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Sound Radiation from a Roundabout

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Sound propagation from the vehicles moving on the city roundabout, with taking into account the wind is investigated. Solution of the problem for one moving sound source is found by means of the integral Fourier transforms extended over space variables and time. Inverse transforms are calculated approximately, using a stationary phase method and iterative technique. The solution for a general problem is obtained as a superposition of many partial solutions. The numerical analysis of noise characteristics is performed for the three-way Korfanty roundabout case in Łódź.

Keywords: roundabout, car sound radiation, stationary phase method, acoustic pressure, power flow density.

1. Introduction

The city roundabout becomes the most expanded project decision for traffic intersection of several roads (Brilon, 1988; Stuwe, 1992). Now the tendency appears to build the small roundabouts with radii 26–40 m for decreasing of vehicle average velocities and, respectively, for reduction of the car noise level [(1)]. At the same time, theoretical investigations of the noise emitted by these road elements are developed.

The structure of roadway noise, as it was shown in earlier author's publication (Piddubniak *et al.*, 2007), strongly depends on the transport facilities type, velocities of motion and frequencies of sound generation by single sources. In real situation many natural factors such as inhomogeneity, temperature and moisture

[(1)] Modern roundabouts: An informational guide – http://www.tfhrc.gov/safety/00068.htm of air, or the roadbed state, have influence on this structure formation. Recently, velocity and direction of wind were taken into consideration in the mathematical modeling of noise generated from the elements of city roadway as a part of multilane road (Piddubniak *et al.*, 2009b) and traffic cross-road (Piddubniak *et al.*, 2009a).

In this paper, the velocity and direction of wind are also taken into account in studying of the city roundabout noise generation. Blokhintzev (1981) was the first who described the sound radiation from a moving source in a moving medium. Wells, Han (1995) developed this theory for the case of propeller noise. A simple engineering model of noise propagation above a flat ground surface for stationary and moving point sources was proposed by Gołębiewski (2008). The model includes air absorption and ground effect in presence of the turbulence. Grigor'eva (1991), Chang, Ho Bong (1995) and Godin (1997) investigated the energy balance when an acoustic wave propagated through a moving medium. In addition, influence of thermal diffusion, particle collision and medium drift was analyzed (Chang, Ho Bong, 1995). Antes, Baaran (2001) used the boundary element method for determination of radiation, reflection and diffraction of sound around several independent moving bodies, with surfaces producing sound. In these and some other papers it was shown that problem of sound generation by the moving bodies for different moving media types should be considered.

The main goal of this paper is to obtain the analitycal algorithm for determination of acoustic pressure and noise power characteristics, caused by many vehicles moving along the three-way roundabout in presence of wind.

2. Mathematical formulation of problem and its analytical solution

Let us consider the problem of sound emission from motor vehicles moving on a roundabout with the radius $\xi = a$. The two types of vehicles are involved in investigations, namely, the automobiles (L) and trucks (C). Each of them moves with constant velocities, v_L and v_C. The intervals between discrete vehicles are, respectively, Δ_L and Δ_C. The roundabout is three-way, with one-directional, counter-clockwise direction of motion for automobiles and trucks on circles, where $\xi = \xi_{L1}$, $\xi = \xi_{L2}$ and $\xi = \xi_C$ respectively. The vehicles, as carriers of noise, are determined by the point sources with the intensities F_L and F_C located at the heights of $z = h_L$ and $z = h_C$ ($h_L < h_C$). We also assume that acoustical medium (air) moves parallel to the road plane with constant velocity vector $\mathbf{v}_w$ in direction θ_w to axis $0x$: $\mathbf{v}_w = (v_{wx}, v_{wy}, 0)$; $v_{wx} = v_w \cos\theta_w$, $v_{wy} = v_w \sin\theta_w$, $v_w = |\mathbf{v}_w|$.

The basic relations for linear acoustics of moving media are the Euler equation (equation of motion) and the mass balance equation (Brekhovskikh, Godin, 1989):

$$\rho \frac{\mathrm{d}\mathbf{v}(\mathbf{x},t)}{\mathrm{d}t} = -\nabla p(\mathbf{x},t) + \mathbf{F}(\mathbf{x},t), \tag{1}$$

$$\nabla \cdot \mathbf{v}(\mathbf{x},t) = -\frac{1}{\rho c^2}\frac{\mathrm{d}p(\mathbf{x},t)}{\mathrm{d}t}, \tag{2}$$

where $p(\mathbf{x},t)$ is the acoustic pressure, $\mathbf{v}(\mathbf{x},t)$ is the velocity vector of particles, ρ is the acoustic density, c is the sound velocity, t is the time, ∇ is the Hamilton operator, $\nabla = \nabla_\perp + \mathbf{i}_z\partial/\partial z$, where $\nabla_\perp = \mathbf{i}_x\partial/\partial x + \mathbf{i}_y\partial/\partial y$. The derivative with respect to time is determined as $\mathrm{d}/\mathrm{d}t \equiv \partial/\partial t + \mathbf{v}_w \cdot \nabla_\perp$. Also it must be noted that particle velocity vector is connected with particle displacement vector in moving acoustical medium:

$$\mathbf{v}(\mathbf{x},t) = \frac{\mathrm{d}\mathbf{u}(\mathbf{x},t)}{\mathrm{d}t} = \frac{\partial \mathbf{u}(\mathbf{x},t)}{\partial t} + \mathbf{v}_w \cdot \nabla_\perp \mathbf{u}(\mathbf{x},t). \tag{3}$$

In Eq. (1) $\mathbf{F}(\mathbf{x},t)$ is the complex mass load vector, determining the moving point source power, $\mathbf{F}(\mathbf{x},t) = \mathbf{F}_0 G(\boldsymbol{\xi},t)\delta(z-z_0)$, where $\mathbf{F}_0 = (F_{0x}, F_{0y}, F_{0z})$ is the vector of constant mass load real amplitudes, $\delta(z)$ is the Dirac's function, $G(\boldsymbol{\xi},t)$ is the function characterizing complex mass load distribution in plane $0xy$; $\mathbf{x} = \boldsymbol{\xi} + \mathbf{i}_z z$, $\boldsymbol{\xi} = \mathbf{i}_x x + \mathbf{i}_y y$ are radius-vectors in space $0xyz$ and in plane $0xy$, respectively.

On the wayside interface $z = 0$, between the acoustic and solid elastic half-spaces, the following conditions must be satisfied:

$$\sigma_z + p_{\mathrm{tot}} = 0, \qquad \tau_{xz} = 0, \qquad \tau_{yz} = 0, \qquad u_{sz} = u_{\mathrm{tot},z}, \tag{4}$$

where $p_{\mathrm{tot}} = p_{\mathrm{rad}} + p_{\mathrm{ref}}$, $u_{\mathrm{tot},z} = u_{\mathrm{rad},z} + u_{\mathrm{ref},z}$ are, respectively, the total acoustic pressure and the total normal component of particle displacement vector in acoustic medium, σ_z, τ_{xz}, τ_{yz} are the stress tensor components in solid, u_{sz} is the component of elastic displacement vector. The indices "rad" and "ref" denote the waves radiated by sound source and those reflected from a plane $z = 0$ in the acoustic medium. The source term in Eq. (1) for a reflected wave may be neglected.

