PAN
ARCHIVES OF ACOUSTICS
Vol. **44**, No. 3, pp. 447–457 (2019)
DOI: 10.24425/aoa.2019.129260

A Key-Finding Algorithm Based on Music Signature

Dariusz KANIA[(1)*], Paulina KANIA[(2)]

[(1)] *Institute of Electronics*
Silesian University of Technology
Akademicka 16, 44-100 Gliwice; Poland
*Corresponding Author e-mail: dariusz.kania@polsl.pl

[(2)] *Faculty of Physics*
Adam Mickiewicz University
Umultowska 85, 61-614 Poznań, Poland; e-mail: paukan@st.amu.edu.pl

(*received December 3, 2017; accepted May 13, 2019*)

The paper presents the key-finding algorithm based on the music signature concept. The proposed music signature is a set of 2-D vectors which can be treated as a compressed form of representation of a musical content in the 2-D space. Each vector represents different pitch class. Its direction is determined by the position of the corresponding major key in the circle of fifths. The length of each vector reflects the multiplicity (i.e. number of occurrences) of the pitch class in a musical piece or its fragment. The paper presents the theoretical background, examples explaining the essence of the idea and the results of the conducted tests which confirm the effectiveness of the proposed algorithm for finding the key based on the analysis of the music signature. The developed method was compared with the key-finding algorithms using Krumhansl-Kessler, Temperley and Albrecht-Shanahan profiles. The experiments were performed on the set of Bach preludes, Bach fugues and Chopin preludes.

Keywords: music information retrieval; computational music cognition; music data mining; music visualisation.

1. Introduction

The major and minor scales are the foundation of the Western tonal music. The tonality of the music allows creation of music notations, construction of chords, the analysis of their interdependence, or various aspects of harmony and arrangement. The theory of tonality allows us to determine the tonal similarities of particular musical scales and chords. It also allows evaluating the auditory reception of the content of musical pieces, the perception of which relates to aspects of psychology and aesthetics.

The analysis of tonality of pieces usually includes three aspects of evaluating a musical work. They are as follows: the frequency analysis of individual sounds, or signals, the complexity analysis of harmonic structure of chords, or the sequence of successive chords and the analysis of audio signals, using various types of tonality models. Many different tonal models have been developed and tested for many years. They are, among others, the geometrically-regular helical models proposed by Shepard (1982); the very mathematically complex Spiral Array model, that represents the interrelations among musical pitches (Chew, 2000; 2014); the model of diatonic space that quantified intuitions of the relative distance of pitches, chords and keys (Bernardes *et al.*, 2016; Lerdahl, 2005); or the orbifold which is a way of geometrical representation of musical chord (Tymoczko, 2006).

The tonal analysis of a piece is often based on the analysis of its harmonic structure. Several such methods based on music theory are proposed in (Papadopoulos, Peeters, 2012). Very often, systems for chord recognition rely on different types of Markov model or neural networks (McVicar *et al.*, 2014; Chen, Su, 2018; Sigtia *et al.*, 2015; Wu, Li, 2018; Zhou, Lerch, 2015). In the chord identification process, various types of histograms are often used (Harte, Sandler, 2005; Osmalsky *et al.*, 2012). The methods, which are oriented to data mining from audio files (Dorochowicz, Kostek, 2018; Gómez, 2006; Papadopoulos, Peeters, 2012; Reljin, Pokrajac, 2017; Sigtia *et al.*, 2015), are particularly valuable. The method to extract a tonal description of an audio content is presented in (Gómez, 2006). The description is validated by estimating the key of a piece and tonal representation of the polyphonic audio signal. System for real-time description and visualisation of the tonality of audio signals and tonal uncertainty over time are presented in (Gómez, Bonada, 2005; Martorell, Gómez, 2011; Tverdokhleb *et al.*, 2017). Sometimes, the presence of higher harmonics of pitch notes is used in the tonal analysis (Papadopoulos, Peeters, 2012). The tonal analysis is successfully used in the process of generating structured music with constrained patterns (Herremans, Chew, 2017; Roig *et al.*, 2014), creating the model of music tension which can be used for detection of music emotion (Grekow, 2017b; Lerdahl, 2005; Lerdahl, Krumhansl, 2007), systems of music visualisation (Cancino-Chacón *et al.*, 2014; Grekow, 2017a), computer-aided composition software (Huang *et al.*, 2016; Sabathé *et al.*, 2017), automated music analysis (Cancino-Chacón *et al.*, 2017), and music genre recognition (Anglade *et al.*, 2010; Bhalke *et al.*, 2017; Perez-Sanchio *et al.*, 2010; Rosner *et al.*, 2014; Rosner, Kostek, 2018).

A number of tonality models have addressed the problem of key-finding. The objective of key-finding algorithms is to take several tones in a melody or chords and assign a key to this fragment of a musical piece. The input for all key-finding algorithms can be acoustic signal or symbolically coded music which is represented in MIDI format. Considerable interest in proposing methods and computer algorithms for automatically key-finding can be observed (Albrecht, Huron, 2014; Albrecht, Shanahan, 2013; Handelman Sigler, 2013; Quinn, White, 2017; Shmulevich, Yli-Harja, 2000; Temperley, Marvin, 2008).

The first key-finding algorithm which was implemented on a computer is presented in (Longuet-Higgins, Steedman, 1971). The results of the experiments were encouraging and provided motivation for further work. Krumhansl (1990) proposed a key-finding algorithm using the summed duration times of individual pitch classes in the analysed segment of music. This algorithm is based on correlation coefficients of the resulting input vector with all major and minor key profiles (Krumhansl, Kessler, 1982). The results of experiments for Bach Preludes and Fugues in the Well-Tempered Clavier were favourable and, therefore, dynamic models of tonality induction were proposed (Toiviainen, Krumhansl, 2003). Temperley (2002; 2004) presented a new approach to key-finding. His Bayesian approach is based on the probability of sequences of tones. Some researchers have proposed improvements of the Krumhansl-Kessler algorithm. In general, different key-profiles are used (Albrecht, Shanahan, 2013; Temperley, 2004). The utilised modifications of key profiles lead to better accuracy of key-finding algorithms (Dawson, 2018). An important observation about the existing key-finding algorithms is the disparity between accuracy in determining the key of major- and minor-mode works (Albrecht, Shanahan, 2013).

Methods for determining the key presented in (Albrecht, Shanahan, 2013; Krumhansl, 1990; Temperley, 2007) are characterised by a very large computational complexity, which, in the case of willingness of their hardware implementation in musical instruments, becomes extremely inconvenient.

