional flows, or in a baroclinic fluid. In the flows exceeding one dimension, the buoyancy waves appear (Brekhovskikh, Godin, 1990; Pedloski, 1987). This work considers volumes of an ideal gas with variable equilibrium temperature, affected by a constant mass force. The first results that allow to distinguish modes due to relations of specific perturbations, have been obtained relative to the motion of an exponentially stratified ideal gas in the constant gravitational field (Leble, 1990; Leble, Vereschagina, 2016).

In this study, which develops ideas of Brezhnev, Kshevetsky, and Leble (1994), the modes of a planar flow are determined by means of relations between specific perturbations that are time independent. They are valid for arbitrary dependence of the equilibrium temperature on a coordinate. These relations give ability to distinguish modes from the total field analytically at any instant, to predict their dynamics, and to conclude about energy of any of them (which remains constant in time). This is undoubtedly of importance in applications of meteorology and diagnostics of atmospheric dynamics, including understanding of such phenomena as variations of the equilibrium temperature of the stratosphere, e.g. so-called warming (Sun *et al.*, 2012). It may be explained in the framework of non-linear interaction of sound and entropy modes (Karpov *et al.*, 2016; Perelomova, 1998; 2000; 2009). The whole exposition is also important in the diagnostics of wave and non-wave modes in order to follow experimental observations and numerical simulations (Leble, Vereschagina, 2016).

The case of non-exponential atmosphere in equilibrium permits to fix the entropy and acoustic modes without subdivision into "upwards" and "downwards" directed waves (Brezhnev *et al.*, 1994). The particular case of the equilibrium constant temperature, which has been investigated by the authors, is provided in Sec. 2, whereas Sec. 3 discusses typical examples of gas perturbations, relating to linear dependence of the background temperature upon the vertical coordinate, and in the field of constant gravity force.

2. Conservation equations and modes of a flow

2.1. Basic equations

The equations governing fluid dynamics in absence of the first, second viscosity, and thermal conduction, in fact manifest conservation of momentum, energy, and mass. They determine dynamics of all possible types of motion which may occur in a fluid and are in general non-linear. We start from the linearised conservation equations describing one-dimensional flow along the vertical axis z in terms of deviations of pressure and density, p' and ρ', from equilibrium stationary values $\overline{p}$, $\overline{\rho}$, which are no longer constants, but some functions of the coordinate:

$$\begin{aligned}\frac{\partial v}{\partial t} &= -\frac{1}{\overline{\rho}}\frac{\partial p'}{\partial z} - \frac{\rho'}{\overline{\rho}}g,\\ \frac{\partial p'}{\partial t} &= -v\frac{d\overline{p}}{dz} - \gamma\overline{p}\frac{\partial v}{\partial z},\\ \frac{\partial \rho'}{\partial t} &= -v\frac{d\overline{\rho}}{dz} - \overline{\rho}\frac{\partial v}{\partial z}.\end{aligned} \tag{1}$$

The bulk flow is absent, so that the mean velocity equals zero, $\overline{v} \equiv 0$. The external force associates with the constant gravity acceleration g which is directed opposite to axis z, though it may refer to other mass forces including non-inertial ones. The flow of an ideal gas is considered, whose internal energy e in terms of pressure and density takes the form

$$e = \frac{p}{(\gamma-1)\rho}, \tag{2}$$

where $p = \overline{p} + p'$, $\rho = \overline{\rho} + \rho'$, and $\gamma = C_p/C_v$ denotes the specific heats ratio. Equations (1) describe perturbations of a small magnitude. The relation between the equilibrium pressure and density follows from the zero order stationary equality,

$$\frac{\mathrm{d}\overline{p}(z)}{\mathrm{d}z} = -g\overline{\rho}(z). \tag{3}$$

The background density which supports the equilibrium distribution of temperature $\overline{T}(z)$, takes the form

$$\overline{\rho}(z) = \frac{\overline{\rho}(0)H(0)}{H(z)}\exp\left(-\int\limits_0^z \frac{\mathrm{d}z'}{H(z')}\right), \tag{4}$$

where the pressure scale height is given by the formula

$$H(z) = \frac{\overline{p}}{\overline{\rho}g} = \frac{\overline{T}(z)(C_p - C_v)}{g}. \tag{5}$$