For an elastic half-space $-\infty < z < 0$ we have the following wave equations (Achenbach, 1973):

$$\nabla^2\varphi - \frac{1}{c_L^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{5}$$

$$\nabla^2\boldsymbol{\psi} - \frac{1}{c_T^2}\frac{\partial^2\boldsymbol{\psi}}{\partial t^2} = 0, \tag{6}$$

$$\nabla \cdot \boldsymbol{\psi} = 0 \qquad \left(\nabla^2 \equiv \nabla \cdot \nabla\right), \tag{7}$$

where $c_L = \sqrt{(\lambda + 2\mu)/\rho_s}$ and $c_T = \sqrt{\mu/\rho_s}$ are the velocities of longitudinal and transversal waves, λ and μ are the elastic Lamé parameters, ρ_s is the material density of solid. The relations between the elastic stresses tensor $\boldsymbol{\sigma}(\mathbf{x},t)$, displacement vector $\mathbf{u}_s(\mathbf{x},t)$ and wave potentials φ and $\boldsymbol{\psi}$ are well-known in the literature (see, e.g. Achenbach, 1973):

$$\sigma_z(\mathbf{x},t) = \lambda\nabla^2\varphi(\mathbf{x},t) + 2\mu\left\{\frac{\partial^2\varphi(\mathbf{x},t)}{\partial z^2} + \frac{\partial}{\partial z}\left[\frac{\partial\psi_y(\mathbf{x},t)}{\partial x} - \frac{\partial\psi_x(\mathbf{x},t)}{\partial y}\right]\right\}, \tag{8}$$

$$\tau_{xz}(\mathbf{x},t) = \mu\left[2\frac{\partial^2\varphi(\mathbf{x},t)}{\partial x\partial z} - 2\frac{\partial^2\psi_x(\mathbf{x},t)}{\partial x\partial y} + \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right)\psi_y(\mathbf{x},t)\right], \tag{9}$$

$$\tau_{yz}(\mathbf{x},t) = \mu\left[2\frac{\partial^2\varphi(\mathbf{x},t)}{\partial y\partial z} + \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi_x(\mathbf{x},t) + 2\frac{\partial^2\psi_y(\mathbf{x},t)}{\partial x\partial y}\right], \tag{10}$$

$$u_{sz}(\mathbf{x},t) = \frac{\partial\varphi(\mathbf{x},t)}{\partial z} + \frac{\partial\psi_y(\mathbf{x},t)}{\partial x} - \frac{\partial\psi_x(\mathbf{x},t)}{\partial y}. \tag{11}$$

It should be noted that in these relations, the condition (7) is taken into account.

To solve the problem we apply the complex integral Fourier transforms over space variables x, y and time t (Felsen, Markuvitz, 1973):

$$f^F(\boldsymbol{\kappa},z,\omega) = \iiint\limits_{-\infty}^{\infty} f(\mathbf{x},t)e^{i(\omega t-\boldsymbol{\kappa}\cdot\xi)}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}t, \tag{12}$$

$$f(\mathbf{x},t) = \frac{1}{(2\pi)^3}\iiint\limits_{-\infty}^{\infty} f(\mathbf{x},t)e^{-i(\omega t-\boldsymbol{\kappa}\cdot\boldsymbol{\xi})}\,\mathrm{d}\boldsymbol{\kappa}\,\mathrm{d}\omega, \tag{13}$$

where $\boldsymbol{\kappa} = (k_x,k_y)$, $\mathrm{d}\boldsymbol{\kappa} = \mathrm{d}k_x\,\mathrm{d}k_y$, $\mathrm{d}\boldsymbol{\xi} = \mathrm{d}x\,\mathrm{d}y$.

Then in the Fourier transforms, the differential equations for the acoustical pressure in the radiation and reflection sound fields are expressed as:

$$\left(\frac{\partial^2}{\partial z^2} + k_{zw}^2\right)p_{\mathrm{rad}}^F(\boldsymbol{\kappa},z,\omega) = \mathbf{F}_0\cdot\left(i\boldsymbol{\kappa} + \mathbf{i}_z\frac{\partial}{\partial z}\right)G^F(\boldsymbol{\kappa},\omega)\delta(z-z_0), \tag{14}$$

$$\left(\frac{\partial^2}{\partial z^2} + k_{zw}^2\right)p_{\mathrm{ref}}^F(\boldsymbol{\kappa},z,\omega) = 0, \tag{15}$$

for $0 < z < \infty$, where

$$\begin{aligned} k_{zw} &= \sqrt{k_w^2-\kappa^2}, & \mathrm{Im}k_{zw} &\geq 0,\\ k_w &= k - \mathbf{M}_w\cdot\boldsymbol{\kappa}, & k &= \omega/c,\\ \mathbf{M}_w &= \mathbf{v}_w/c, & \kappa &= |\boldsymbol{\kappa}|. \end{aligned} \tag{16}$$

Here k is the wave number in the acoustical medium, $\mathbf{M}_w$ is the Mach number vector for the wind.

For the elastic wave potentials $\varphi(\mathbf{x},t)$ and $\boldsymbol{\psi}(\mathbf{x},t)$, the differential equations of motion (5) and (6) in the Fourier transforms take the forms:

$$\left(\frac{\partial^2}{\partial z^2}+k_{zL}^2\right)\varphi^F(\boldsymbol{\kappa},z,\omega)=0, \tag{17}$$

$$\left(\frac{\partial^2}{\partial z^2}+k_{zT}\right)\boldsymbol{\psi}^F(\boldsymbol{\kappa},z,\omega)=0, \tag{18}$$

for $-\infty<z<0$, where

$$k_{zA}=\sqrt{k_A^2-\kappa^2},\qquad \mathrm{Im}k_{zA}\geq 0,\qquad k_A=\omega/c_A\qquad (A=L,T). \tag{19}$$

Here k_L and k_T are the wave numbers for an elastic material.

The solutions of Eqs. (14) and (15) are obtained with the assumption that $p^F_{\text{rad}}(\boldsymbol{\kappa},z,\omega)$ and $p^F_{\text{ref}}(\boldsymbol{\kappa},z,\omega)$ are bounded as $z\to\infty$:

$$p^F_{\text{rad}}(\boldsymbol{\kappa},z,\omega)=i\mathbf{F}_0\cdot[\boldsymbol{\kappa}+\mathbf{i}_z k_{zw}\mathrm{sgn}(z-z_0)]G^F(\boldsymbol{\kappa},\omega)g(\boldsymbol{\kappa},z,\omega), \tag{20}$$

$$p^F_{\text{ref}}(\boldsymbol{\kappa},z,\omega)=p^F_{\text{ref}}(\boldsymbol{\kappa},\omega)e^{ik_{zw}z}, \tag{21}$$

for $0\leq z<\infty$, where

$$g(\boldsymbol{\kappa},z,\omega)=\frac{1}{2ik_{zw}}e^{ik_{zw}|z-z_0|}. \tag{22}$$

Similarly, the solutions of Eqs. (17) and (18), bounded for $z\to-\infty$, are obtained in the form

$$\varphi^F(\boldsymbol{\kappa},z,\omega)=\varphi^F(\boldsymbol{\kappa},\omega)e^{-ik_{zL}z}, \tag{23}$$

$$\boldsymbol{\psi}^F(\boldsymbol{\kappa},z,\omega)=\boldsymbol{\psi}^F(\boldsymbol{\kappa},\omega)e^{-ik_{zT}z}, \tag{24}$$

for $-\infty<z\leq 0$.

The functions $p^F_{\text{ref}}(\boldsymbol{\kappa},\omega)$, $\varphi^F(\boldsymbol{\kappa},\omega)$ and $\boldsymbol{\psi}^F(\boldsymbol{\kappa},\omega)$ are unknown and must be found.

Substituting the solutions (20)–(24) in the Fourier transformed Eqs. (1), (3), (8)–(11), we obtain the Fourier transforms for the particle displacement and velocity vectors in an acoustical medium and for the components of stress tensor and displacement vector in an elastic solid:

$$\mathbf{u}^F_{\text{rad}}(\boldsymbol{\kappa},z,\omega)=-\frac{1}{\rho c^2k_w^2}\mathbf{F}_0\cdot\{[\boldsymbol{\kappa}+\mathbf{i}_z k_{zw}\mathrm{sgn}(z-z_0)][\boldsymbol{\kappa}+\mathbf{i}_z k_{zw}\mathrm{sgn}(z-z_0)]$$
$$\times\, g(\boldsymbol{\kappa},z,\omega)+(\mathbf{i}_x\mathbf{i}_x+\mathbf{i}_y\mathbf{i}_y)\delta(z-z_0)\}G^F(\boldsymbol{\kappa},\omega), \tag{25}$$

$$\mathbf{u}^F_{\text{ref}}(\boldsymbol{\kappa},z,\omega)=\frac{i}{\rho c^2k_w^2}p^F_{\text{ref}}(\boldsymbol{\kappa},\omega)(\boldsymbol{\kappa}+\mathbf{i}_z k_{zw})e^{ik_{zw}z}, \tag{26}$$

$$\mathbf{v}_j^F(\boldsymbol{\kappa}, z, \omega) = -ick_w \mathbf{u}_j^F(\boldsymbol{\kappa}, z, \omega), \tag{27}$$