The aim of the paper is to present a simple key-finding algorithm based on the proposed form of piece description called the music signature. The algorithm can be used to determine the key of the whole piece or its fragment. Experiments that proved its usefulness were performed on the preludes and fugues of Bach and preludes of Chopin.

The paper consists of the following: theoretical background presenting the fundamental concepts illustrating the way of creating the music signature (Sec. 2); the method for tonality analysis based on the music signature (Sec. 3); the proposed key-finding algorithm (Sec. 4); the results and discussion (Sec. 5); and a short summary.

2. Theoretical background

A piece of music can be treated as a set of notes occurring at specific points in time. By omitting the time aspect and the octave ranges of individual notes, the content of the a music piece can be associated with elements of the twelve-element set of pitch classes {C, C♯/D♭, D, D♯/E♭, E, F, G, G♯/A♭, A, A♯/B♭, B}. According to the enharmonic equivalents, we assume that C♯ ≡ D♭, D♯ ≡ E♭, etc.

Let x_i denote the multiplicity of a pitch class i in a music composition, where i = C, C♯, ..., B. In order to make a statistical description of the content of a music piece independent of its length, we will normalise the multiplicities of pitch classes according to the formula (1):

$$k_i = \frac{x_i}{x_{\max}}, \tag{1}$$

where i = C, C♯, D, ..., B, and the value $x_{\max} = \max(x_{\mathrm{C}}, x_{\mathrm{C}\sharp}, x_{\mathrm{D}}, x_{\mathrm{D}\sharp}, x_{\mathrm{E}}, x_{\mathrm{F}}, x_{\mathrm{F}\sharp}, x_{\mathrm{G}}, x_{\mathrm{G}\sharp}, x_{\mathrm{A}}, x_{\mathrm{A}\sharp}, x_{\mathrm{B}})$.

Let $\mathbf{K}$ be the vector of the normalised multiplicities of individual pitch-classes. This vector is arranged in accordance with the sequence of the 12 pitch classes of the chromatic scale in the circle of fifths. Assuming that the first element of vector $\mathbf{K}$ refers to the pitch class A, and choosing the positive direction of rotation, we get:

$$\mathbf{K} = [k_{\mathrm{A}}\ k_{\mathrm{D}}\ k_{\mathrm{G}}\ k_{\mathrm{C}}\ k_{\mathrm{F}}\ k_{\mathrm{B}\flat}\ k_{\mathrm{E}\flat}\ k_{\mathrm{A}\flat}\ k_{\mathrm{D}\flat}\ k_{\mathrm{F}\sharp}\ k_{\mathrm{B}}\ k_{\mathrm{E}}]. \tag{2}$$

Definition 1: The **music signature** of a piece is the set of vectors $\{\mathbf{k}_i$: i = A, D, G, C, F, B♭, E♭, A♭, D♭, F♯, B, E$\}$, whose polar coordinates (r_i, φ_i) are defined according to the following rules:

- the length of each vector is equal to the normalised multiplicity of the corresponding pitch class, i.e. $r_i = |\mathbf{k}_i|$,
- the direction of each vector is determined according to the equation $\varphi_i = j \cdot \frac{360^\circ}{12}$, where $j = 0$ for i = A, $j = 1$ for i = D, etc.

The music signature of a piece or its fragment is a geometric reflection of its content. According to definition 1 it can be interpreted as rays extending from the centre of the circle of fifths towards the notes lying on its perimeter, whose lengths represent the normalised multiplicities of individual pitch classes in a given piece of music.

Example 1: Let us consider a fragment of a piece presented in Fig. 1, for which we want to create a vector of the pitch class multiplicities $\mathbf{K}$.

Fig. 1. A fragment of a musical piece.

Based on the notation of the analysed piece fragment, it is possible to determine the multiplicities of the individual pitch classes, which are, respectively:

$$x_{\mathrm{A}} = 7; \quad x_{\mathrm{D}} = 8; \quad x_{\mathrm{G}} = 9; \quad x_{\mathrm{C}} = 10;$$

$$x_{\mathrm{F}} = 4; \quad x_{\mathrm{B}\flat} = 0; \quad x_{\mathrm{E}\flat} = 0; \quad x_{G\sharp} = 1;$$

$$x_{\mathrm{C}\sharp} = 1; \quad x_{\mathrm{F}\sharp} = 0; \quad x_{\mathrm{B}} = 5; \quad x_{\mathrm{E}} = 7.$$

Knowing the multiplicities of the individual pitch classes, the $x_{\max}$ value can be determined.

$$\begin{aligned} x_{\max} &= \max(x_{\mathrm{A}},\ x_{\mathrm{D}},\ x_{\mathrm{G}},\ x_{\mathrm{C}},\ x_{\mathrm{F}},\ x_{\mathrm{B}\flat}, \\ &\qquad x_{\mathrm{E}\flat},\ x_{\mathrm{G}\sharp},\ x_{\mathrm{C}\sharp},\ x_{\mathrm{F}\sharp},\ x_{\mathrm{B}},\ x_{\mathrm{E}}) \\ &= \max(7, 8, 9, 10, 4, 0, 0, 1, 1, 0, 5, 7) = 10. \end{aligned}$$

The values of the normalised multiplicity coefficients k_i are, respectively:

$$k_{\mathrm{A}} = 0.7; \quad k_{\mathrm{D}} = 0.8; \quad k_{\mathrm{G}} = 0.9; \quad k_{\mathrm{C}} = 1;$$

$$k_{\mathrm{F}} = 0.4; \quad k_{\mathrm{B}\flat} = 0; \quad k_{\mathrm{E}\flat} = 0; \quad k_{\mathrm{G}\sharp} = 0.1;$$

$$k_{\mathrm{C}\sharp} = 0.1; \quad k_{\mathrm{F}\sharp} = 0; \quad k_{\mathrm{B}} = 0.5; \quad k_{\mathrm{E}} = 0.7.$$

Taking into account the values of multiplicity coefficients, for the analysed fragment the pitch class multiplicities vector takes the following form:

$$\mathbf{K} = [0.7 \;\; 0.8 \;\; 0.9 \;\; 1 \;\; 0.4 \;\; 0 \;\; 0 \;\; 0.1 \;\; 0.1 \;\; 0 \;\; 0.5 \;\; 0.7].$$