It is convenient to introduce the quantity φ' instead of perturbation in density,

$$\varphi' = p' - \gamma\frac{\overline{p}}{\overline{\rho}}\rho'. \tag{6}$$

In a flow with $g = 0$, φ' represents deviation of the thermodynamic process occurring in a fluid from the adiabatic one. The variation in specific entropy s of a fluid's element in a time unit is described by the conservation of its energy along its trajectory:

$$T\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}e}{\mathrm{d}t} - \frac{p}{\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}t}. \tag{7}$$

In the case of infinitely-small magnitude flow of an ideal gas in the absence of an external force, Eq. (7) takes the form

$$\overline{T}(0)\frac{\partial s'}{\partial t} = \frac{1}{\overline{\rho}(0)(\gamma-1)}\frac{\partial \varphi'}{\partial t}.$$

This makes evident $\varphi' = 0$ for the isentropic processes with zero local variation in entropy, $s' = 0$. The total energy per unit cross section of a gas cylinder of the height h

$$\varepsilon = \frac{1}{2}\int\limits_0^h \left(\overline{\rho}v^2 + \frac{p'^2}{\gamma\overline{p}} + \frac{\varphi'^2}{\gamma\nu(z)\overline{p}}\right)\mathrm{d}z \tag{8}$$

is constant, where h may be infinite. The energy density in (8) is positive if the parameter $\nu(z)$ is positive:

$$\nu(z) = \gamma - 1 + \gamma\frac{\mathrm{d}H(z)}{\mathrm{d}z} > 0.$$

It readily follows from Eqs. (1)–(6), (8), that

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = 0.$$

The independence of ε on time reflects conservation of the total energy of a gas volume, which includes kinetic, barotropic, and thermal parts. The energy flow through the total cylinder's surface is zero. That imposes appropriate boundary conditions at its upper and lower boundaries. For ε to be constant, there is a certain freedom to establish the boundary conditions at $z = 0$ and $z = h$: $v(z=0) = v(z=h) = 0$, p' is any smooth function (condition of impermeability across the boundaries), or, for example, $v(z=0) = 0$, $p'(z=h) = 0$. Introducing the new set of variables,

$$\begin{aligned}
P &= p' \cdot \exp\left(\int\limits_0^z \frac{\mathrm{d}z'}{2H(z')}\right),\\
\Phi &= \varphi' \cdot \exp\left(\int\limits_0^z \frac{\mathrm{d}z'}{2H(z')}\right),\\
U &= v \cdot \exp\left(-\int\limits_0^z \frac{\mathrm{d}z'}{2H(z')}\right),
\end{aligned} \tag{9}$$

one may rearrange Eqs. (1) into the following set:

$$\begin{aligned}
\frac{\partial U}{\partial t} &= \frac{1}{\overline{\rho}(0)}\left(\frac{\gamma-2}{2\gamma H(0)} - \eta(z)\frac{\partial}{\partial z}\right)P + \frac{\Phi}{\gamma H(0)\overline{\rho}(0)},\\
\frac{\partial P}{\partial t} &= -\gamma g H(0)\overline{\rho}(0)\frac{\partial U}{\partial z} - g\overline{\rho}(0)\frac{\gamma-2}{2\eta(z)}U,\\
\frac{\partial \Phi}{\partial t} &= -\frac{\nu(z)}{\eta(z)}g\overline{\rho}(0)U,
\end{aligned} \tag{10}$$

where

$$\eta(z) = \frac{H(z)}{H(0)}.$$

The analytical analysis of the dispersion relations and modes determined by them, may be proceeded in the case of constant $\overline{T}$ and hence, constant H. This case, corresponding to $\eta = 1$ and $\nu = \gamma - 1$, has been considered in detail by Leble, Perelomova (2013). In this case of "exponential" atmosphere, the dispersion relations which follow from the system (10), determine three modes, or, in other words, possible motions of a gas. Two of them are acoustic, differing in direction of propagation, and the last one is the entropy mode. In the absence of mass force, this mode is isobaric.