for $0 \le z < \infty$; $j =$ rad, ref,

$$\sigma_z^F(\boldsymbol{\kappa}, z, \omega) = -(\lambda k_L^2 + 2\mu k_{zL}^2)\varphi^F(\boldsymbol{\kappa}, \omega)e^{-ik_{zL}z} + 2\mu k_{zT}[k_x \psi_y^F(\boldsymbol{\kappa}, \omega) - k_y \psi_x^F(\boldsymbol{\kappa}, \omega)]e^{-ik_{zT}z}, \tag{28}$$

$$\tau_{xz}^F(\boldsymbol{\kappa}, z, \omega) = \mu\{2k_x k_{zT}\varphi^F(\boldsymbol{\kappa}, \omega)e^{-ik_{zL}z} + [2k_x k_y \psi_x^F(\boldsymbol{\kappa}, \omega) + (-k_x^2 + k_y^2 + k_{zT}^2)\psi_y^F(\kappa, \omega)]e^{-ik_{zT}z}\}, \tag{29}$$

$$\tau_{yz}^F(\boldsymbol{\kappa}, z, \omega) = \mu\{2k_y k_{zT}\varphi^F(\boldsymbol{\kappa}, \omega)e^{-ik_{zL}z} + [(-k_x^2 + k_y^2 - k_{zT}^2)\psi_x^F(\boldsymbol{\kappa}, \omega) - 2k_x k_y \psi_y^F(\boldsymbol{\kappa}, \omega)]e^{-ik_{zT}z}\}, \tag{30}$$

$$u_{sz}^F(\boldsymbol{\kappa}, z, \omega) = -ik_{zL}\varphi^F(\boldsymbol{\kappa}, \omega)e^{-ik_{zL}z} + i[k_x \psi_x^F(\boldsymbol{\kappa}, \omega) - k_y \psi_y^F(\boldsymbol{\kappa}, \omega)]e^{-ik_{zT}z}, \tag{31}$$

for $-\infty < z \le 0$.

Satisfying the boundary conditions (40) in the Fourier transforms we obtain the following system of four algebraic equations for the unknown functions $p_{\text{ref}}^F(\boldsymbol{\kappa}, \omega)$, $\varphi^F(\boldsymbol{\kappa}, \omega)$, $\psi_x^F(\boldsymbol{\kappa}, \omega)$ and $\psi_y^F(\boldsymbol{\kappa}, \omega)$:

$$\begin{pmatrix} -\lambda k_L^2 - 2\mu k_{zL}^2 & 2\mu k_x k_{zT} & 2\mu k_y k_{zT} & 1 \\ 2k_x k_{zT} & 2k_x k_y & -k_x^2 + k_y^2 + k_{zT}^2 & 0 \\ 2k_y k_{zT} & -k_x^2 + k_y^2 - k_{zT}^2 & 2k_x k_y & 0 \\ -k_{zL} & -k_y & k_x & -\dfrac{k_{zw}}{\rho c^2 k_w^2} \end{pmatrix} \begin{pmatrix} \varphi^F(\boldsymbol{\kappa}, \omega) \\ \psi_x^F(\boldsymbol{\kappa}, \omega) \\ \psi_y^F(\boldsymbol{\kappa}, \omega) \\ p_{\text{ref}}^F(\boldsymbol{\kappa}, \omega) \end{pmatrix} = -p_{\text{rad}}^F(\boldsymbol{\kappa}, \omega) \begin{pmatrix} 1 \\ 0 \\ 0 \\ \dfrac{k_{zw}}{\rho c^2 k_w^2} \end{pmatrix}, \tag{32}$$

where

$$p_{\text{rad}}^F(\boldsymbol{\kappa}, \omega) = \mathbf{F}_0 \cdot (\boldsymbol{\kappa} - \mathbf{i}_z k_{zw})G^F(\boldsymbol{\kappa}, \omega)\frac{e^{-ik_{zw}z_0}}{2k_{zw}}. \tag{33}$$

Solving this system of equations for the function $p_{\text{ref}}^F(\boldsymbol{\kappa}, \omega)$, we find

$$p_{\text{ref}}^F(\boldsymbol{\kappa}, \omega) = R(\boldsymbol{\kappa}, \omega)p_{\text{rad}}^F(\boldsymbol{\kappa}, \omega), \tag{34}$$

where $R(\boldsymbol{\kappa}, \omega)$ is the spectral coefficient of sound wave reflection from the surface of an elastic half-space:

$$R(\boldsymbol{\kappa},\omega) = W^{-}(\boldsymbol{\kappa},\omega)/W^{+}(\boldsymbol{\kappa},\omega) \tag{35}$$

and

$$\begin{aligned} W^{\pm}(\boldsymbol{\kappa},\omega) &= (k_T^2 - 2\kappa^2)^2 + 4\kappa^2 k_{zL}k_{zT} \pm N_s k_T^2 k_w^2 (c/c_T)^2 k_{zL}/k_{zw}, \\ N_s &= \rho/\rho_s. \end{aligned} \tag{36}$$

Using the inverse integral Fourier transforms (14) and their properties, we obtain

$$p_{\text{rad}}(\mathbf{x},t) = \mathbf{F}_0 \cdot \nabla P_{\text{rad}}(\mathbf{x},t), \tag{37}$$

$$p_{\text{ref}}(\mathbf{x},t) = \mathbf{F}_0 \cdot \widetilde{\nabla} P_{\text{ref}}(\mathbf{x},t), \tag{38}$$

where $\widetilde{\nabla} = \nabla_{\perp} - \mathbf{i}\partial/\partial z$ and

$$P_{\text{rad}}(\mathbf{x},t) = -\frac{1}{8\pi^2}\iiint\limits_{-\infty}^{\infty} G(\boldsymbol{\xi}-\boldsymbol{\xi}', t-t')\varPhi_{\text{rad}}(\boldsymbol{\xi}', |z-z_0|)\,\mathrm{d}\boldsymbol{\xi}'\,\mathrm{d}t', \tag{39}$$

$$P_{\text{ref}}(\mathbf{x},t) = -\frac{1}{8\pi^2}\iiint\limits_{-\infty}^{\infty} G(\boldsymbol{\xi}-\boldsymbol{\xi}', t-t')\varPhi_{\text{ref}}(\boldsymbol{\xi}', z+z_0)\,\mathrm{d}\boldsymbol{\xi}'\,\mathrm{d}t', \tag{40}$$

$$\varPhi_j(\mathbf{x},t) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \varPhi_j^F(\mathbf{x},\omega)e^{-i\omega t}\,\mathrm{d}\omega \qquad (j = \text{rad},\ \text{ref}), \tag{41}$$

$$\varPhi_{\text{rad}}^F(\mathbf{x},\omega) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} e^{i(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}+k_{zw}z)}\frac{\mathrm{d}\boldsymbol{\kappa}}{k_{zw}}, \tag{42}$$

$$\varPhi_{\text{ref}}^F(\mathbf{x},\omega) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} R(\boldsymbol{\kappa},\omega)e^{i(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}+k_{zw}z)}\frac{\mathrm{d}\boldsymbol{\kappa}}{k_{zw}}. \tag{43}$$

In the Eqs. (39) and (40), $\boldsymbol{\xi}' = (x', y'),\ \mathrm{d}\boldsymbol{\xi}' = \mathrm{d}x'\,\mathrm{d}y'$.