According to the definition of the music signature, the polar coordinates of vectors $\mathbf{k}_i = (k_i, \phi_i)$ are, respectively:

$$\mathbf{k}_{\mathrm{A}} = (0.7; 0^\circ); \quad \mathbf{k}_{\mathrm{D}} = (0.8; 30^\circ); \quad \mathbf{k}_{\mathrm{G}} = (0.9; 60^\circ);$$

$$\mathbf{k}_{\mathrm{C}} = (1; 90^\circ); \quad \mathbf{k}_{\mathrm{F}} = (0.4; 120^\circ); \quad \mathbf{k}_{\mathrm{B}\flat} = (0; 150^\circ);$$

$$\mathbf{k}_{\mathrm{E}\flat} = (0; 180^\circ); \quad \mathbf{k}_{\mathrm{A}\flat} = (0.1; 210^\circ); \quad \mathbf{k}_{\mathrm{D}\flat} = (0.1; 240^\circ);$$

$$\mathbf{k}_{\mathrm{F}\sharp} = (0; 270^\circ); \quad \mathbf{k}_{\mathrm{B}} = (0.5; 300^\circ); \quad \mathbf{k}_{\mathrm{E}} = (0.7; 330^\circ).$$

Knowing the polar coordinates of the individual vectors, it is possible to draw a music signature, which is presented in Fig. 2.

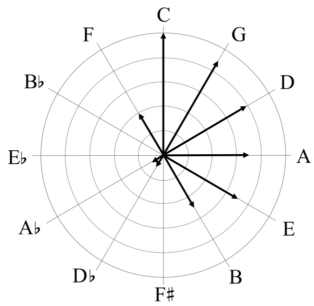


Fig. 2. The music signature of the fragment of a piece analysed in Example 1.

Let Y, Z ∈ {C, C♯/D♭, D, D♯/E♭, E, F, F♯/G♭, G, G♯/A♭, A, A♯/B♭, B}. A straight line Y–Z passing through the centre of the circle of fifths and two tones Y and Z can be referred to as an **axis of the circle of fifths**. In the circle of fifths it is possible to distinguish 6 different axes, which are marked as: C–F♯, G–D♭, D–A♭, A–E♭, E–B♭, and B–F. We assume that the F♯–C axis is identical with the C–F♯ axis, D♭–G with the G–D♭ axis, etc.

An axis of the circle of fifths divides the set of vectors $\{\mathbf{k}_i : i$ = A, D, G, C, F, B♭, E♭, A♭, D♭, F♯, B, E$\}$ of the music signature into two subsets of vectors located on the opposite sides of the axis. For example, for the axis of the circle of fifths C–F♯, the first subset contains vectors $\mathbf{k}_{\mathrm{B}}$, $\mathbf{k}_{\mathrm{E}}$, $\mathbf{k}_{\mathrm{A}}$, $\mathbf{k}_{\mathrm{D}}$, $\mathbf{k}_{\mathrm{G}}$, and the second one: $\mathbf{k}_{\mathrm{F}}$, $\mathbf{k}_{\mathrm{B}\flat}$, $\mathbf{k}_{\mathrm{E}\flat}$, $\mathbf{k}_{\mathrm{A}\flat}$, $\mathbf{k}_{\mathrm{D}\flat}$.

Let [C–F♯] denote the absolute value of the difference in summed lengths of vectors located on the opposite sides of the axis of the circle of fifths C–F♯, i.e.

$$\begin{aligned} [\mathrm{C{-}F}\sharp] = \big|&(|\mathbf{k}_{\mathrm{B}}| + |\mathbf{k}_{\mathrm{E}}| + |\mathbf{k}_{\mathrm{A}}| + |\mathbf{k}_{\mathrm{D}}| + |\mathbf{k}_{\mathrm{G}}|) \\ &- (|\mathbf{k}_{\mathrm{F}}| + |\mathbf{k}_{\mathrm{B}\flat}| + |\mathbf{k}_{\mathrm{E}\flat}| + |\mathbf{k}_{\mathrm{A}\flat}| + |\mathbf{k}_{\mathrm{D}\flat}|)\big|. \end{aligned}$$

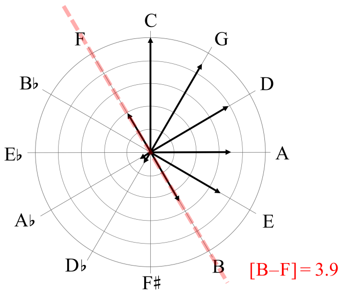
Of course, the value [C–F♯] = [F♯–C], [G–D♭] = [D♭–G], etc.

Definition 2: The axis of the circle of fifths Y–Z, for which [Y–Z] reaches the maximum value is called the **main axis of the music signature**.

Example 2: Let us look at the music signature obtained in Example 1. The table below shows the values [Y–Z] for individual axes of the circle of fifths. The maximum value occurs for the B–F axis, which therefore is the main axis of this music signature.

C–F♯	G–D♭	D–A♭	A–E♭	E–B♭	B–F
3	1.2	0.3	1.7	3.1	3.9

A graphic representation of the main axis of the signature is presented in Fig. 3.



$$[\mathrm{B{-}F}] = \left|(|\mathbf{k}_{\mathrm{C}}| + |\mathbf{k}_{\mathrm{G}}| + |\mathbf{k}_{\mathrm{D}}| + |\mathbf{k}_{\mathrm{A}}| + |\mathbf{k}_{\mathrm{E}}|) - (|\mathbf{k}_{\mathrm{B}\flat}| + |\mathbf{k}_{\mathrm{E}\flat}| + |\mathbf{k}_{\mathrm{A}\flat}| + |\mathbf{k}_{\mathrm{D}\flat}| + |\mathbf{k}_{\mathrm{F}\sharp}|)\right|$$

Fig. 3. The example of a music signature and its main axis.

Let Y → Z denotes a **directed axis of the circle of fifths**. The Y → Z axis coincides with the Y–Z axis of the circle of fifths, but its additional feature is the direction from Y to Z. Each axis of the circle of fifths can be associated with two directed axes. The directions of these two axes are opposite. For example, the C–F♯ axis is associated with two directed axes: C → F♯ and F♯ → C.

We can distinguish 12 directed axes in the circle of fifths, which are: C → F♯; G → D♭; D → A♭; A → E♭; E → B♭; B → F; F♯ → C; D♭ → G; A♭ → D; E♭ → A; B♭ → E; F → B. Each directed axis divides the vectors of the music signature into two subsets.