2.2. Diagnostic relations in the general case

The difficulty of the case of $\overline{T}$ dependent on z is that the algebraic dispersion equation, valid over the total wave-length domain, can no longer be introduced. Equations (10), after the Fourier transformation to frequency domain ω, determine the spectral problem with the frequency as a spectral parameter. Generally, the estimation of spectrum is fairly difficult. Nevertheless, we can extract important information about modes from the conservation equations immediately. The modes may be determined by relations linking perturbations in the general case as well. The completeness of the set of eigenvectors of the system Eqs. (10) allows to represent the total vector of perturbations as a sum of acoustic and entropy contributions at any instant (Brezhnev *et al.*, 1994). For convenience, we reproduce the modified relations in which directed acoustic branches are not subdivided:

$$\begin{aligned}
\Psi(z,t) &= \begin{pmatrix} U \\ P \\ \Phi \end{pmatrix} = \Psi_a(z,t) + \Psi_0(z,t)\\
&= \begin{pmatrix} U_a \\ \left(\frac{\gamma-2}{2\eta(z)} + \gamma H(0)\frac{\partial}{\partial z}\right)\frac{\eta(z)}{\nu(z)}\Phi_a \\ \Phi_a \end{pmatrix}\\
&\quad + \begin{pmatrix} 0 \\ P_0 \\ \left(-\frac{\gamma-2}{2} + \gamma H(0)\eta(z)\frac{\partial}{\partial z}\right)P_0 \end{pmatrix}.
\end{aligned} \tag{11}$$

We avoid a transition to the spectral problem as it was undertaken by Brezhnev, Kshevetsky, and Leble (1994), extracting the diagnostic relations from Eqs. (10) directly. Expressing

$$U = -\frac{\eta(z)}{g\nu(z)\overline{\rho}(0)}\frac{\partial \Phi}{\partial t}, \tag{12}$$

we plug it in the second equation of (10) with the result:

$$\frac{\partial P}{\partial t} = \left(\gamma g H(0)\overline{\rho}(0)\frac{\partial}{\partial z} + g\overline{\rho}(0)\frac{\gamma-2}{2\eta(z)}\right) \cdot \frac{\eta(z)}{g\nu(z)\overline{\rho}(0)}\frac{\partial \Phi}{\partial t}. \tag{13}$$

Integrating it and keeping in mind temporal independence of the entropy part on the vector of state (this part corresponds to zero frequency of dispersion relation in the conventional description and hence it is stationary), we arrive at

$$P_a = \left(\gamma H(0)\frac{\partial}{\partial z} + \frac{\gamma-2}{2\eta(z)}\right)\frac{\eta(z)}{\nu(z)}\Phi_a + C(z). \tag{14}$$

The choice of the initial condition $C(z) = 0$ yields the relation connecting $P(z,t)$ and $\Phi(z,t)$ in the joint acoustic mode,

$$P_a = \left(\frac{\gamma-2}{2\eta(z)} + \gamma H(0)\frac{\partial}{\partial z}\right)\frac{\eta(z)}{\nu(z)}\Phi_a. \tag{15}$$

The first equation in the system (10) for $U_0 = 0$ fixes the diagnostic link in the stationary entropy mode,

$$\Phi_0 = \left(-\frac{\gamma-2}{2} + \gamma H(0)\eta(z)\frac{\partial}{\partial z}\right)P_0. \tag{16}$$

In the case of non-zero mass force, in contrast to the case $g = 0$, the non-zero Φ_a may represent perturbation of φ'. It is connected with P_a by means of Eq. (14) and propagates with the sound speed. Plugging the total variable $P = P_a + P_0$, we arrive at the alternative local diagnostic relation

$$\begin{aligned}\frac{\partial U_a}{\partial t} &= -\frac{\eta(z)}{g\nu(z)\overline{\rho}(0)}\frac{\partial^2 \Phi_a}{\partial t^2} \\ &= \left[\left(\frac{\gamma-2}{2\gamma H(0)} - \eta(z)\frac{\partial}{\partial z}\right)\left(\frac{\gamma-2}{2\eta(z)} + \gamma H(0)\frac{\partial}{\partial z}\right) \cdot \frac{\eta(z)}{\nu(z)} + \frac{1}{\gamma H(0)}\right]\frac{\Phi_a}{\overline{\rho}(0)},\end{aligned} \tag{17}$$

which allows to extract the joint acoustic contribution by measuring either the derivative of U with respect to time or the second derivative of Φ with respect to time along with derivatives of Φ with respect to z. The wave equation

$$\frac{H(0)}{\gamma g\nu(z)}\frac{\partial^2\Phi_a}{\partial t^2} = -\left[\left(\frac{\gamma-2}{2\gamma\eta(z)} - H(0)\frac{\partial}{\partial z}\right)\left(\frac{\gamma-2}{2\gamma\eta(z)} + H(0)\frac{\partial}{\partial z}\right)\frac{\eta(z)}{\nu(z)} + \frac{1}{\gamma^2\eta(z)}\right]\Phi_a, \tag{18}$$

points a way to establish the diagnostics of individual directed acoustic waves. It rearranges into the conventional wave equation in the case of exponentially stratified atmosphere in the limits $\eta = 1$, $\nu = \gamma - 1$.