Note that the Eq. (42) may be expressed by the standard integral representation for the spherical wave as a superposition of the plane waves (Brekhovskikh, Godin, 1989), by changing the variables of integration. This means that integrals (42) are calculated exactly:

$$\varPhi_{\text{rad}}^F(\mathbf{x},\omega) = -\frac{2\pi i}{R_w(\mathbf{x})}e^{i(k/\alpha)[R_w(\mathbf{x})-\mathbf{M}_w\cdot\boldsymbol{\xi}]}, \tag{44}$$

where

$$
\begin{aligned}
R_w(\mathbf{x}) &= \sqrt{\alpha_y x^2 + \alpha_x y^2 + \beta x y + \alpha z^2},\\
\alpha_x &= 1 - M_{wx}^2,\\
\alpha_y &= 1 - M_{wy}^2,\\
\alpha &= 1 - M_w^2,\\
\beta &= 2 M_{wx} M_{wy}.
\end{aligned}
\tag{45}
$$

For calculation of integrals (43) we apply the stationary phase approximation (Felsen, Markuvitz, 1973), without taking into account the small contribution of the Rayleigh surface wave (Überall, 1973). After some operations we finally obtain the main term of asymptotic

$$
\Phi_{\text{ref}}^F(\mathbf{x},\omega) = -\frac{2\pi i R(\mathbf{x})}{R_w(\mathbf{x})} e^{i(k/\alpha)[R_w(\mathbf{x}) - \mathbf{M}_w \cdot \boldsymbol{\xi}]},
\tag{46}
$$

where

$$
\begin{aligned}
R(\mathbf{x}) &= V^-(\mathbf{x})/V^+(\mathbf{x}),\\
V^{\pm}(\mathbf{x}) &= [S_T^2 - 2S^2(\mathbf{x})]^2 + 4S^2(\mathbf{x}) S_{zL}(\mathbf{x}) S_{zT}(\mathbf{x})\\
&\quad \pm N_s S_T^2 (c/c_T)^2 [1 - \mathbf{M}_w \cdot \boldsymbol{\xi}/R_w(\mathbf{x})]^2 S_{zL}(\mathbf{x})/S_z(\mathbf{x}),\\
S(\mathbf{x}) &= |\mathbf{S}(\mathbf{x})|,\\
\mathbf{S}(\mathbf{x}) &= \mathbf{r}(\boldsymbol{\xi})/R_w(\mathbf{x}) - \mathbf{M}_w,\\
\mathbf{r}(\boldsymbol{\xi}) &= (\alpha_y x + \beta y,\ \beta x + \alpha_x y),\\
S_{zA}(\mathbf{x}) &= \sqrt{S_A^2 - S^2(\mathbf{x})},\\
S_A &= \alpha c/c_A \quad (A = L, T),\\
S_z(\mathbf{x}) &= \alpha z / R_w(\mathbf{x}).
\end{aligned}
\tag{47}
$$

Here the new form of the reflection coefficient $R(\mathbf{x})$ is caused by contributions of the stationary phase point values. In fact, $R(\mathbf{x})$ is the function of spherical angle coordinates and physical-mechanical parameters of the acoustic and elastic media.

Substituting the expressions (44) and (46) into Eqs. (41), we obtain the potential functions for acoustic medium in the following form:

$$
\Phi_{\text{rad}}(\mathbf{x},t) = -\frac{2\pi i}{R_w(\mathbf{x})} \delta \left\{ t - \frac{1}{c\alpha} [R_w(\mathbf{x}) - \mathbf{M}_w \cdot \boldsymbol{\xi}] \right\},
\tag{48}
$$

$$\Phi_{\text{ref}}(\mathbf{x},t) = -\frac{2\pi i R(\mathbf{x})}{R_w(\mathbf{x})}\delta\left\{t - \frac{1}{c\alpha}[R_w(\mathbf{x}) - \mathbf{M}_w \cdot \boldsymbol{\xi}]\right\}. \tag{49}$$

Similarly, by substituting these last functions into Eqs. (39), (40) and using the Dirac function property (Kecs, Teodorescu, 1978)

$$\int_0^\infty \delta(t-t')f(t')\,\mathrm{d}t' = f(t) \qquad (0 < t < \infty), \tag{50}$$

we obtain the following results:

$$P_{\text{rad}}(\mathbf{x},t) = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G\left\{\boldsymbol{\xi}-\boldsymbol{\xi}', t - \frac{1}{c\alpha}\left[R_w(\boldsymbol{\xi}', z-z_0) - \mathbf{M}_w\cdot\boldsymbol{\xi}'\right]\right\} \times \frac{\mathrm{d}\boldsymbol{\xi}'}{R_w(\boldsymbol{\xi}', z-z_0)}, \tag{51}$$

$$P_{\text{ref}}(\mathbf{x},t) = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G\left\{\boldsymbol{\xi}-\boldsymbol{\xi}', t - \frac{1}{c\alpha}\left[R_w(\boldsymbol{\xi}', z+z_0) - \mathbf{M}_w\cdot\boldsymbol{\xi}'\right]\right\} \times \frac{R(\boldsymbol{\xi}', z+z_0)\,\mathrm{d}\boldsymbol{\xi}'}{R_w(\boldsymbol{\xi}', z+z_0)}. \tag{52}$$

Consider now the motion of a point source along circle $\xi = \xi_0$, $0 \le \theta \le 2\pi$. Then function $G(\boldsymbol{\xi}, t)$ may be represented as

$$G(\boldsymbol{\xi},t) = \delta(\xi-\xi_0)\delta(\xi_0\theta - v_0 t)e^{-i\Omega_0 t} \qquad (0 \le \theta \le 2\pi), \tag{53}$$

where v_0 is the velocity of source motion in positive direction of angular variable θ (for positive time t) at the height of $z = z_0$, Ω_0 is the circular frequency of sound radiation.

Calculating the integrals in Eqs. (51) and (52) we use the polar coordinates ξ, θ and ξ', θ': $x = \xi\cos\theta$, $y = \xi\sin\theta$, $x' = \xi'\cos\theta'$, $y' = \xi'\sin\theta'$ ($0 \le \xi < \infty$, $0 \le \theta \le 2\pi$, $0 \le \xi' < \infty$, $0 \le \theta' \le 2\pi$). Then, using the Dirac's function properties (50) and $\delta(\xi_0\theta - v_0 t) = \xi_0^{-1}\delta\left(\theta - v_0 t/\xi_0\right)$ (Kecs, Teodorescu, 1978) in the Eqs. (51) and (52), we obtain:

$$P_{\text{rad}}(\mathbf{x},t) = -\frac{1}{4\pi}e^{-i\Omega_0 t}\int_0^{2\pi}\exp\left\{iK_{w0}\left[R_w\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z-z_0\right) - \mathbf{M}_w\cdot\left(\boldsymbol{\xi}-\boldsymbol{\xi}'\right)\right]\right\} \times \delta\left\langle\theta' - \frac{v_0}{\xi_0}\left\{t - \frac{1}{c\alpha}\left[R_w\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z-z_0\right) - \mathbf{M}_w\cdot\left(\boldsymbol{\xi}-\boldsymbol{\xi}'\right)\right]\right\}\right\rangle \times \left.\frac{\mathrm{d}\theta'}{R_w\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z-z_0\right)}\right|_{\xi'=\xi_0}, \tag{54}$$

$$P_{\mathrm{ref}}(\mathbf{x},t) = -\frac{1}{4\pi}e^{-i\Omega_0 t}\int\limits_0^{2\pi} \exp\left\{iK_{w0}\left[R_w\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z+z_0\right) - \mathbf{M}_w\cdot\left(\boldsymbol{\xi}-\boldsymbol{\xi}'\right)\right]\right\}$$
$$\times\,\delta\left\langle\theta' - \frac{v_0}{\xi_0}\left\{t - \frac{1}{c\alpha}\left[R_w\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z+z_0\right) - \mathbf{M}_w\cdot\left(\boldsymbol{\xi}-\boldsymbol{\xi}'\right)\right]\right\}\right\rangle$$
$$\times\,\frac{R\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z+z_0\right)}{R_w\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z+z_0\right)}\,\mathrm{d}\theta'\Bigg|_{\xi'=\xi_0}, \tag{55}$$

where $K_{w0} = K_0/\alpha$, $K_0 = \Omega_0/c$.

Here the following property of δ-function must be used (Kecs, Teodorescu, 1978):

$$\delta[f(\theta')] = \sum_j \frac{\delta(\theta'-\theta_j)}{|f'(\theta_j)|}, \tag{56}$$

where θ_j $(j = 1,2,3,...)$ are the zeros of function $f(\theta')$.

In our case,

$$f(\theta') = \theta' - \frac{v_0}{\xi_0}\left\{t - \frac{1}{c\alpha}\left[R_w\left(\boldsymbol{\xi}-\boldsymbol{\xi}', z\right) - \mathbf{M}_w\cdot\left(\boldsymbol{\xi}-\boldsymbol{\xi}'\right)\right]_{\xi'=\xi_0}\right\}, \tag{57}$$

$$\begin{aligned}
f'(\theta_j) &= 1 - M_{w0}\left\langle M_w\sin(\theta_j-\theta_w) + \frac{1}{R_w(\boldsymbol{\xi}-\boldsymbol{\xi}_{0j}, z)}\,\{\xi\sin(\theta-\theta_j)\right.\\
&\qquad \left. + M_w^2\cos(\theta_j-\theta_w)\left[\xi\sin(\theta-\theta_w) - \xi_0\sin(\theta_j-\theta_w)\right]\}\right\rangle,\\
\boldsymbol{\xi}_{0j} &= \xi_0(\mathbf{i}_x\cos\theta_j + \mathbf{i}_y\sin\theta_j) \qquad (j=1,2,3,...),\\
M_{w0} &= M_0/\alpha,\\
M_0 &= v_0/c,
\end{aligned} \tag{58}$$

where M_0 is the Mach number for a moving source.