Let [Y → Z] denote the difference in value of the summed lengths of vectors located on the right and the left side of the Y–Z axis, viewed in the direction of the Y → Z directed axis, for example:

$$[\mathrm{F}\sharp \to \mathrm{C}] = (|\mathbf{k}_{\mathrm{B}}| + |\mathbf{k}_{\mathrm{E}}| + |\mathbf{k}_{\mathrm{A}}| + |\mathbf{k}_{\mathrm{D}}| + |\mathbf{k}_{\mathrm{G}}|) - (|\mathbf{k}_{\mathrm{F}}| + |\mathbf{k}_{\mathrm{B}\flat}| + |\mathbf{k}_{\mathrm{E}\flat}| + |\mathbf{k}_{\mathrm{A}\flat}| + |\mathbf{k}_{\mathrm{D}\flat}|),$$

$$[\mathrm{C} \to \mathrm{F}\sharp] = (|\mathbf{k}_{\mathrm{F}}| + |\mathbf{k}_{\mathrm{B}\flat}| + |\mathbf{k}_{\mathrm{E}\flat}| + |\mathbf{k}_{\mathrm{A}\flat}| + |\mathbf{k}_{\mathrm{D}\flat}|) - (|\mathbf{k}_{\mathrm{B}}| + |\mathbf{k}_{\mathrm{E}}| + |\mathbf{k}_{\mathrm{A}}| + |\mathbf{k}_{\mathrm{D}}| + |\mathbf{k}_{G}|).$$

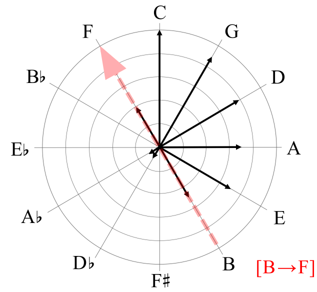
Of course, [C → F♯] = −[F♯ → C], [G → D♭] = −[D♭ → G], etc.

Definition 3: A directed axis of the circle of fifths Y → Z, for which [Y → Z] reaches the maximum value is called the **main directed axis of a music signature**.

Example 3: Let us consider the music signature of the fragment of a piece analysed in Example 1. The table below shows the values [Y → Z] for individual directed axes of the circle of fifths. The maximum value occurs for the [B → F] axis, which, therefore, is the main directed axis of this music signature.

C → F♯	G → D♭	D → A♭	A → E♭	E → B♭	B → F
3	−1.2	0.3	1.7	3.1	3.9
F♯ → C	D♭ → G	A♭ → D	E♭ → A	B♭ → E	F → B
3	1.2	−0.3	−1.7	−3.1	−3.9

A graphic illustration of the main directed axis of an example of music signature is presented in Fig. 4.



$$[\mathrm{B} \to \mathrm{F}] = (|\mathbf{k}_{\mathrm{C}}| + |\mathbf{k}_{\mathrm{G}}| + |\mathbf{k}_{\mathrm{D}}| + |\mathbf{k}_{\mathrm{A}}| + |\mathbf{k}_{\mathrm{E}}|) - (|\mathbf{k}_{\mathrm{B}\flat}| + |\mathbf{k}_{\mathrm{E}\flat}| + |\mathbf{k}_{\mathrm{A}\flat}| + |\mathbf{k}_{\mathrm{D}\flat}| + |\mathbf{k}_{\mathrm{F}\sharp}|)|$$

Fig. 4. Music signature with its main directed axis.

For the analysed fragment of a piece, the B–F axis is the main axis of the music signature (Example 2, Fig. 3). The process of searching for the main directed axis of the music signature can be simplified by considering only two directed axes of the circle of fifths B → F and F → B coinciding with the main axis of the B–F music signature. Because [B → F] is positive while [F → B] is negative, the B → F axis is the main directed axis of the music signature.

Having found the main axis B–F of the music signature of the analysed fragment (Example 2, Fig. 3) we need only to choose between the two directed axes B → F and F → B to find the main directed axis.

3. Tonality analysis based on music signature

The dominant part of the Western musical pieces is based on the major-minor scales system. The circle of fifths is associated with 12 pairs of relative minor/major keys: 12 major keys and 12 minor keys. Each major key has its relative minor key. The tonic of a major is the third degree of its relative minor scale, while the tonic of a minor scale is the sixth scale degree of its relative major scale, e.g. C–a, G–e, D–b. For simplicity, let us limit the considerations to major scales for now.

When analysing the distribution of keys in the circle of fifths, it becomes possible to determine the pitches of individual scales. For example, the pitches of G major scale: G–A–B–C–D–E–F♯–G, are located on the F♯ → C axis and on the right side of the circle of fifths looking in the direction from F♯ to C. Therefore, the F♯ → C axis can be associated with the G major scale contrary to the C → F♯ axis, whose right side (looking from the C position towards F♯) contains the pitches of the D♭ major scale (D♭–E♭–F–G♭–A♭–B♭–C–D♭). The above observations are illustrated in Fig. 5.

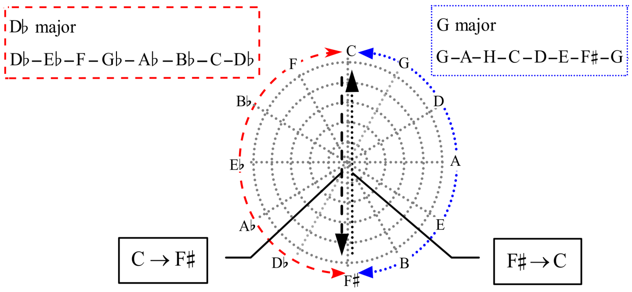


Fig. 5. The circle of fifths with the pitches for D♭ major and G major scales.

Each directional axis divides the circle of fifths into two parts. The pitches located on the right side (looking in the axial direction) along with the pitches on the axis create particular scale degrees. The first scale degree (tonic) is located on the circle of fifths on the first position to the right in the clockwise direction.

In accordance with the above principle, each directional axis can be assigned its associated major scale.

C → F♯	⇒	D♭ major
G → D♭	⇒	A♭ major
D → A♭	⇒	E♭ major
A → E♭	⇒	B♭ major
E → B♭	⇒	F major
B → F	⇒	C major
F♯ → C	⇒	G major
D♭ → G	⇒	D major
A♭ → D	⇒	A major
E♭ → A	⇒	E major
B♭ → E	⇒	B major
F → B	⇒	F♯ major

The considerations presented above refer to the analysis of pieces created in major keys. Generally, the methodology presented above allows determining the key signature of the analysed piece. The sets of tones of the major scale and the relative minor scale are very similar. Taking into account the natural minor scale, they are identical, e.g. C major (C–D–E–F–G–A–B–C) and relative natural minor (A–B–C–D–E–F–G–A). In this situation, it is possible to generalise the above considerations to minor scales. The circle of the fifths with the major and minor keys is presented in Fig. 6. For example, the main directed axis F♯ → C is the main axis of the G major or e minor scale.