The acoustic variable Φ_a determines P_a and stationary quantity $\Phi_0 = \Phi - \Phi_a$ uniquely along with P_0 in accordance to Eqs. (11), (16). Equations (15)–(18) point a way to extract acoustic perturbations from the total ones. The procedure is valid at any instant. It may be readily applied in evaluations of the contributions of acoustic and entropy parts in the total energy. In contrast to the illustrations of Brezhnev, Kshevetsky, and Leble (1994), we demonstrate the structure of the localised perturbations. The initial ratio of these energies keeps constant in time. As a reference perturbation, we use φ' instead of excess density ρ'. The reasons for this are following: in the case with $g = 0$, the acoustic part of φ' is proportional to the perturbation in acoustic entropy, $\varphi'_a = (\gamma-1)\rho_0\overline{T}(0)s'_a$ and measures deviation of a thermodynamic process from isentropic (that is not longer valid in the general case if $g \neq 0$). Also, in terms of φ, the link between acoustic perturbations (15) is simple and local. This is the second reason to choose perturbation of φ instead of excess density ρ'.

2.3. Comments on the wave modes

The relation between P_a and R_a in the case of isothermal gas in equilibrium (where $R = \rho'\exp(z/2H)$), has been derived by Perelomova (1998; 2000). This relation is integro-differential with some kernel which represents dispersive properties of sound waves in inhomogeneous medium. In contrast to acoustic waves, the entropy mode possesses stationary perturbation in entropy. In the flows without an external force, the entropy mode is isobaric with any smooth perturbation of mass density, but if $g \neq 0$, it is not, in accordance to the relation between Φ_0 and P_0, Eq. (16). In order to conclude about velocity which specifies the sound, the knowledge of its relation with Φ_a is required. The relations differ in sign for various acoustic modes that differ by direction of propagation (index 1 denotes the wave propagating in the positive direction of axis z, and index 2 refers to the wave propagating in the negative direction). It follows from the conservation system, that the variables are connected by the relations (10): $U_1(z,t) = \widehat{K}\Phi_1(z,t)$, $U_2(z,t) = -\widehat{K}\Phi_2(z,t)$, where $\widehat{K}$ is some integro-differential operator (that is due to asymmetry of acoustic branches $\omega_1 = -\omega_2$). This remark gives a hint how to subdivide the total velocity into "upward" and "downward" part in the general case of z-dependent height scale. Each such link between U_i and Φ_i would be expressed by integral operators, parametrised by z, so that $U_1 + U_2 = U$. This problem will be considered in a forthcoming study.

Concluding, we claim that the relations linking U with P and Φ in a fluid's flow which is affected by the constant mass force, are integro-differential in general. The exact links of excess pressure, density, and velocity in unbounded volumes of gas with constant H, are derived exactly with regard to one-dimensional flow by Perelomova (1998). In the case of dynamics of fluid different from an ideal gas, the equation of state (2) should be corrected. Deviation of the thermodynamic state from that for an ideal gas also corrects the dynamic equation governing an excess pressure, the second one from Eqs. (1), and hence, definitions of P, Φ and U (Eqs. (9)) and definition of modes (Perelomova, 2000). The linear projecting is helpful also in studies of weakly non-linear dynamics of a fluid and, in particular, in investigations of non-linear interaction of modes. Application of the corresponding projector at the system of conservative equations (Eqs. (11), supplemented by non-linear terms), allows to derive coupled dynamic equations for interacting modes. The example of that in unbounded volume of gas was considered by Perelomova (1998; 2000). Perelomova (2009) applied the theory to short acoustic perturbartions. The method proposed by the authors is successful in the solution of some problems of fluids flows in wave-guides (Leble, 1990).