The transcendental equation $f(\theta') = 0$ is solved approximately under the conditions: $R_w(\boldsymbol{\xi}-\boldsymbol{\xi}', z) > 0$ for $\xi' = \xi_0$ and $0 \le \theta_j \le 2\pi$ $(j = 1,2,3,...)$. The roots θ_j are obtained by the method of successive approximations, assuming the Mach numbers M_w and M_{w0} to be small values. Then from this procedure, the following iterative algorithm is found:

$$\theta^{(n)} = \frac{v_0 t}{\xi_0} - \frac{M_{w0}}{\xi_0}\{R_w^{(n-1)}(\mathbf{x},t) - M_w[\xi\cos(\theta-\theta_w) - \xi_0\cos(\theta^{(n-1)}-\theta_w)]\}$$
$$(n = 1,2,3,...), \tag{59}$$

where $R_w^{(n-1)}(\mathbf{x},t) = R_w(\boldsymbol{\xi}-\boldsymbol{\xi}', z)$ for $\xi' = \xi_0$, $\theta' = \theta^{(n)}(\mathbf{x},t)$, $0 \le \theta^{(n)}(\mathbf{x},t) \le 2\pi$ $(n = 0,1,2,...)$ and $\theta^{(0)} = v_0 t/\xi_0$, i.e. $0 \le t \le 2\pi\xi_0/v_0$. In particular, for $j = 1$ we obtain $\theta_1 = \theta^{(n)}(\mathbf{x},t)$ $(n = 0,1,2,...)$.

By eliminating the first-order approximation ($n = 1$) in Eq. (59), we obtain the far-field asymptotic expressions for integrals (54) and (55):

$$P_{\mathrm{rad}}(\mathbf{x},t) = \frac{-1}{4\pi R_{w0}(\boldsymbol{\xi}-\boldsymbol{\xi}_0^-, z-z_0)} \times \exp\left\{iK_{w0}\left[R_w(\boldsymbol{\xi}-\boldsymbol{\xi}_0^-, z-z_0) - \mathbf{M}_w\cdot(\boldsymbol{\xi}-\boldsymbol{\xi}_0^-)\right]\right\}, \quad (60)$$

$$P_{\mathrm{ref}}(\mathbf{x},t) = -\frac{R(\boldsymbol{\xi}-\boldsymbol{\xi}_0^+, z+z_0)}{4\pi R_{w0}(\boldsymbol{\xi}-\boldsymbol{\xi}_0^+, z+z_0)} \times \exp\left\{iK_{w0}\left[R_w(\boldsymbol{\xi}-\boldsymbol{\xi}_0^+, z+z_0) - \mathbf{M}_w\cdot(\boldsymbol{\xi}-\boldsymbol{\xi}_0^+)\right]\right\}, \quad (61)$$

where

$$\begin{aligned} R_{w0}[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0] = {} & R_w[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0] \\ & \times\{1 + M_{w0}\xi_0^{-1}[x_0^\pm(\mathbf{x},t)M_{wy} - y_0^\pm(\mathbf{x},t)M_{wx}]\} \\ & + M_{w0}\xi_0^{-1}\{y_0^\pm(\mathbf{x},t)r_x[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t)] \\ & - x_0^\pm(\mathbf{x},t)r_y[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t)]\}, \end{aligned}$$

$$\begin{aligned} \boldsymbol{\xi}_0^\pm(\mathbf{x},t) &= (x_0^\pm(\mathbf{x},t), y_0^\pm(\mathbf{x},t)), \\ x_0^\pm(\mathbf{x},t) &= \xi_0\cos[\theta_0^\pm(\mathbf{x},t)], \\ y_0^\pm(\mathbf{x},t) &= \xi_0\sin[\theta_0^\pm(\mathbf{x},t)], \\ \theta_0^\pm(\mathbf{x},t) &= \theta_0(t) - M_{w0}\xi_0^{-1}\{R_w[\boldsymbol{\xi}-\boldsymbol{\xi}_0(t), z\pm z_0] \\ &\qquad - \mathbf{M}_w\cdot[\boldsymbol{\xi}-\boldsymbol{\xi}_0(t)]\}, \\ \boldsymbol{\xi}_0(t) &= (x_0(t), y_0(t)), \\ x_0(t) &= \xi_0\cos[\theta_0(t)], \\ y_0(t) &= \xi_0\sin[\theta_0(t)], \\ \theta_0(t) &= v_0 t\xi_0^{-1}, \\ & 0 \le \theta_0(t) \le 2\pi, \\ & 0 \le \theta_0^\pm(\mathbf{x},t) \le 2\pi. \end{aligned} \quad (62)$$

Now by substitution of the functions $P_{\mathrm{rad}}(\mathbf{x},t)$ and $P_{\mathrm{ref}}(\mathbf{x},t)$ in Eqs. (37) and (38), the solutions for acoustical pressure in waves, radiated by a single moving source and reflected from the surface of elastic half-space, are obtained in forms:

$$p_{\mathrm{rad}}(\mathbf{x},t,\Omega_0) = K_0\mathbf{F}_0\cdot\mathbf{B}^-(\mathbf{x},t)P_{\mathrm{rad}}(\mathbf{x},t)e^{-i\Omega_0 t}, \tag{63}$$

$$p_{\mathrm{ref}}(\mathbf{x},t,\Omega_0) = K_0\mathbf{F}_0\cdot\mathbf{B}^+(\mathbf{x},t)P_{\mathrm{ref}}(\mathbf{x},t)e^{-i\Omega_0 t}, \tag{64}$$

where

$$\mathbf{B}^\pm(\mathbf{x},t) = \frac{i}{\alpha}\left\{\mathbf{I}[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0] - \frac{\mathbf{J}[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0]}{iK_{w0}R_{w0}[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0]}\right\},$$

$$\begin{aligned}
&\mathbf{I}[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0] \\
&\quad= \frac{\mathbf{S}^\pm[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0]}{1+M_{w0}\xi_0^{-1}\{y_0^\pm(\mathbf{x},t)S_x[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t)] - x_0^\pm(\mathbf{x},t)S_y[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t)]\}}, \\
&\mathbf{J}[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0] = \mathbf{M}_w + M_{w0}\xi_0^{-1}[\mathbf{i}_x y_0^\pm(\mathbf{x},t) - \mathbf{i}_y x_0^\pm(\mathbf{x},t)] \\
&\qquad + \mathbf{I}[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0]\left\langle 1 - 2M_{w0}\xi_0^{-1}[M_{wx}y_0^\pm(\mathbf{x},t) - M_{wy}x_0^\pm(\mathbf{x},t)]\right. \\
&\qquad\qquad - (M_{w0}\xi_0^{-1})^2\{\alpha\xi_0^2 + x_0^\pm(\mathbf{x},t)S_x[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t)] \\
&\qquad\qquad \left. + y_0^\pm(\mathbf{x},t)S_y[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t)])R_w[\boldsymbol{\xi}-\boldsymbol{\xi}_0^\pm(\mathbf{x},t), z\pm z_0]\}\right\rangle,
\end{aligned} \tag{65}$$

$$\mathbf{S}^\pm(\mathbf{x}) = \mathbf{S}(\mathbf{x}) \mp \mathbf{i}_z S_z(\mathbf{x}).$$

In the case of a single sound source localized at the initial moment $t = 0$ in point $\theta = \theta_j \neq 0$, θ must be replaced by $\theta - \theta_j$ in the above formulae. When such sources are more than one and their distribution is uniform along the circle $\xi = \xi_0$, then the interval from point to point is $\Delta\theta = \theta_{j+1} - \theta_j$, i.e. $\theta_{j+1} = \theta_j + \Delta\theta$. Choosing $\Delta\theta = 2\pi/N$, we have N sound sources placed along the circle with radius $\xi = \xi_0$. Then $\theta_j = j\Delta\theta$ ($j = 0, 1, 2, ...N-1$). Therefore, the function $G(\boldsymbol{\xi}, t)$ (53) may be described in the form

$$G(\boldsymbol{\xi},t) = \delta(\xi-\xi_0)\sum_{j=0}^{N-1}\delta[\xi_0(\theta - j\Delta\theta) - v_0 t]e^{-i\Omega_0 t}, \tag{66}$$

for $0 \le \theta \le 2\pi$.