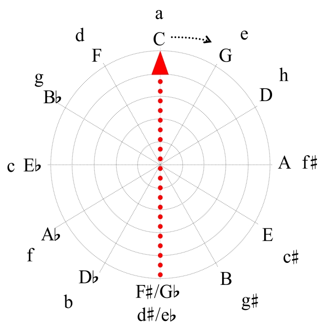


Fig. 6. The circle of fifths containing the major and minor keys and an exemplary main directed axis (here F♯ → C).

After finding the main directional axis of a music signature, major or relative minor scale should be chosen. The algorithm for choosing between two relative major/minor keys can be based on Pearson's correlation of the pitch class multiplicities vector with the key profiles proposed by Krumhansl-Kessler's (Krumhansl, Kessler, 1982) or others (Albrecht, Shanahan, 2013; Temperley, 2004). The analysis of the value of the Pearson's correlation coefficients for the relative major and minor profiles corresponding to the main directed axis of the music signature allows choosing one of the two relative keys. The key with a higher value of Pearson's correlation coefficient is selected.

Thus, after determining the main directional axis of the music signature, the key selection is reduced to a decision whether the analysed piece is in the major scale, or in the relative minor scale.

Example 4: Let us consider the fragment of a piece presented in Fig. 1 again. The B → F axis is the main directed axis of the music signature (see Example 3). Hence, the sought key of the piece is placed in the circle of fifths in the first position to the right in relation to the arrowhead of the main directed axis. It is the C major or the A minor scale. The Pearson correlation coefficients between the multiplicities vector of the analysed fragment of the piece and the Krumhansl-Kessler's major and minor key profiles are, respectively, $r_{\text{C major}} = 0.88$ and $r_{\text{A minor}} = 0.71$. The graphs illustrating the correlations are shown in Fig. 7. Because $r_{\text{C major}} > r_{\text{A minor}}$, we conclude that the C major scale is the sought scale of the piece.

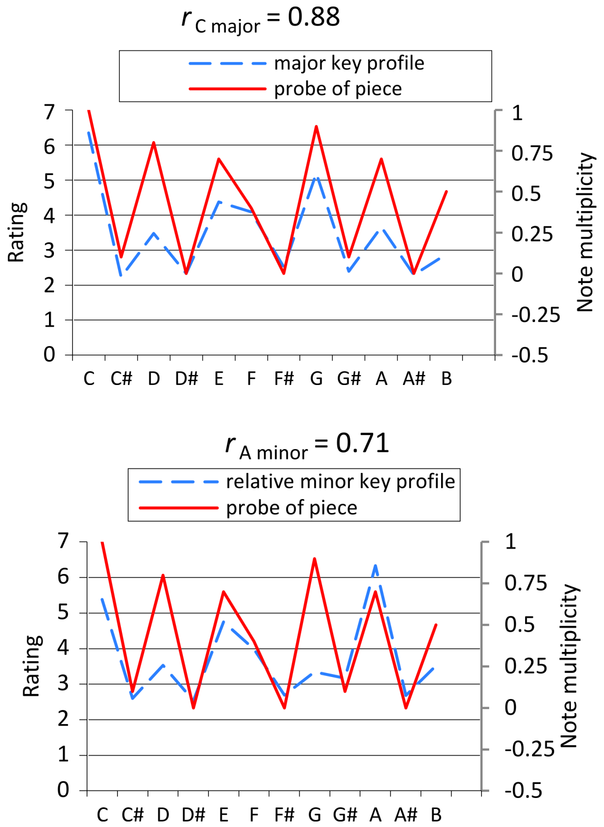


Fig. 7. The correlation of probe of piece with C major and A minor key profiles.

4. The key-finding algorithm

The theoretical basis presented in the previous sections led to the algorithm for finding the key signature of the analysed piece and detecting its minor or major scale.

The key-finding algorithm based on music signature consists of the following steps:

1) The note multiplicities vector K is created for the piece being considered.
2) The music signature is created based on the multiplicities vector.
3) The main axis of the music signature is determined. If it is impossible to determine the main axis of music signature for the analysed sample of the piece, the sample is extended. Successive extensions of a sample by one note take place until the clear selection of the main axis of the music signature can be done.
4) The main directed axis of the music signature is determined (i.e. the direction of the previously found main axis is chosen).
5) The key signature of the analysed piece is determined based on the circle of fifths. This narrows the choice of keys to the corresponding pair of relative major/minor ones.
6) The Pearson correlation coefficients between the multiplicities vector of the analysed piece (or its fragment) and major and minor key profiles are determined for the two keys resulting from the previous step. Krumhansl-Kessler's or others key profiles may be used.
7) The key of the analysed piece is determined by selecting this key from the relative major/minor pair for which the value of the Pearson correlation coefficient is higher.

If only the key signature (number of sharps or flats) of a piece is to be found the steps from 1 to 5 of the above algorithm can be used. All steps of the proposed algorithm were thoroughly discussed in the examples in Secs 2 and 3.

5. Results and discussion

In order to confirm the effectiveness of the proposed key-finding algorithm, a number of experiments were performed. The developed key-finding algorithm based on the analysis of the music signature of a piece was compared to the key-finding algorithm based on the Krumhansl's and Kessler's profiles (Krumhansl, Kessler, 1982), Temperley profiles (Temperley, 2004) and Albrecht's and Shanahan's profiles (Albrecht, Shanahan, 2013). In these algorithms the key of a piece is determined by finding the maximum values of the Pearson correlation coefficients with 24 profiles of major and minor profiles. As in (Krumhansl, 1990), in all approaches the keys of the pieces were determined based on the first 4 notes. If it was impossible to separate the first four notes in time, the smallest possible number of notes, greater than four, was taken into account. Similar experiments were performed using the proposed key-finding algorithm based on the music signature analysis. The key was also determined based on the first 4 notes of the piece. If the result of the analysis did not indicate the main directed axis of the signature, consecutive notes of the piece were added, until the main directed axis of the music signature was clearly identified.