3. The particular case of linear dependence H on z

The complete investigation of wave and entropy modes spectra and evolution for arbitrary dependence H on z is fairly difficult, hence we fix our attention on the simplest non-trivial case of linear function, which is a particular case of constant parameters η and ν. This case is realistic and shows how a diagnostics could be performed. Illustrations from Sec. 3 correspond to $\alpha H(0)$, which takes values -0.1, 0 or 0.1. In the model of the standard atmosphere (U.S. Standard Atmosphere, 1976) there are some extended domains of almost linear dependence of H on z: $\alpha H(0)$ is about -0.2 over the domain of z between 0 and 10 kilometers, $\alpha H(10) \approx 0$ between 10 and 20 kilometers, and $\alpha H(30) \approx 0.1$ between 30 and 45 kilometers, see Fig. 39 of U.S. Standard Atmosphere (1976). These quantities and domains may considerably vary depending on season, daytime, and meteorological conditions. In this case, $\eta = 1 + \alpha z$, where α is some non-zero constant, $\nu = 1 - \gamma + \gamma\alpha H(0)$, and R_1, R_2 take the following forms:

$$R_1(z) = \left(\frac{1}{1+\alpha z}\right)^{1/2\alpha H(0)},$$
$$R_2(z) = H(0)\left(\frac{1}{1+\alpha z}\right)^{1/2\alpha H(0)} \cdot \frac{(1+\alpha z)^{1+1/\alpha H(0)} - 1}{1+\alpha H(0)}. \tag{19}$$

We make use of Eqs. (11) in order to separate acoustic and entropy contributions in any vector of total perturbations. Some simple conclusions illustrate the application of the theory.

3.1. Contribution of only entropy mode in the total perturbations

Commencing with isolated entropy mode, we rearrange the relation (16) for the parameters in the form (19)

$$\Phi \equiv \Phi_0 = \left(-\frac{\gamma-2}{2} + \gamma H(0)(1+\alpha z)\frac{\partial}{\partial z}\right) P \tag{20}$$

at any instant. In evaluations, we consider the stationary perturbation $P \equiv P_0(z)$ as a Gaussian impulse (a) or its derivative multiplied by $H(0)\beta$ (b),

$$\begin{aligned}(a)\quad & P \equiv P_0 = \Pi \exp\left(-\frac{(z-z_0)^2}{\beta^2 H(0)^2}\right),\\ (b)\quad & P \equiv P_0 = -\frac{2\Pi(z-z_0)}{H(0)\beta}\exp\left(-\frac{(z-z_0)^2}{\beta^2 H(0)^2}\right),\end{aligned} \tag{21}$$

where β, Π denote the characteristic dimensionless width (in units H) and magnitude of an impulse. It would be superfluous to mention that relations between field perturbations specifying every mode, Eqs. (15), (16), are valid at any instant. So, we do not determine the time which the samples of perturbations (21) correspond to. For definiteness, $z_0 = 3H(0)$, $\beta = 0.3$, and $\gamma = 1.4$. Figure 1 shows dimensionless perturbations specifying sound and entropy modes.

3.2. Contribution of sound only in the total perturbations

In accordance to Eq. (15),

$$P \equiv P_a = \left(\frac{\gamma-2}{2(1+\alpha z)} + \gamma H(0)\frac{\partial}{\partial z}\right) \cdot \frac{1+\alpha z}{1-\gamma+\gamma H(0)\alpha}\Phi. \tag{22}$$