If each source makes $2M+1$ rotations in positive direction of angle θ for positive and negative values of time, then coordinate θ must be replaced with $\theta + 2\pi m$ ($m = 0, \pm1, \pm2, ..., \pm M$):

$$G(\boldsymbol{\xi},t) = \delta(\xi-\xi_0)\sum_{m=-M}^{M}\sum_{j=0}^{N-1}\delta[\xi_0(\theta - j\Delta\theta + 2\pi m) - v_0 t]e^{-i\Omega_0 t}, \tag{67}$$

for $0 \le \theta \le 2\pi$.

This leads to replacement of $\theta \to \theta - j\Delta\theta + 2\pi m$ or $t \to t + (\xi_0/v_0)(j\Delta\theta - 2\pi m)$ in Eqs. (63)–(65) and to summation of the j-th and m-th terms, similarly to Eq. (67).

If the circles, along which the sound sources move, are more than one, then we perform the replacement of $\xi_0 \to \xi_n$, $\Delta\theta \to \Delta\theta_n$, $v_0 \to v_n$, $\Omega_0 \to \Omega_n$, and summation of the final n-th expressions. Furthermore, the amplitudes of vibrations of each source, i.e. their vectors $\mathbf{F}_0$ may be taken into attention. In the case of a three-way roundabout ($n = 1, 2, 3$), the final formula for the acoustical pressure becomes:

$$p_{\text{tot}}(\mathbf{x}, t) = \sum_{n=1}^{3} \sum_{A=L,C} \sum_{m=-M_{An}}^{M_{An}} \sum_{j=0}^{N_{An}} p_{\text{tot},Aj}(\xi - \xi_n, \theta - j\Delta\theta_{An} + 2\pi m, z, t, h_A, \Omega_{An}) \times [H(\theta) - H(\theta - j\Delta\theta_{An} + 2\pi m - v_A t/\xi_{An})], \qquad (68)$$

where the type of vehicle was taken into account; $H(t)$ is the Heaviside's step function. Subscript at p_{tot} under the summation sign indicates the amplitude values of waves radiated by sources $\mathbf{F}_{Aj}$.

3. Numerical analysis of acoustic characteristics

The calculations of acoustic pressure are performed by means of Eqs. (63)–(65), (68). We admit that components of the load vectors $\mathbf{F}_{Aj}$ are the same in all directions for all transport facilities, i.e. $F_{Ajx} = F_{Ajy} = F_{Ajz} = F_A$ $(A = L, C)$ for each j. The product $F_A K_A$, appearing in the formulae, may be calculated from relation

$$F_A K_A = 8\pi \cdot 10^{0.05(I_A - 100)} \quad (A = L, C), \qquad (69)$$

where I_A is the average intensity measured at the one-metre distance from individual A-type sound source $(A = L, C)$. We put in this expression $I_L = 75$ dB and $I_C = 85$ dB (Engel, 2001). The vehicles move in air, with density $\rho = 1.293$ kg/m^3 and sound velocity $c = 331$ m/s (Artobolevski, 1976), along the road coated by asphalt, with material density $\rho_s = 2000$ kg/m^3 and velocities of longitudinal and transversal waves $c_L = 3468$ m/c and $c_T = 1667$ m/c (Kettil *et al.*, 2005). The frequencies of vibrations are $\Omega_L = 300$ Hz and $\Omega_C = 250$ Hz (Engel, 2001), the source heights are $h_L = 1$ m and $h_C = 2$ m. We consider the roundabout with two inner ways, $\xi_{L1} = 25$ m, $\xi_{L2} = 28$ m, for automobiles and one outer way, $\xi_C = 32$ m, for trucks moving with the same velocity $v_{L1} = v_{L2} = v_C = 30$ km/h (i.e. $n = 1, 2$ for $A = L$ and $n = 3$ for $A = C$). This simulates the situation on the Korfanty roundabout, at an uncontrolled intersection of Textile Workers Avenue and Limanowski Street in Łódź. From observations made between 10–11 AM, 8.XI.2008, were calculated

5040 automobiles and 600 trucks moving on this roundabout[(2)]. The periods between car appearances at a fixed point in certain circle are, respectively, such as $\Delta_{tL1} = 3600/N_{L1}$, $\Delta_{tL2} = 3600/N_{L2}$, $\Delta_{tC} = 3600/N_C$ (in seconds), where $N_{L1} = N_{L2} = 2520$, $N_C = 600$. Then the arc intervals between consecutive vehicles are $\Delta_{sL1} = v_L\Delta_{tL1}$, $\Delta_{sL2} = v_L\Delta_{tL2}$, $\Delta_{sC} = v_C\Delta_{tC}$ (in metres), and angle intervals are $\Delta_{\theta L1} = \Delta_{sL1}/\xi_{L1}$, $\Delta_{\theta L2} = \Delta_{sL2}/\xi_{L2}$, $\Delta_{\theta C} = \Delta_{sC}/\xi_C$ (in radians). From calculations it follows that at the same time, on the roundabout may be 13 vehicles in the inside circle, 15 – in the middle circle and 4 – in the outer circle. It means that transport facilities perform 193, 168 and 150 rotations per hour, respectively.

The structure of total acoustical pressure $p(t) = \mathrm{Re}(p_{\mathrm{tot}})$ (in Pascals), calculated in centre of roundabout $x = y = 0$ at the height of $z = 3$ m, after 41 rotations of all vehicles ($M_{L1} = M_{L2} = M_C = 20$) in windless conditions, is presented in Fig. 1. The number of transport units in the circles for arbitrary time instants is invariable. The signal is subjected to an amplitude modulation as a result of sources symmetry with respect to the roundabout centre and superposition of waves, radiated by sources and reflected from the elastic solid half-space. The signal cardinally changes its structure, if point of observation will be replaced from the centre of roundabout to periphery. This case is shown in Fig. 2. Here, the observation point is located at coordinates $\xi = 40$ m, $\theta = 0°$ (at a distance of 8 m from roundabout in radial direction). The vehicles perform 11 rotations. The influence of wind was not taken into consideration. The small and large amplitudes on this figure correspond, respectively, to noise emitted by automobiles and trucks.

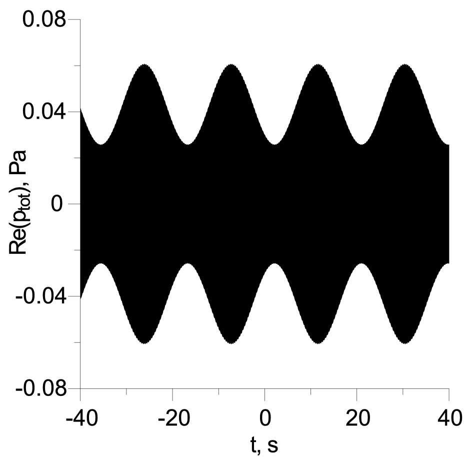


Fig. 1. Acoustical pressure $p(t) = \mathrm{Re}(p_{\mathrm{tot}})$ in the centre of roundabout $\xi = 0$ after 41 rotations of all cars in windless conditions ($v_w = 0$).

[(2)] The observations were done by students of the Technical University of Łódź: Tomasz Adamczak and Łukasz Kurek.

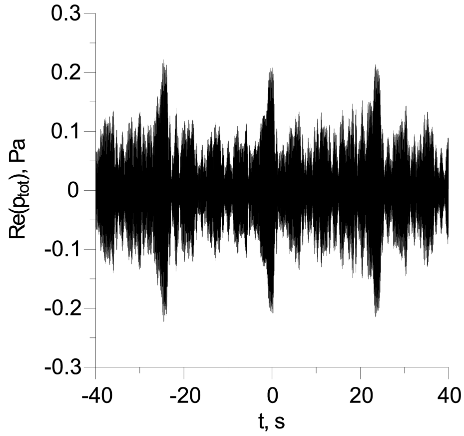


Fig. 2. Acoustical pressure $p(t) = \mathrm{Re}(p_{\mathrm{tot}})$ on distance of $\xi = 40$ m, $\theta = 0°$ from the centre of roundabout after 11 rotations of all cars in windless conditions ($v_w = 0$).