The experiments on the set of Preludes and Fugues of the Bach's Well-Tempered Clavier – Book 1 and Frederic Chopin's preludes were performed. The results are presented in Tables 1, 2, and 3. The respective columns of the tables contain: the prelude/fugue number, its key and partial results of the performed analyses for the methods for determining the key based on the Krumhansl's and Kessler's profiles (K-K profile), Temperley profiles (T profile), Albrecht's and Shanahan's profiles (A-S profile) and the proposed method using the music signature (Music signature). The K-K profile, T profile and A-S profile parts include, respectively: in the column marked $r_{\max}$ – maximum value of

Table 1. Comparison of the key-finding algorithms for Bach Well Tempered Clavier preludes (Book 1).

No.	Key	K-K profile		T profile		A-S profile		Music signature		
		$r_{\max}$	key ($r_{\max}$)	$r_{\max}$	key ($r_{\max}$)	$r_{\max}$	key ($r_{\max}$)	[X → Y]	r_{major}; r_{minor}	key$_{m_s}$
1	C	0.81	C	0.69	C	0.76	C	[B → F]	$r_{\text{major}} > r_{\text{minor}}$	C
2	c	0.92	c	0.75	c	0.86	c	[D → A♭]	$r_{\text{major}} < r_{\text{minor}}$	c
3	C♯	0.83	C♯	0.65	C♯	0.79	C♯	[C → F♯]	$r_{\text{major}} > r_{\text{minor}}$	C♯
4	c♯	0.87	c♯	0.72	c♯	0.83	c♯	[E♭ → A]	$r_{\text{major}} < r_{\text{minor}}$	c♯
5	D	0.73	D	0.61	D	0.69	D	[D♭ → G]	$r_{\text{major}} > r_{\text{minor}}$	D
6	d	0.81	d	0.58	d	0.73	d	[E → B♭]	$r_{\text{major}} < r_{\text{minor}}$	d
7	E♭	0.87	E♭	0.68	E♭	0.79	E♭	[G → D♭]	$r_{\text{major}} > r_{\text{minor}}$	A♭
8	e♭	0.92	e♭	0.75	e♭	0.78	e♭	[F → B]	$r_{\text{major}} < r_{\text{minor}}$	e♭
9	E	0.83	E	0.65	E	0.79	E	[E♭ → A]	$r_{\text{major}} > r_{\text{minor}}$	E
10	e	0.92	e	0.75	e	0.86	e	[F♯ → C]	$r_{\text{major}} < r_{\text{minor}}$	e
11	F	0.83	F	0.74	F	0.80	F	[E → B♭]	$r_{\text{major}} > r_{\text{minor}}$	F
12	f	0.85	f	0.58	C♯	0.61	f	[G → D♭]	$r_{\text{major}} < r_{\text{minor}}$	f
13	F♯	0.83	b♭	0.72	F♯	0.69	F♯	[F → B]	$r_{\text{major}} > r_{\text{minor}}$	F♯
14	f♯	0.93	f♯	0.77	f♯	0.85	f♯	[A♭ → D]	$r_{\text{major}} < r_{\text{minor}}$	f♯
15	G	0.83	G	0.65	G	0.79	G	[F♯ → C]	$r_{\text{major}} > r_{\text{minor}}$	G
16	g	0.85	g	0.63	g	0.77	g	[A → E♭]	$r_{\text{major}} < r_{\text{minor}}$	g
17	A♭	0.87	A♭	0.80	A♭	0.87	A♭	[G → D♭]	$r_{\text{major}} > r_{\text{minor}}$	A♭
18	g♯	0.83	g♯	0.61	g♯	0.64	g♯	[B♭ → E]	$r_{\text{major}} < r_{\text{minor}}$	g♯
19	A	0.73	A	0.59	A	0.69	A	[A♭ → D]	$r_{\text{major}} > r_{\text{minor}}$	A
20	a	0.82	a	0.58	a	0.66	a	[B → F]	$r_{\text{major}} < r_{\text{minor}}$	a
21	B♭	0.88	B♭	0.71	B♭	0.85	B♭	[A → E♭]	$r_{\text{major}} > r_{\text{minor}}$	B♭
22	b♭	0.92	b♭	0.75	b♭	0.86	b♭	[C → F♯]	$r_{\text{major}} < r_{\text{minor}}$	b♭
23	B	0.68	B	0.63	B	0.65	B	[B♭ → E]	$r_{\text{major}} > r_{\text{minor}}$	B
24	b	0.83	b	0.84	b	0.93	b	[A♭ → D]	$r_{\text{major}} < r_{\text{minor}}$	f♯

Table 2. Comparison of the key-finding algorithms for Bach Well-Tempered Clavier fugues (Book 1).