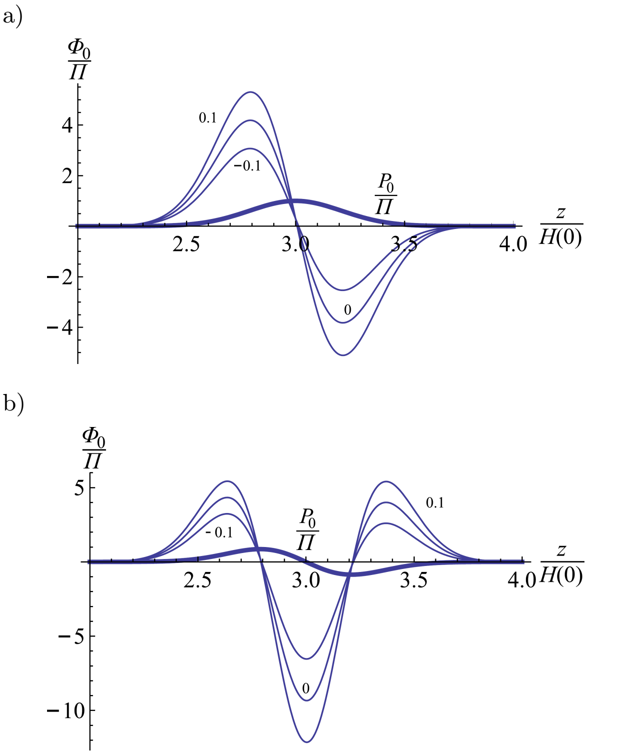


Fig. 1. Case of exclusive contribution of the entropy mode in the total perturbation. Dimensionless excess pressure P_0/Π (bold line) and Φ_0/Π (normal lines) in the entropy mode for different $\alpha H(0)$ $(-0.1, 0, 0.1)$. Cases of symmetric (a) and asymmetric (b) impulses.

$\Phi \equiv \Phi_a$ is taken in the particular case of symmetric or asymmetric impulses,

$$\begin{aligned} &(a)\ \ \Phi \equiv \Phi_a = \Pi \exp\left(-\frac{(z-z_0)^2}{\beta^2 H(0)^2}\right), \\ &(b)\ \ \Phi \equiv \Phi_a = -\frac{2\Pi}{H(0)\beta} \exp\left(-\frac{(z-z_0)^2}{\beta^2 H(0)^2}\right). \end{aligned} \tag{23}$$

The values of z_0, β, and γ are the same as in the above subsection. Figure 2 represents dimensionless perturbations which correspond to the case of zero entropy mode contribution.

3.3. Zero total Φ

Specific perturbations in pressure for the stationary mode are taken in the form (a,b) of Eq. (23), with the correspondent Φ_0 which is determined by Eq. (16). As for sound, $\Phi_a = -\Phi_0$, and the excess pressure relates to Φ_a by means of Eq. (15). Figure 3 shows excess pressure in acoustic and entropy modes in the cases (a) and (b) for different α and for the same β and γ as in the previous subsections.

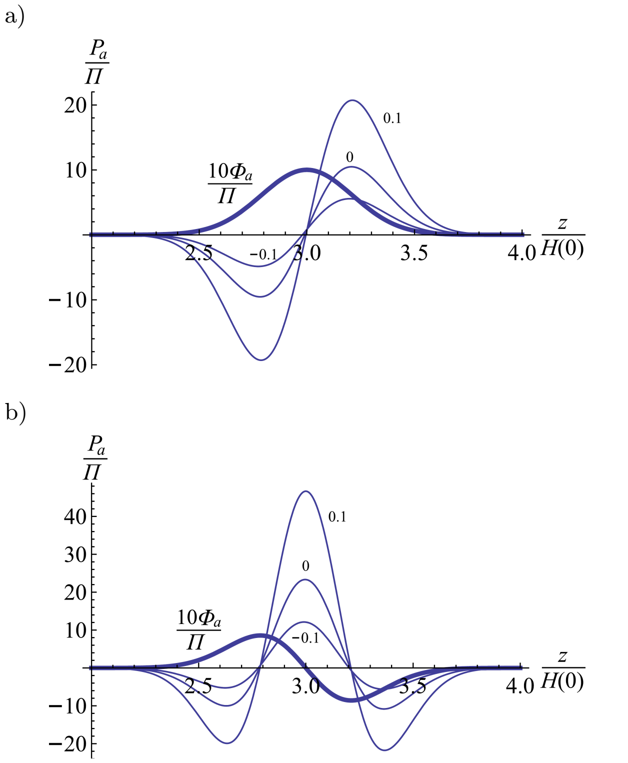


Fig. 2. Case of zero contribution of the entropy mode in the total perturbation. Dimensionless perturbations P_a/Π (normal lines), Φ_a/Π (bold line) for different $\alpha H(0)$ $(-0.1, 0, 1)$. Cases of symmetric (a) and asymmetric (b) impulses.