Figures 3 demonstrate the influence of wind velocity ($v_w = 20$ km/h and $v_w = 40$ km/h; $\theta_w = 0°$) on the structure of acoustical pressure, calculated in the same point. It is shown that with increasing wind velocity signal "wash upon" in time, increasing thus amplitude of acoustical pressure generated by automobiles.

a) b)

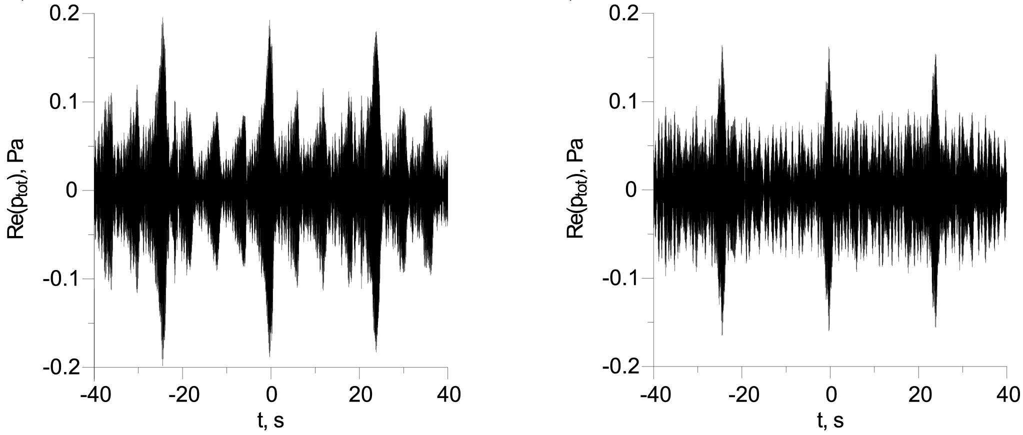


Fig. 3. Acoustical pressure $p(t) = \mathrm{Re}(p_{\mathrm{tot}})$ on distance of $\xi = 40$ m, $\theta = 0°$ from the centre of roundabout after 11 rotations of all cars in windy conditions ($\theta_w = 0°$): a) $v_w = 20$ km/h; b) $v_w = 40$ km/h.

Figure 4 illustrates the distribution of instantaneous sound intensity $I(x, y) = 20 \log[|p_{\mathrm{tot}}(\mathbf{x}, t)/p_0|]$ (in decibels) at time $t = 10$ s and at the height of $z = 3$ m with $v_w = 40$ km/h, $\theta_w = 45°$, where $p_0 = 2 \cdot 10^{-5}$ Pa is the threshold pressure. The radiation is characterized by a high level of acoustical pressure (near 80 dB) on the roundabout, where noise concentration is displayed as a ring, and by insignificant decreasing to 60–70 dB at a far distance from the roundabout. More details of numerical calculations for different values of wind velocity and direction are presented in Figs. 5 for the same space intensity distribution, in the form of isolines (isophones) for moment $t = 10$ s and at the height of $z = 3$ m. In windless conditions (Fig. 5a), the isolines are non-symmetrical with respect to roundabout centre, because at this moment the space symmetry of sources localization is absent. The introduction of wind parameters causes considerable changes of this distribution on the roundabout periphery.

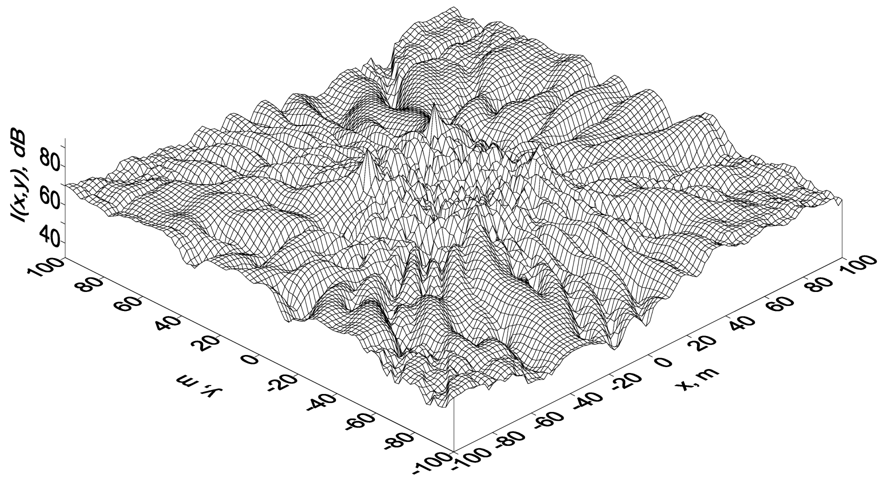
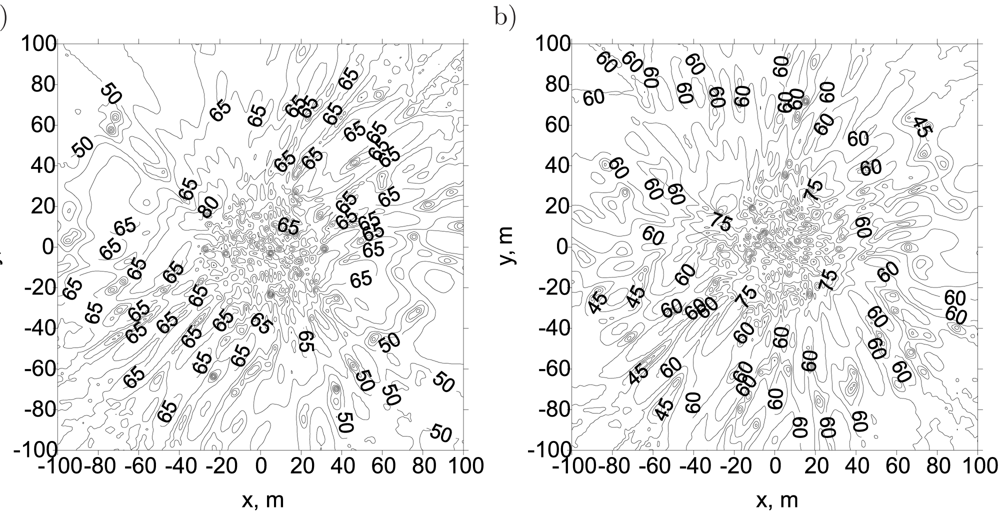


Fig. 4. Space distribution of instantaneous sound intensity $I(x, y)$ in moment $t = 10$ s at the height of $z = 3$ m in windy conditions ($v_w = 40$ km/h, $\theta_w = 45°$).

a) b)



[Fig. 5 a, b]