No.	Key	K-K profile		T profile		A-S profile		Music signature		
		$r_{\max}$	key ($r_{\max}$)	$r_{\max}$	key ($r_{\max}$)	$r_{\max}$	key ($r_{\max}$)	[X → Y]	r_{major}; r_{minor}	key$_{m_s}$
1	C	0.68	F	0.60	F	0.70	F	[E → B♭]	$r_{\text{major}} > r_{\text{minor}}$	F
2	c	0.79	C	0.66	C	0.81	C	[B → F]	$r_{\text{major}} > r_{\text{minor}}$	C
3	C♯	0.61	g♯	0.46	C♯	0.57	g♯	[F → B]	$r_{\text{major}} > r_{\text{minor}}$	F♯
4	c♯	0.57	c♯	0.54	c♯	0.50	c♯	[B♭ → E]	$r_{\text{major}} < r_{\text{minor}}$	g♯
5	D	0.61	D	0.65	D	0.63	G	[D♭ → G]	$r_{\text{major}} > r_{\text{minor}}$	D
6	d	0.60	d	0.63	C	0.54	d	[B → F]	$r_{\text{major}} > r_{\text{minor}}$	C
7	E♭	0.76	g	0.58	E♭	0.57	g	[D → A♭]	$r_{\text{major}} > r_{\text{minor}}$	E♭
8	e♭	0.64	E♭	0.67	e♭	0.78	e♭	[F → B]	$r_{\text{major}} < r_{\text{minor}}$	e♭
9	E	0.65	f♯	0.66	b	0.72	h	[E♭ → A]	$r_{\text{major}} > r_{\text{minor}}$	E
10	e	0.92	e	0.75	e	0.86	e	[F♯ → C]	$r_{\text{major}} < r_{\text{minor}}$	e
11	F	0.54	c	0.51	B♭	0.53	c	[A → E♭]	$r_{\text{major}} > r_{\text{minor}}$	B♭
12	f	0.43	c	0.33	f	0.41	C	[B → F]	$r_{\text{major}} > r_{\text{minor}}$	C
13	F♯	0.72	F♯	0.62	F♯	0.76	F♯	[F → B]	$r_{\text{major}} > r_{\text{minor}}$	F♯
14	f♯	0.62	A	0.48	f♯	0.58	A	[A♭ → D]	$r_{\text{major}} < r_{\text{minor}}$	f♯
15	G	0.61	G	0.51	G	0.62	G	[F♯ → C]	$r_{\text{major}} > r_{\text{minor}}$	G
16	g	0.49	e♭	0.43	b	0.40	b	[A → E♭]	$r_{\text{major}} < r_{\text{minor}}$	g
17	A♭	0.88	A♭	0.76	A♭	0.86	A♭	[G → D♭]	$r_{\text{major}} > r_{\text{minor}}$	A♭
18	g♯	0.60	A♭	0.49	A♭	0.60	A♭	[G → D♭]	$r_{\text{major}} > r_{\text{minor}}$	A♭
19	A	0.64	A	0.60	A	0.64	A	[A♭ → D]	$r_{\text{major}} > r_{\text{minor}}$	A
20	a	0.55	A	0.51	A	0.59	A	[A♭ → D]	$r_{\text{major}} > r_{\text{minor}}$	A
21	B♭	0.76	B♭	0.73	B♭	0.78	B♭	[A → E♭]	$r_{\text{major}} > r_{\text{minor}}$	B♭
22	b♭	0.67	B♭	0.69	b♭	0.80	b♭	[C → F♯]	$r_{\text{major}} < r_{\text{minor}}$	b♭
23	B	0.48	F♯	0.53	F♯	0.52	B	[B♭ → E]	$r_{\text{major}} > r_{\text{minor}}$	B
24	b	0.86	b	0.77	b	0.77	G	[D♭ → G]	$r_{\text{major}} < r_{\text{minor}}$	b

Table 3. Comparison of the key-finding algorithms for Chopin preludes.

No.	Key	K-K profile		T profile		A-S profile		Music signature		
		$r_{\max}$	key ($r_{\max}$)	$r_{\max}$	key ($r_{\max}$)	$r_{\max}$	key ($r_{\max}$)	[X → Y]	r_{major}; r_{minor}	key$_{m_s}$
1	C	0.81	C	0.81	C	0.85	C	[B → F]	$r_{\text{major}} > r_{\text{minor}}$	C
2	a	0.64	e	0.56	e	0.60	e	[F♯ → C]	$r_{\text{major}} < r_{\text{minor}}$	e
3	G	0.79	G	0.67	g	0.82	g	[F♯ → C]	$r_{\text{major}} > r_{\text{minor}}$	G
4	e	0.68	b	0.67	e	0.74	e	[F♯ → C]	$r_{\text{major}} < r_{\text{minor}}$	e
5	D	0.39	G	0.40	G	0.40	G	[F♯ → C]	$r_{\text{major}} > r_{\text{minor}}$	G
6	b	0.92	b	0.75	b	0.86	b	[D♭ → G]	$r_{\text{major}} < r_{\text{minor}}$	b
7	A	0.65	E	0.56	A	0.57	A	[A♭ → D]	$r_{\text{major}} > r_{\text{minor}}$	A
8	f♯	0.69	c♯	0.58	f♯	0.60	f♯	[A♭ → D]	$r_{\text{major}} < r_{\text{minor}}$	c♯/f♯
9	E	0.88	E	0.76	E	0.86	E	[E♭ → A]	$r_{\text{major}} > r_{\text{minor}}$	E
10	c♯	0.40	a	0.41	c♯	0.44	A	[E♭ → A]	$r_{\text{major}} < r_{\text{minor}}$	c♯
11	B	0.67	F♯	0.50	F♯	0.63	F♯	[F → B]	$r_{\text{major}} > r_{\text{minor}}$	F♯
12	g♯	0.85	g♯	0.79	g♯	0.91	g♯	[B♭ → E]	$r_{\text{major}} < r_{\text{minor}}$	g♯
13	F♯	0.88	F♯	0.76	F♯	0.86	F♯	[F → B]	$r_{\text{major}} > r_{\text{minor}}$	F♯
14	e♭	0.81	E♭	0.69	e♭	0.86	e♭	[F → B]	$r_{\text{major}} < r_{\text{minor}}$	e♭
15	D♭	0.82	D♭	0.78	D♭	0.82	D♭	[C → F♯]	$r_{\text{major}} > r_{\text{minor}}$	D♭
16	b♭	0.43	F	0.49	b♭	0.42	b♭	[C → F♯]	$r_{\text{major}} < r_{\text{minor}}$	b♭
17	A♭	0.76	A♭	0.75	A♭	0.81	A♭	[G → D♭]	$r_{\text{major}} > r_{\text{minor}}$	A♭
18	f	0.55	b♭	0.53	b♭	0.82	b♭	[A → E♭]	$r_{\text{major}} < r_{\text{minor}}$	g
19	E♭	0.72	B♭	0.59	E♭	0.66	E♭	[D → A♭]	$r_{\text{major}} > r_{\text{minor}}$	E♭
20	c	0.88	c	0.75	c	0.89	c	[D → A♭]	$r_{\text{major}} < r_{\text{minor}}$	c
21	B♭	0.68	F	0.54	F	0.64	F	[A → E♭]	$r_{\text{major}} > r_{\text{minor}}$	B♭
22	g	0.44	a	0.46	d	0.43	d	[E → B♭]	$r_{\text{major}} < r_{\text{minor}}$	d
23	F	0.76	F	0.75	F	0.81	F	[E → B♭]	$r_{\text{major}} > r_{\text{minor}}$	F
24	d	0.92	d	0.75	d	0.86	d	[E → B♭]	$r_{\text{major}} < r_{\text{minor}}$	d

the Pearson correlation coefficients obtained for 24 key profiles (12 major profiles, 12 minor profiles) and the key, for which the maximum value of the correlation coefficient was obtained (column marked key ($r_{\max}$)). The Music signature part of the tables contain the main directed axis of the music signature [X → Y], relation between Pearson's correlation coefficients for the indicated major (r_{major}) and minor (r_{minor}) keys, and the result of the analysis indicating the key (column marked key$_{m_s}$).