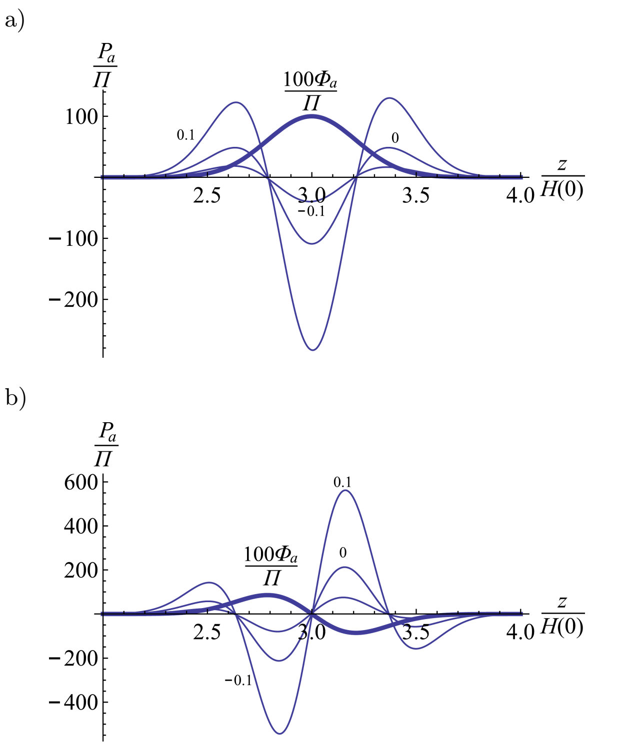
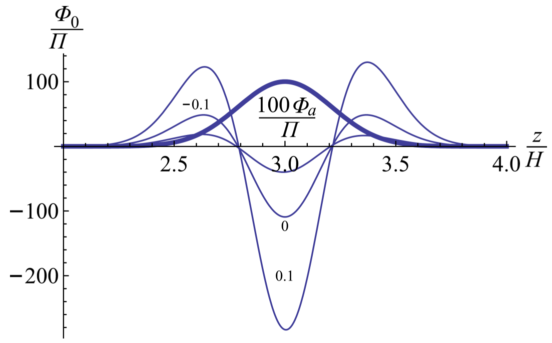


Fig. 3. Case of zero total entropy. Dimensionless perturbations P_a/Π (normal lines), P_0/Π (bold line) for different $\alpha H(0)$ $(-0.1, 0, 1)$. Cases of symmetric (a) and asymmetric (b) impulses.

3.4. Zero total excess pressure

The specific perturbation Φ_a for sound is taken in the form of symmetric or asymmetric impulses in accordance to Eq. (23). P_a relates to Φ_a by means of Eq. (15). Perturbation in pressure for the stationary mode is $P_0 = -P_a$, and Φ_0 is expressed in terms of P_0 in agreement with Eq. (16). Figure 4 exhibits excess pressure for acoustic and entropy modes in the cases (a) and (b) for different α.

a)



b)

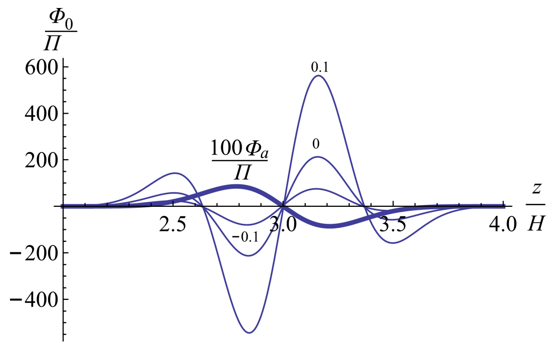


Fig. 4. Case of zero total pressure. Dimensionless perturbations Φ_0/Π (normal lines), Φ_a/Π (bold line) for different $\alpha H(0)$ $(-0.1, 0, 1)$. Cases of symmetric (a) and asymmetric (b) impulses.

The resulting curves for the various modes demonstrate a difference between their vertical profiles for typical modal perturbations, which can help to recognise them during a diagnostics.

4. Concluding remarks

The main result of this study is establishing and illustrating of diagnostic relations, Eqs. (15)–(18), which enable recognition of a mode by means of simultaneous evaluation of perturbations for velocity, pressure, and entropy. The results that correspond to the linear dependence of H on z, are illustrated by the relative plots. The figures show dependences of the principal variables (P, Φ) on z for acoustic and entropy modes. The diagnostic process presented in this work allows for estimation of specific energies of different modes, and thus their contribution in the atmosphere energy balance. The brief description of modes definition and diagnostics process by projecting in the case of constant H is presented in Sec. 2.

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