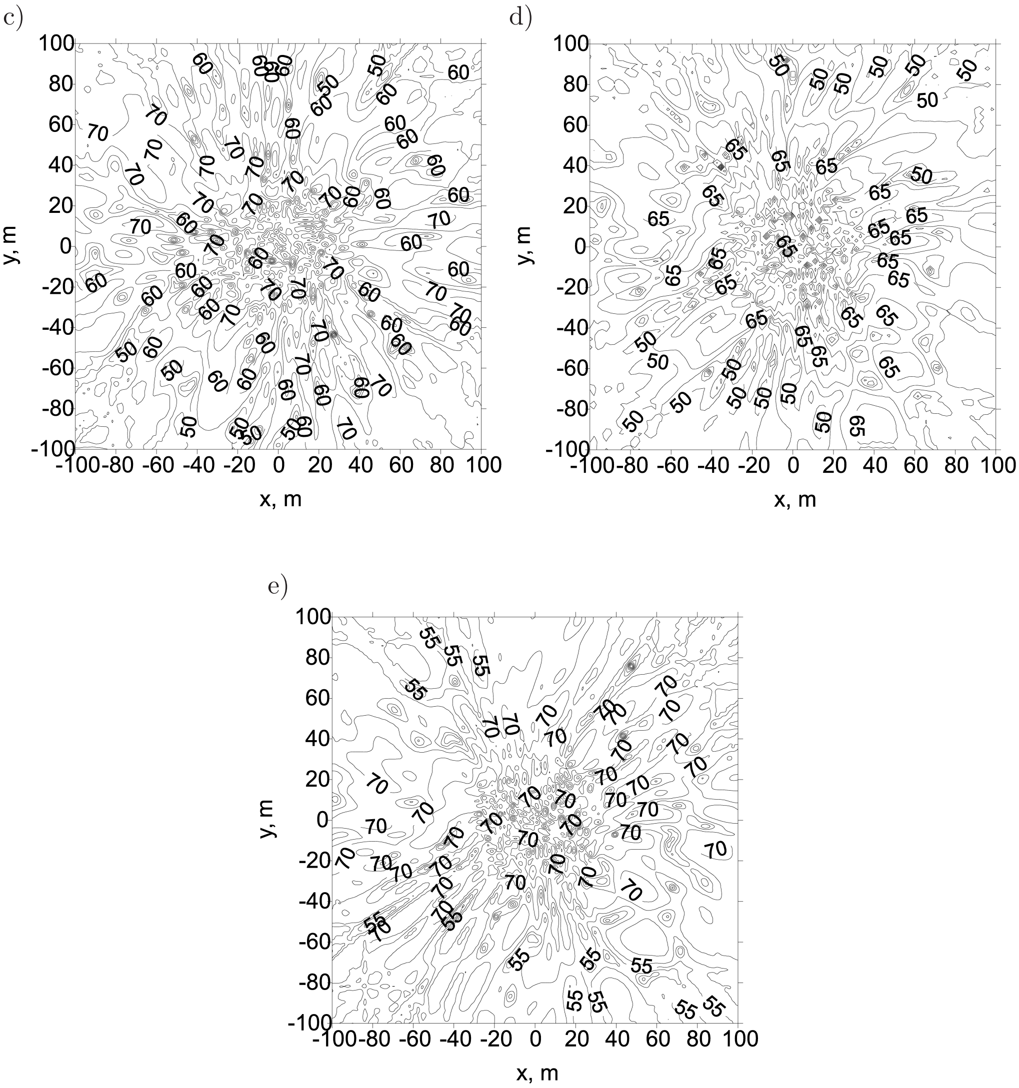


Fig. 5. Isolines of sound intensity $I(x, y)$ (in dB) in moment $t = 10$ s: a) $v_w = 0$, $\theta_w = 0°$; b) $v_w = 20$ km/h, $\theta_w = 0°$; c) $v_w = 40$ km/h, $\theta_w = 0°$; d) $v_w = 20$ km/h, $\theta_w = 90°$; e) $v_w = 20$ km/h, $\theta_w = 180°$.

Figure 6 presents the space distribution of acoustical pressure level

$$I_{\mathrm{av}}(\mathbf{x}) = 10 \log \left\{ \frac{1}{T} \int_0^T \left[\frac{|p_{\mathrm{tot}}(\mathbf{x}, t)|}{p_0} \right]^2 \mathrm{d}t \right\} \tag{70}$$

averaged over time $T = 25$ s in plane $0xy$ at the height of $z = 3$ m in windless conditions (in windy conditions, the figures are similar). Figures 7 concern the same characteristic in isophones both as in windless state (Fig. 7a) and in windy situation (Fig. 7b), when acoustic medium moves in direction $\theta_w = 45°$ to the axis $0x$ with velocity $v_w = 40$ km/h. It is shown that the great concentration of noise level occurs on roundabout during a short time-period. The wind helps some "erosion" of this concentration on side of the road.

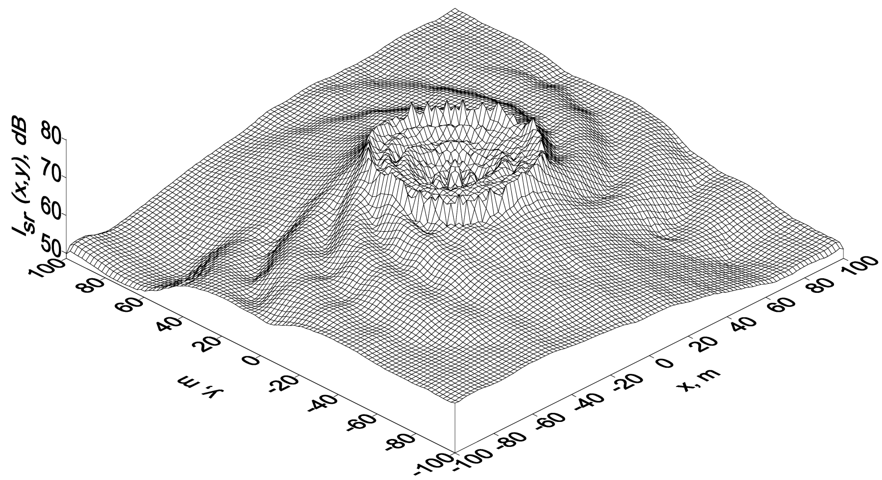


Fig. 6. Space distribution of average sound intensity $I_{\mathrm{av}}(x, y)$ at the height of $z = 3$ m for period $T = 25$ s in windless outer conditions.

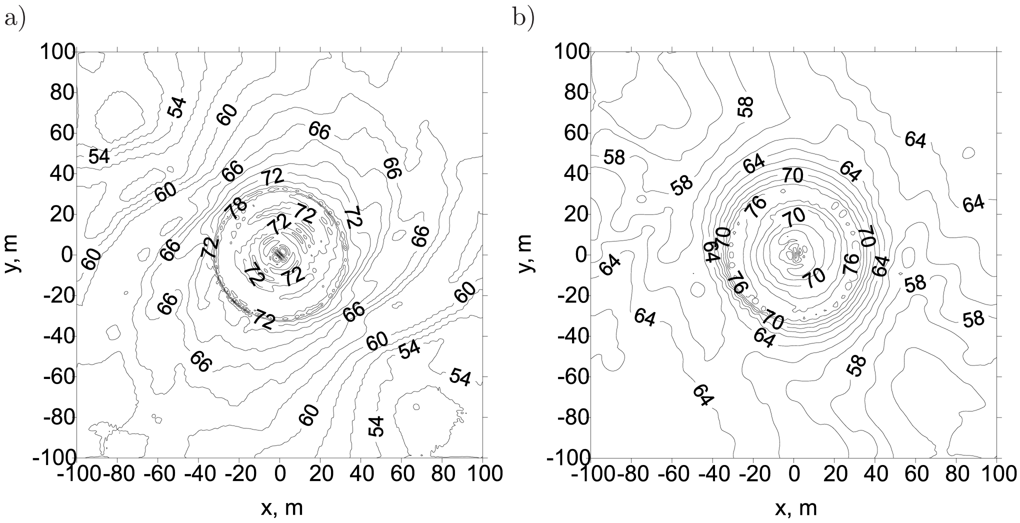


Fig. 7. Isolines of average sound intensity $I_{\mathrm{av}}(x, y)$ (in dB) at the height of $z = 3$ m for period $T = 25$ s: a) motion in windless conditions; b) motion in windy conditions ($v_w = 40$ km/h, $\theta_w = 45°$).

4. Conclusions

The mathematical model of noise radiated from the city uncontrolled roundabout, with taking into account a wind in general case, is proposed in this paper. The solution of problem is obtained as a superposition of a discrete number of point sources moving with constant velocities in closed concentric circles.

The numerical analysis of noise action in the vicinity of roundabout shows that:

1. The acoustical signal structure of a group of vehicles moving on this element of road is very inhomogeneous; the exception is the roundabout enter, where acoustical signal is more or less well-regulated; the signals, registered near the road-side, have the form of quasi-regular impulses with average amplitudes approximately equal to 0.1 Pa for automobiles and 0.2 Pa for trucks; the wind more sufficiently influences the inner structure of acoustical pressure, than its amplitude.
2. The noise, as a function of distance from the roundabout center in radial direction, is characterized by considerably increasing values of sound intensity above way-roads (to 80 dB) and by gradual general decreasing of its level with distance from the roundabout; e.g. at the distance of three roundabout radii, the noise intensity decreases only by 10–15 dB.
3. The numerical calculations show that influence of wind velocity and direction is a very appreciable quality and quantity on sound intensity characteristics both as instantaneous and averaged for some period; in particular, in windless conditions, the average sound intensity is most concentrated near the roundabout; the sharp wind changes space structure of these characteristics, decreasing its level in direction of blowing and increasing in the opposite direction.
4. This study also permits to affirm that the roundabout as element of roads may be treated as an acoustic antenna of ring-rotational type, radiating high-intensive sound.

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