The analysis of the results obtained for 24 Bach's preludes contained in the *Well-Tempered Clavier – Book 1* indicated only four cases, in which the key of the piece was determined incorrectly. The algorithms based on K-K profiles led to a wrong decision in the case of Prelude No. 13, T profiles led to a wrong decision in the case of Prelude No. 12, while the algorithm using the music signature led to a wrong decision in case of two preludes, No. 7 and No. 24. In the case of the proposed algorithm, the incorrectly indicated key was a neighbour of the good key on the circle of fifth. A similar mistake was observed for prelude 13 in the case of using the K-K profiles and for Prelude 12 in the case of using the T profiles. The B♭ minor key indicated by the K-K profile is a relative minor of the D♭ major located in the neighbourhood of the correct key of the prelude, F♯ major (G♭ major). Similarly, the C♯ major key indicated by the T profile is a relative minor of the B♭ minor located in neighbourhood of the correct key of the prelude, F minor.

In the case of the fugues much more faulty results were observed. The K-K profiles, T profiles, A-S profiles led to an incorrect determination of the key, respectively, in 14 (58%), 9 (37.5%), 13 (54%) cases. All profiles led to a wrong key for Fugues 1, 2, 9, 11, 16, 18 and 20.

The proposed algorithm using the music signature has led to a wrong result in nine cases (37.5%) like key-finding algorithm based on A-S profile. The proposed key-finding algorithm led to a wrong decision for Fugues 1, 2, 3, 4, 6, 11, 12, 18, and 20. For five fugues (Nos. 1, 2, 11, 18 and 20) all methods indicated the wrong key. For Fugue No. 4 only the proposed method led to a wrong result.

The highest number of wrong results was observed for the case of the Frederic Chopin's preludes. The algorithm based on K-K profiles led to an incorrect determination of the key in 13 cases (54%). This was observed for Preludes 2, 4, 5, 7, 8, 10, 11, 14, 16, 18, 19, 21, and 22. For T profiles, much better results were observed. Only in the seven (29%) cases (Prelude Nos. 2, 3, 5, 11, 18, 21, and 22) a wrong decision was made.

Algorithm based on A-S profile led to an incorrect determination of the key in eight (33%) cases (Preludes 2, 3, 5, 10, 11, 18, 21, and 22).

The proposed algorithm using the music signature has led to a false result only in five cases (21%) – Preludes 2, 5, 11, 18 and 21. All cases of wrong decisions occurred for the same preludes for which all the other approaches led to an incorrect determination of the key. Analysing these situations in depth, it is important to point out that the reason for a wrong decision is mainly a special way of starting a piece which suggests a key that is different from the one observed by looking at a bigger fragment of the piece. For example, additional sharps and natural symbols appearing in the first notes of the Preludes 2, 11 and 18 make it virtually impossible to properly identify the key.

The most interesting results were observed for Prelude No. 21. The proposed algorithm led to the correct key indication, which was not achieved by any others algorithm. After analysing the sample of the first four (4) notes, the algorithms based on K-K, T, and A-S profiles indicated the wrong key. The proposed algorithm for an identical fragment of piece was unable to indicate the main axis of the signature. This resulted in the need to increase the number of notes what eventually led to the possibility of choosing the main axis of the music signature indicating the right key of the piece. In the case of Prelude No. 21, the determination of the main directed axis of the music signature became possible only after examining the sample containing the first 25 notes. Also for others preludes (Nos. 3, 7, 10, and 14) extension of the analysed fragment of the piece was required. In most preludes, the right decision was made without the need to extend the analysed fragment of the piece. Sometimes extension by one note was required (Prelude No. 10).

In addition to the direct comparison shown in Tables 1, 2 and 3, the comparison of the key-finding algorithms for all approaches is presented synthetically in the form of bar graphs in Fig. 8. The height of the bar shows how many right keys were indicated for each key-finding algorithm. When comparing the number of correct decisions it can be observed that the algorithm based on music signature was especially effective for Chopin's preludes.

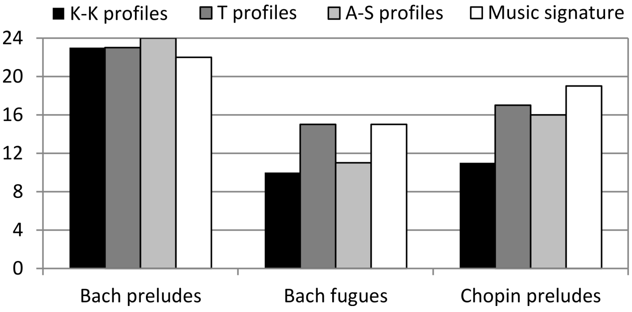


Fig. 8. Comparison of the key-finding algorithms based on Krumhansl-Kessler's, Temperley, Albrecht-Shanahan's profiles and music signature.

6. Conclusions

The concept of describing the musical content by means of the music signature is proposed. Music signature constitutes a compressed form of the representation of the content of a musical piece. It allowed the development of a simple key-finding algorithm. The results presented in the paper confirm the effectiveness of the developed method in case of using a very short fragment of piece in the analysis process. Of course, the analysis of longer fragments of piece, or even the whole piece improves the efficiency of the proposed algorithm. A number of additional experiments, not presented in this study, clearly confirmed this obvious thesis, which is also valid for other well-known key-finding algorithms.

One drawback of the presented idea is the fact that the major/minor key choice procedure is based on the statistical analysis for two relative key profiles. This is done in a typical way, using Pearson correlation coefficients. It can be clearly seen that this is the most complex part of the algorithm in terms of computation. The most valuable aspect of the proposed approach is the original and very simple way of defining the main directed axis of a music signature. This allows an extremely simple determination of the key signature, which indicates a pair of relative major/minor keys. Another advantage of the proposed approach is the possibility of flexible selection of the size of the analysed sample. The extension of the analysed fragment of a piece, occurring in the case of difficulties with determining the main directed axis of the music signature, is the main element, which gave better results than the results obtained with the K-K, T and A-S profiles. In the case of fragments of pieces that were difficult to analyse, the algorithms based on major and minor profiles led to a wrong decision. In the proposed approach, difficult fragments often resulted in lack of decision, which led to the extension of the analysed sample of a piece, until it became possible to determine the key. This feature of the developed algorithm proves its value, giving very favourable results with relatively low computational complexity.

In the future, expanded usage of music signature for musical arrangement classification will be proposed. First works are already done for Polish Christmas carols. Experimental results confirm that the shape of the signature is correlated with the arrangement of a carol. For these experiments the classical and jazz arrangements have been evaluated. It seems that music signature have much more applications than key-finding algorithm.

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